Generalized Timed Büchi Automata (Revisited)

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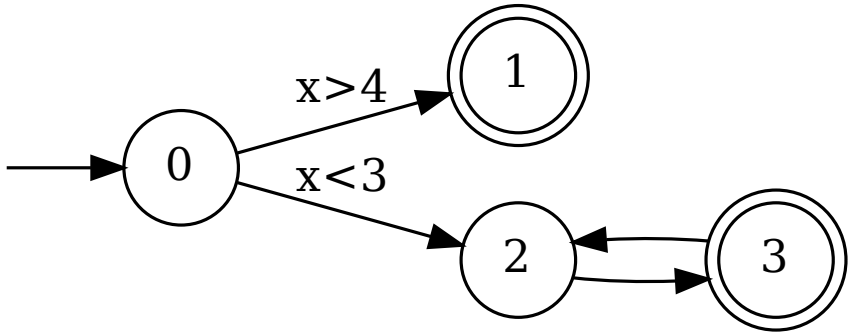
Timed Büchi Automata

Timed Büchi Automata are, like Timed Automata, defined as a tuple

$$\mathcal{A} = (L, l_0, L_F, X, E, \mathrm{Inv})$$

however a run $r = ((l_0, v_0), (l_1, v_1), (l_2, v_2), ...)$ has to visit L_F infinitely often in order to be accepting.



Rejected : $r_{rej} = ((0, 0), (1, 5))$

Accepted : $r_{acc} = ((0, 0), ((2, 2), (3, 2))^{\omega})$

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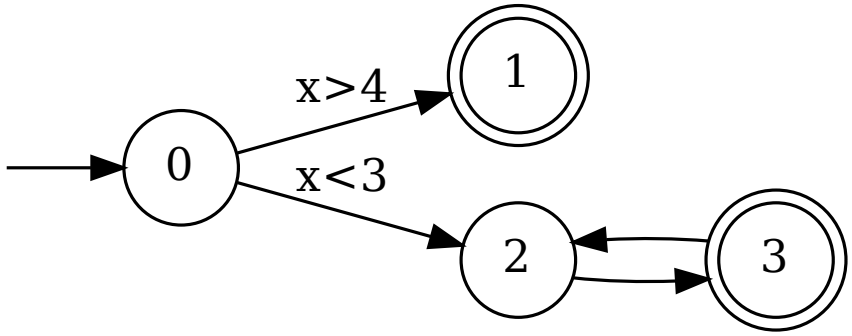

Bonus question : Guarantee that time is allowed to diverge

Accepted : $r_{acc} = ((0,0), (2,2), (3,2), (2,3), (3,3), (2,4), (3,4), ...)$

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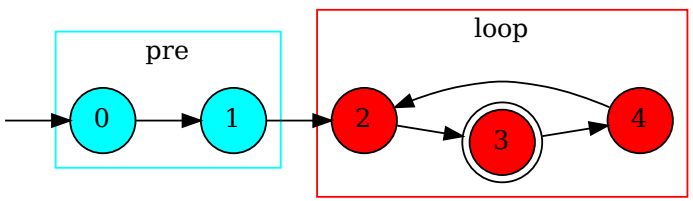

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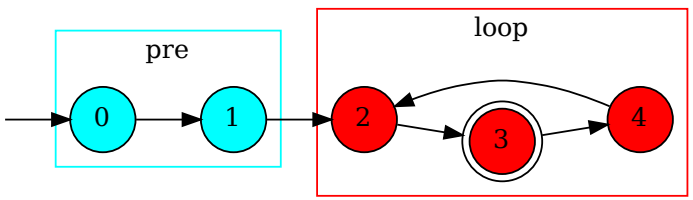

This can be done using a Nested Dept First Search (NDFS).

NDFS

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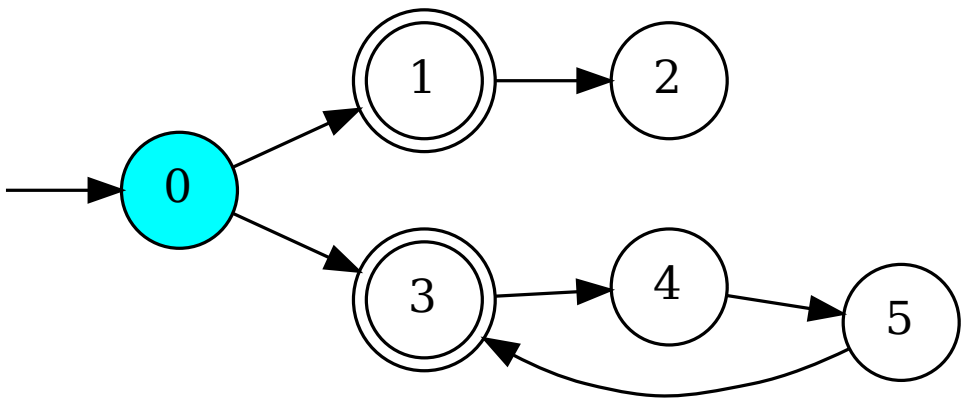
- One to find a prefix ("$dfsBlue$") and one to find a cycle ("$dfsBlue$").
- Needs to maintain three different sets of states :
  Live, Explored, Dead

NDFS

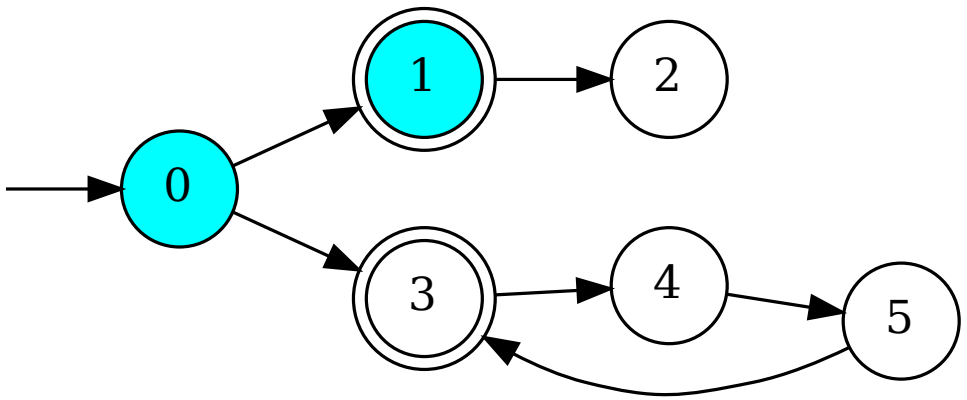


Live=$Cyan$, Explored=$Blue$, Dead=Red

$$Cyan = Blue = Red = \varnothing$$

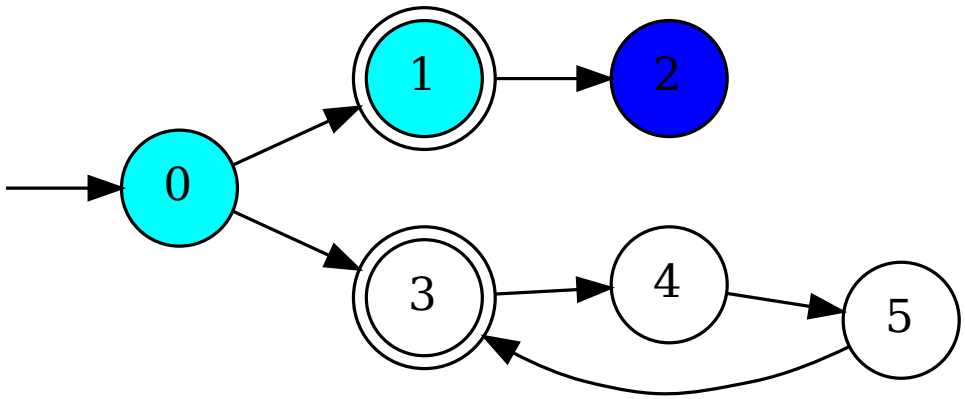
$$dfsBlue(0)$$

NDFS



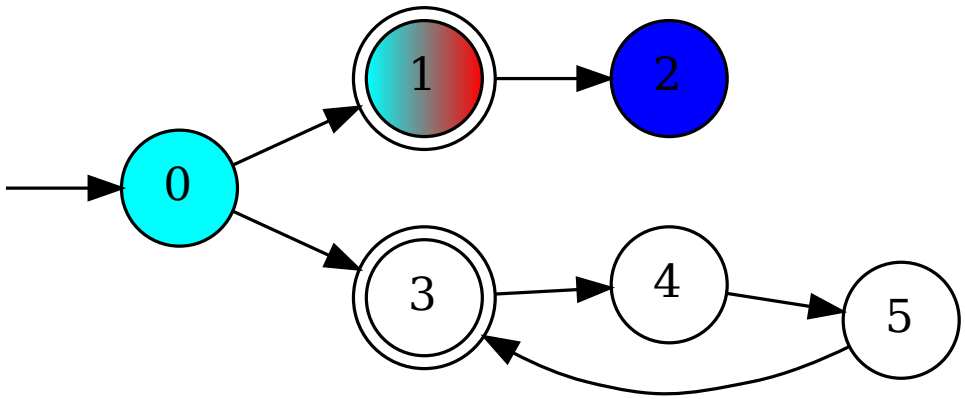
$$succ \notin Blue \wedge succ \notin Cyan \rightarrow dfsBlue(succ)$$

NDFS



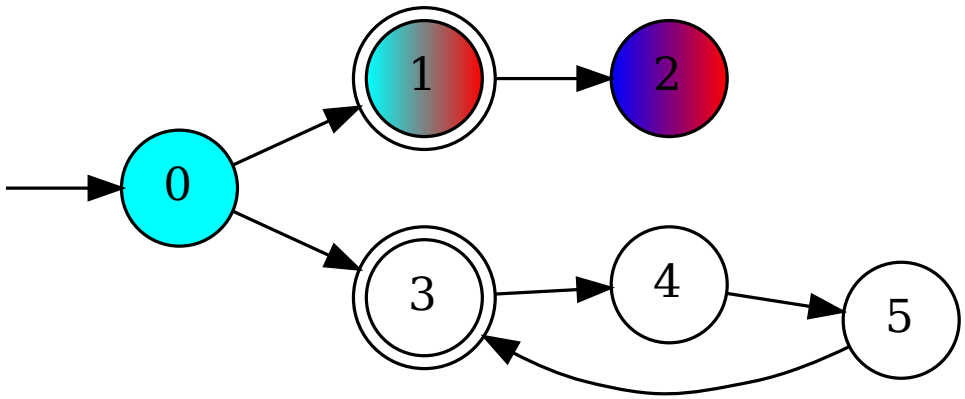
$$s \notin L_F \rightarrow Blue = Blue \cup s; Cyan = Cyan \setminus s$$

NDFS



$$s \in L_F \rightarrow dfsRed(s)$$

NDFS



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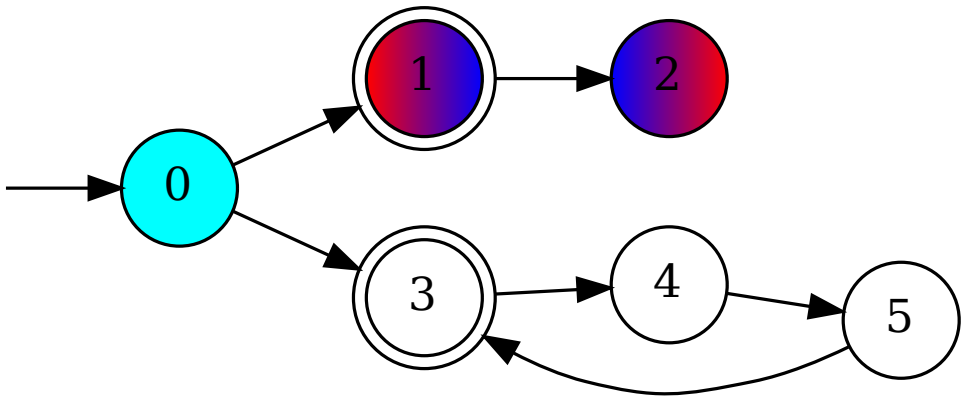

s is explored $\rightarrow Blue = Blue \cup s; Cyan = Cyan \setminus s$

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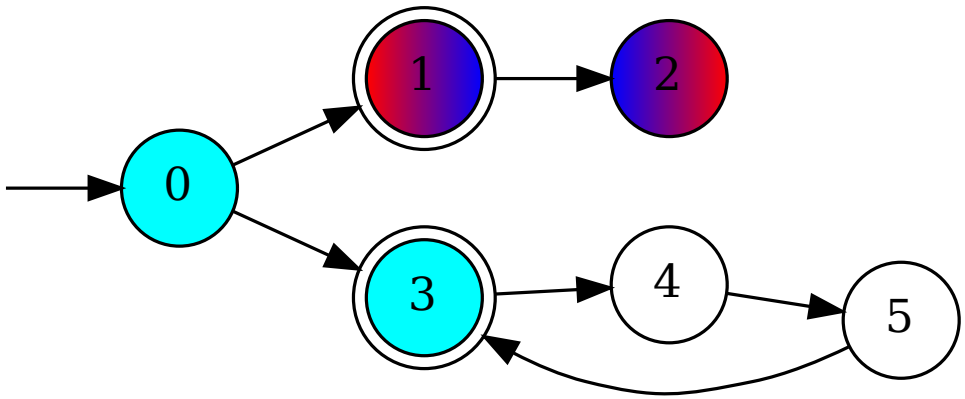

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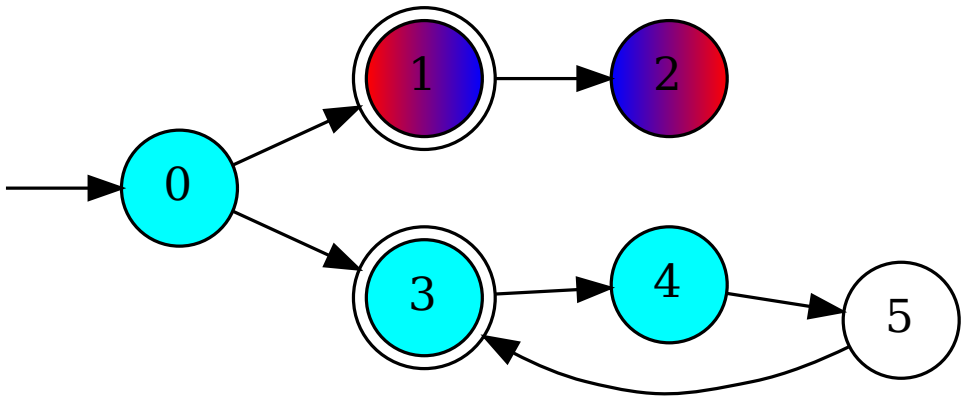

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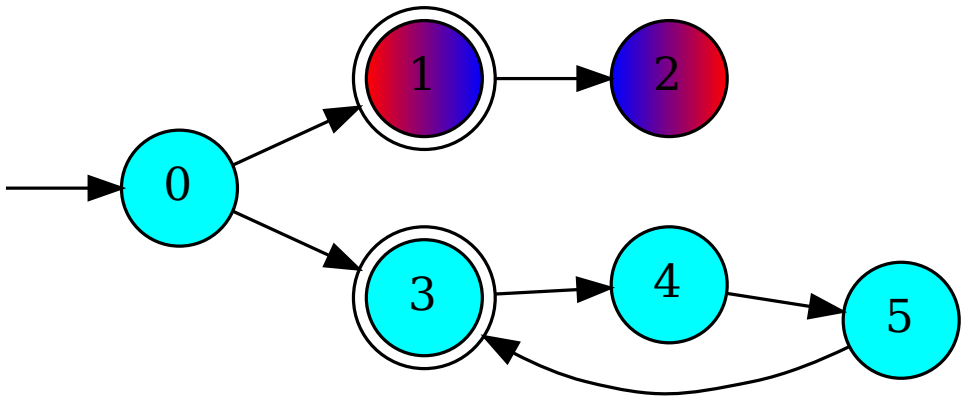

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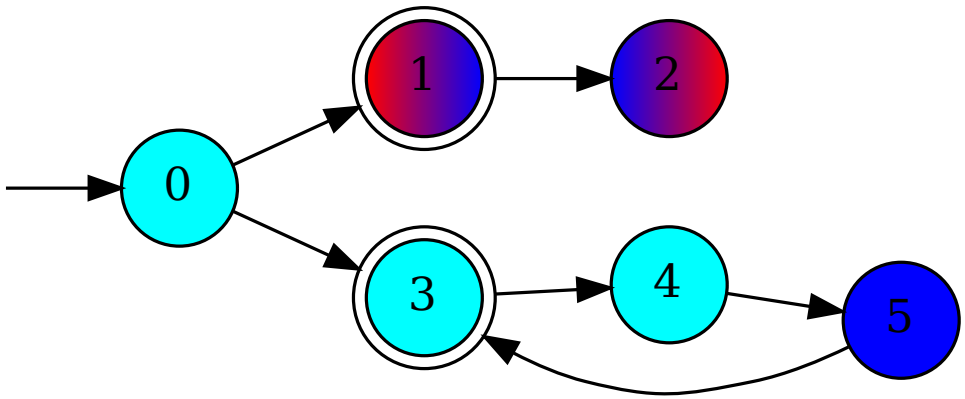

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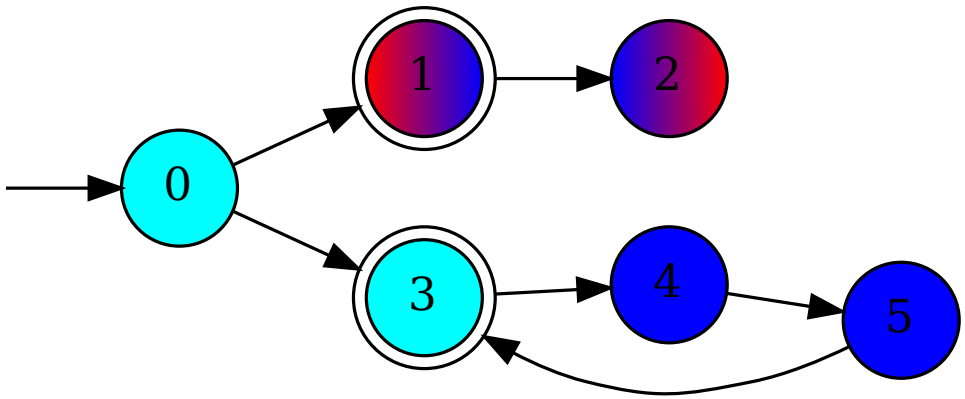

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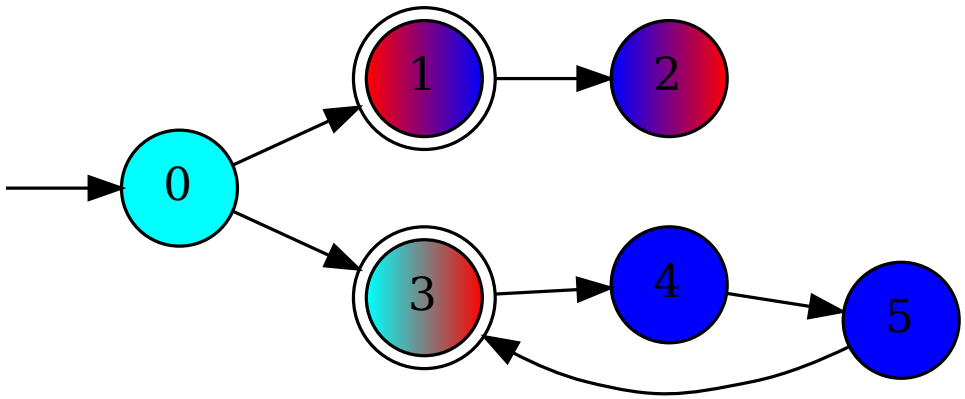
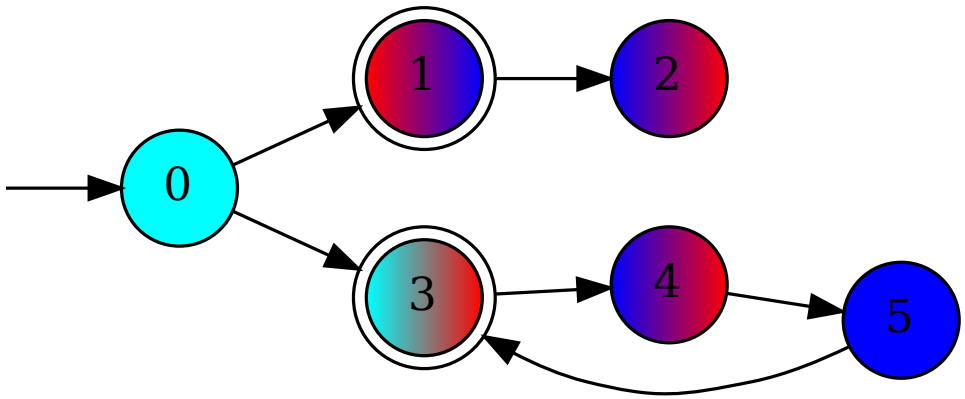


$$s \in L_F \rightarrow dfsRed(succ)$$

NDFS



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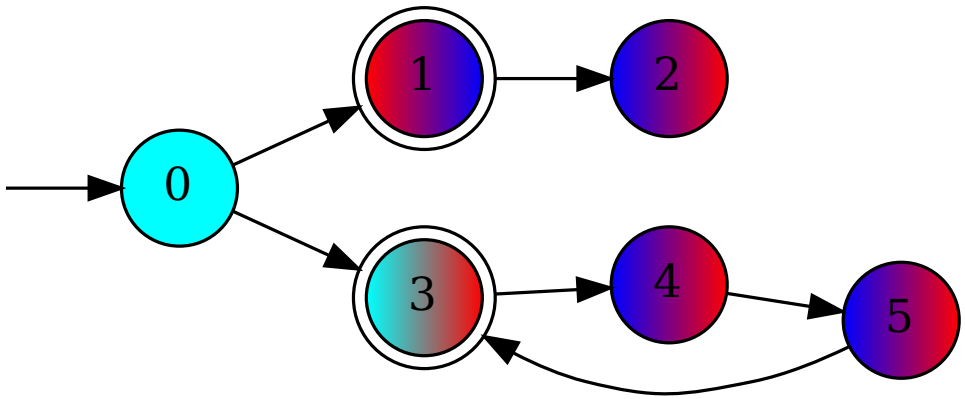

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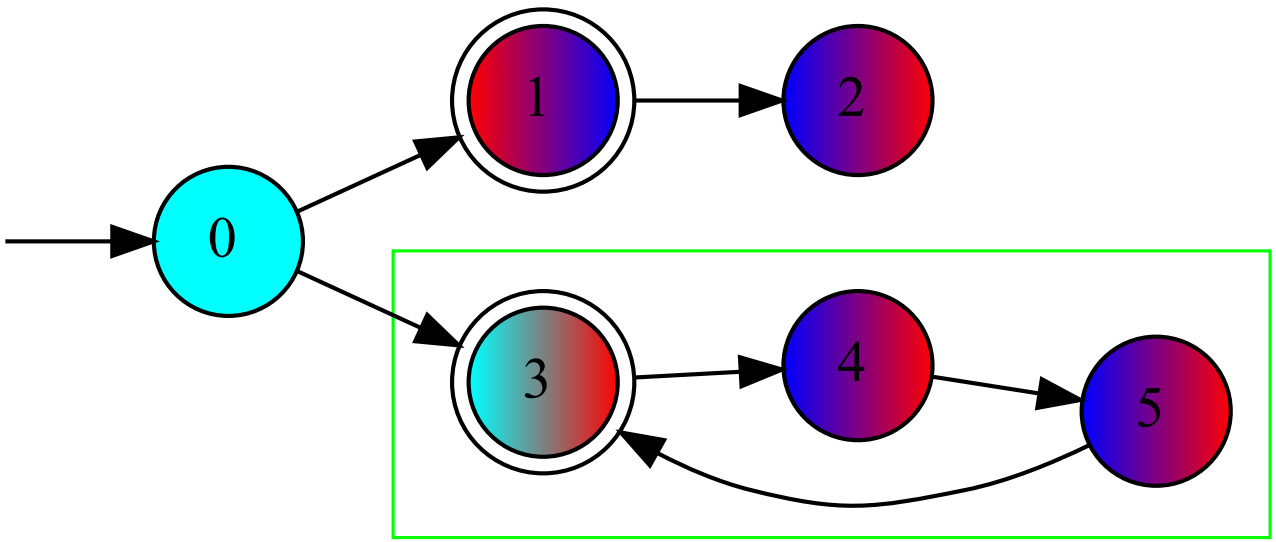

$succ \in Cyan \rightarrow$ report cycle

Couvreur

Another popular approach for checking Büchi emptiness is Couvreur's algorithm [1].

It's main idea is to find an SCC that satisfies the acceptance condition.

1. Couvreur, J. M., Duret-Lutz, A., & Poitrenaud, D. (2005, August). On-the-fly emptiness checks for generalized Büchi automata. In International SPIN Workshop on Model Checking of Software (pp. 169-184). Springer, Berlin, Heidelberg.

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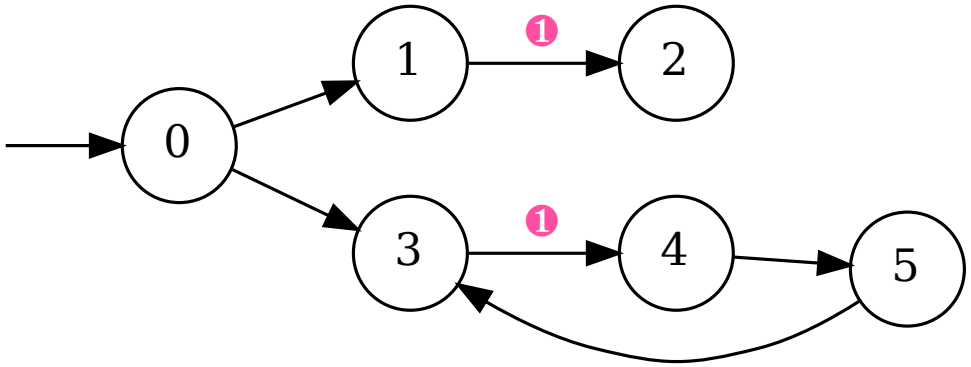
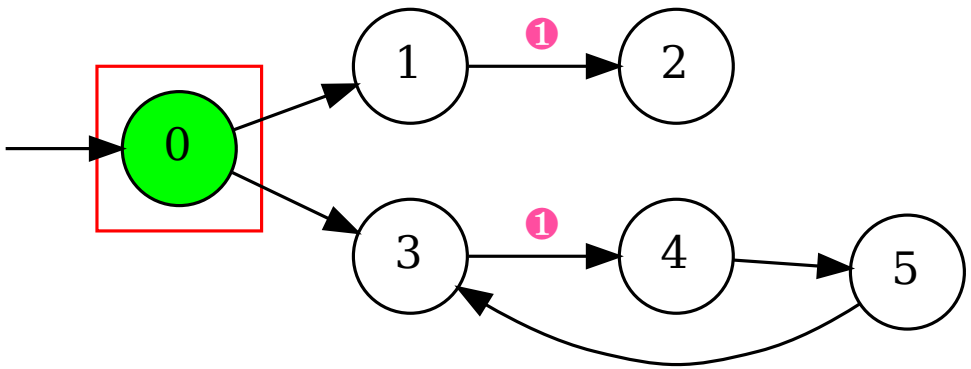


To be coherent with SPOT,
we will use a transition based acceptance here.

Couvreur



When discovering a new state, it gets its own scc and is put on the live stack $Green$

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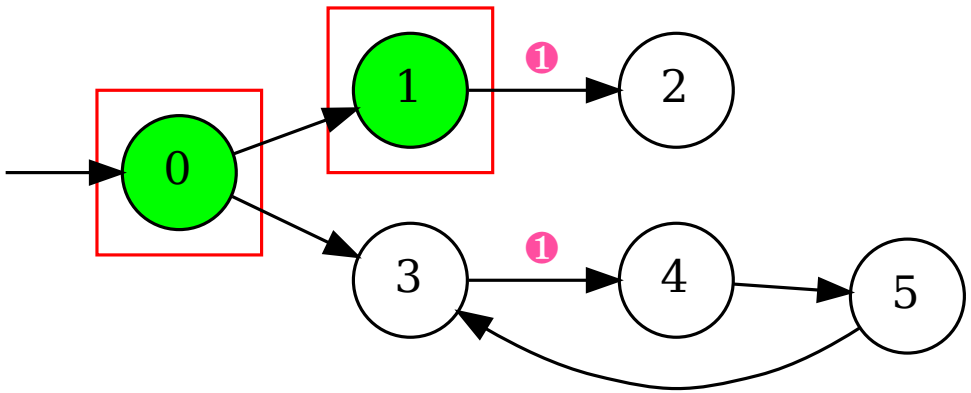

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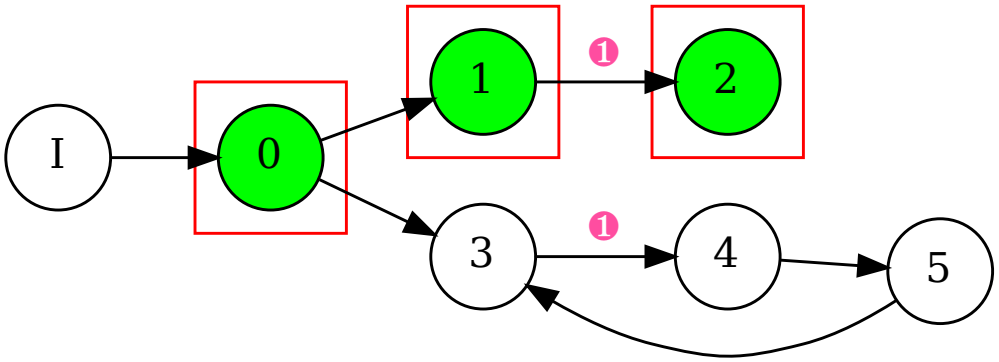

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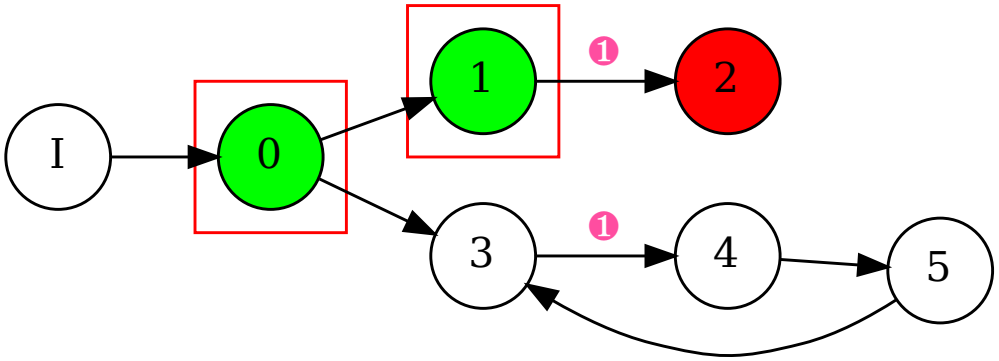

When backtracking the root of an scc,
all of the states are "declared dead" and added to Red.

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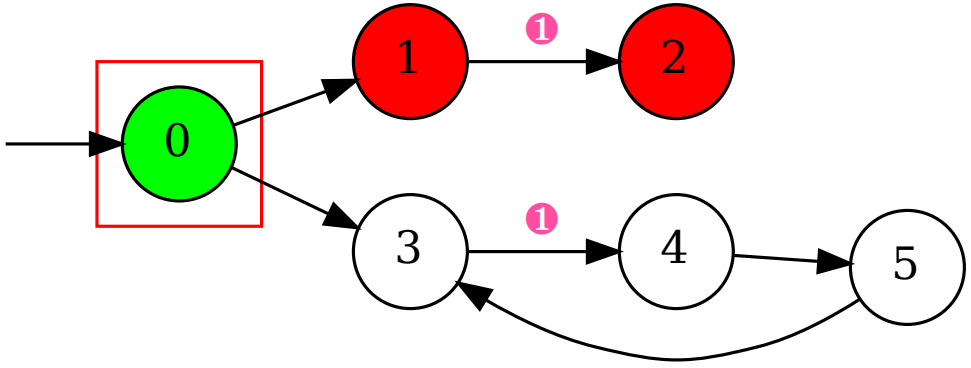

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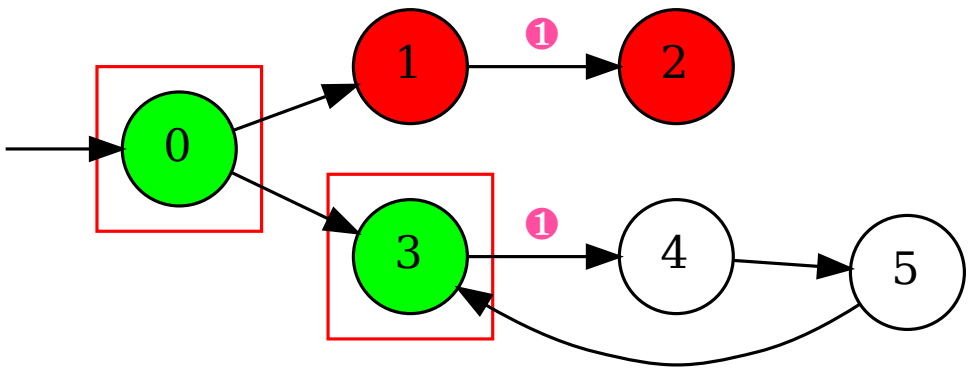
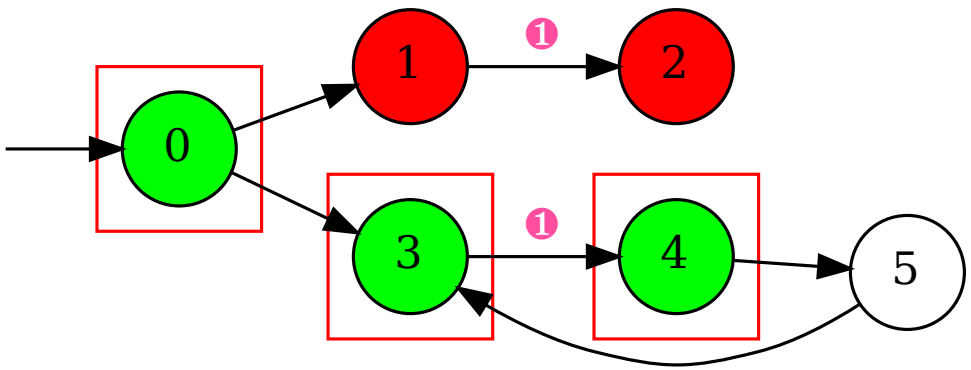


We keep on exploring.

Couvreur



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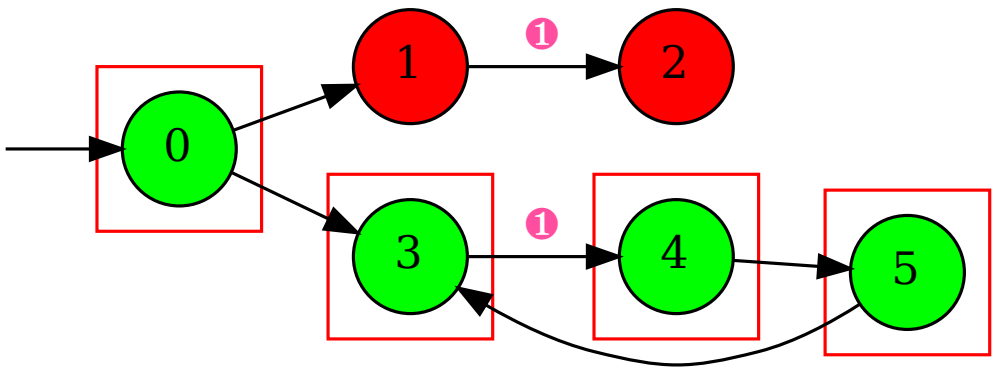

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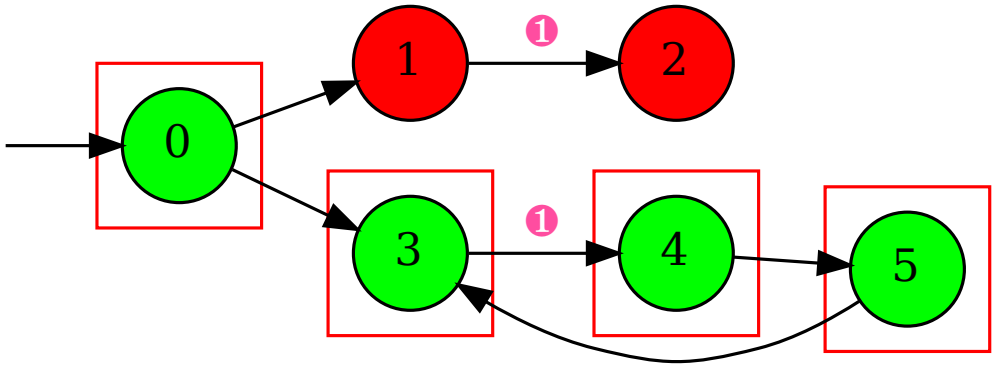
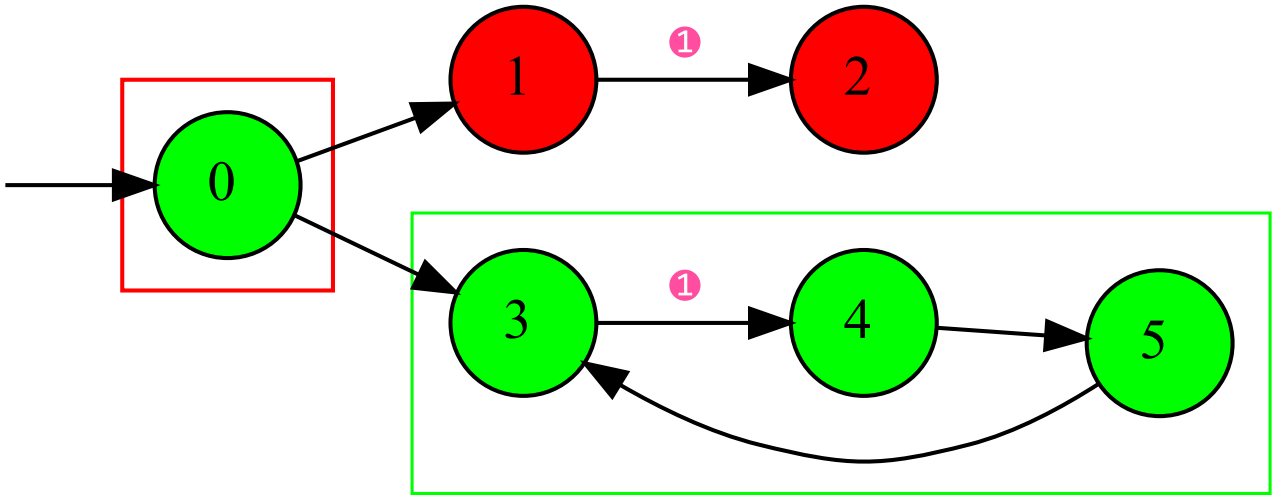


Until we find a back-edge. That is an edge which take us back to a live state (one that is in $Green$)

Couvreur



All scc's between them get merged.
The acceptanace marks are accumulated.

Using subsumption

- Both algorithms handle nodes like discrete states
- Using subsumption where appropriate
  is likely to speed-up calculations

Using subsumption

NDFS

Couvreur

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Dead subsumption :
$s \in Red$
becomes
$\exists s' \in Red\colon\ s \preceq s'$

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$Red \rightarrow Cyan$ subsumption :
$\exists s' \in Cyan\colon s' \preceq s$ report cycle

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Livestack subsumption :
All live states for which
$s' \in Green,\ s' \preceq succ$ holds are valid successors as well

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$Red \to Cyan$ subsumption :
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Equality for $Blue$ and $Cyan$:
Subsumption cannot be used on
$succ \in Blue \cup Cyan$.

Couvreur

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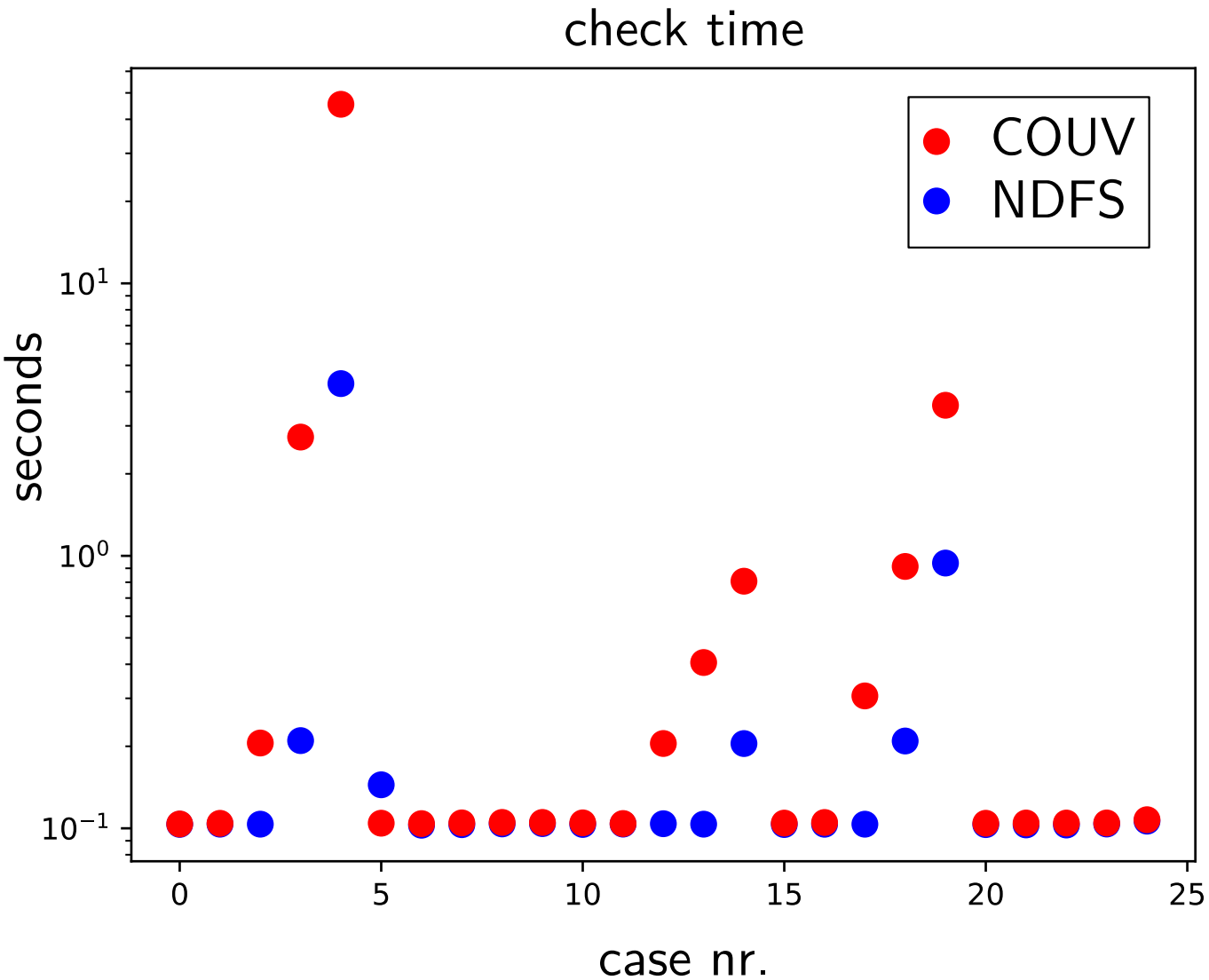
Compairison

Positive and negative aspects

+ Faster : only requires one traversal
+ Faster : usually less compairisons
+ Easier : Only two sets of states
+ Requires less memory
+ Directly applicable to generalized Büchi acceptance
− Provides no actual counterexample

1. Laarman, A., Olesen, M. C., Dalsgaard, A. E., Larsen, K. G., & Van De Pol, J. (2013, July). Multi-core emptiness checking of timed Büchi automata using inclusion abstraction. In International Conference on Computer Aided Verification (pp. 968-983). Springer, Berlin, Heidelberg.

Benchmarks 1



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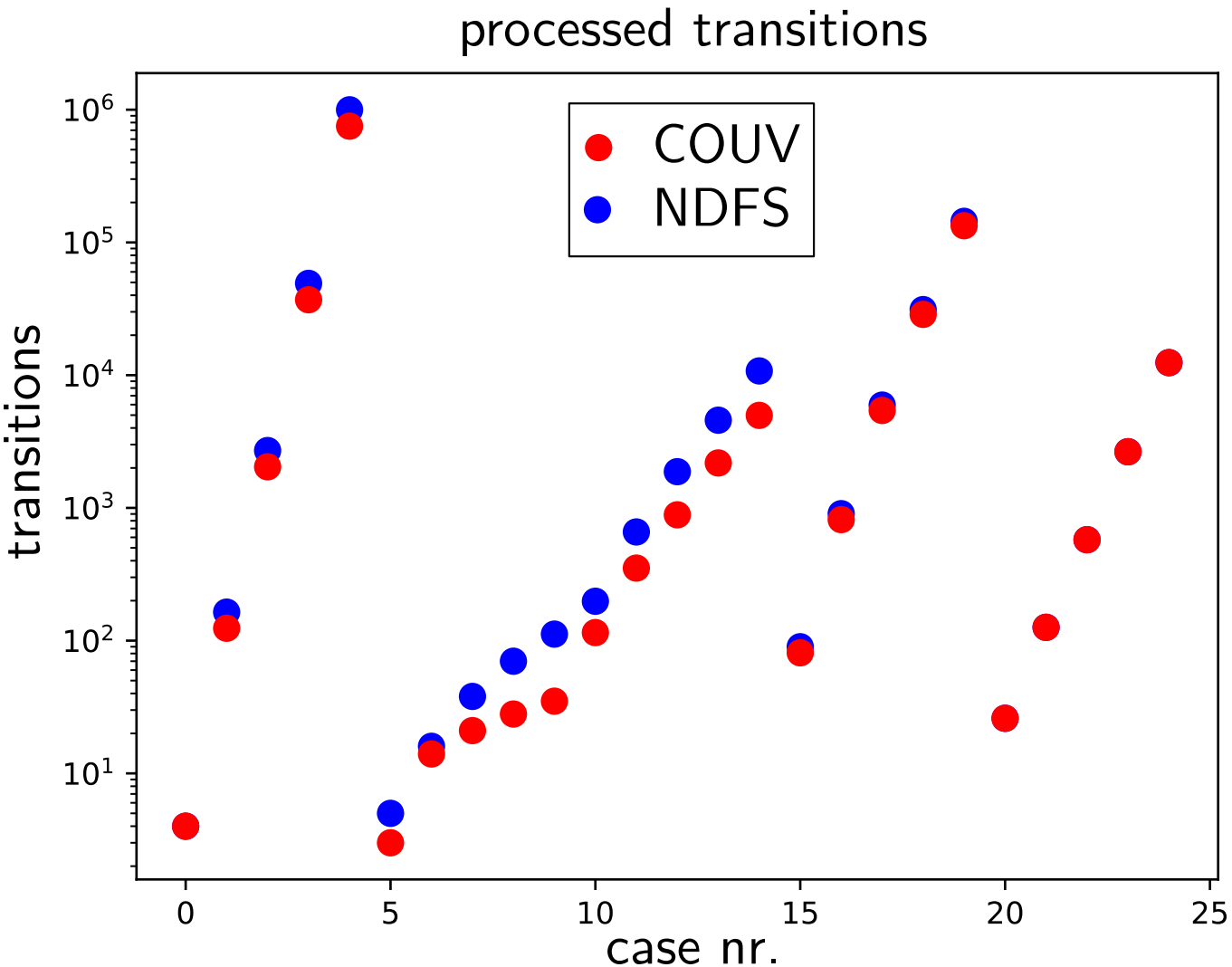

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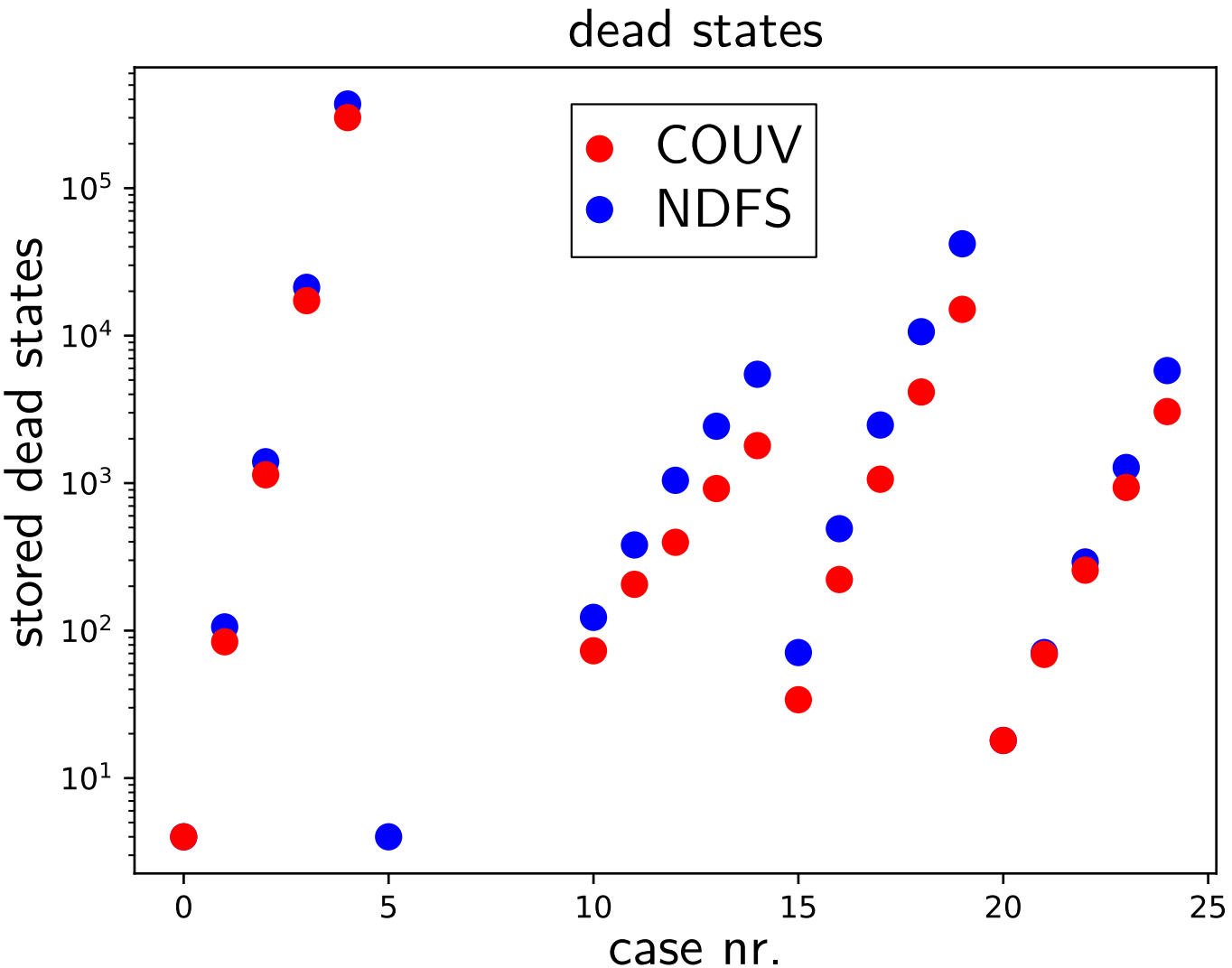
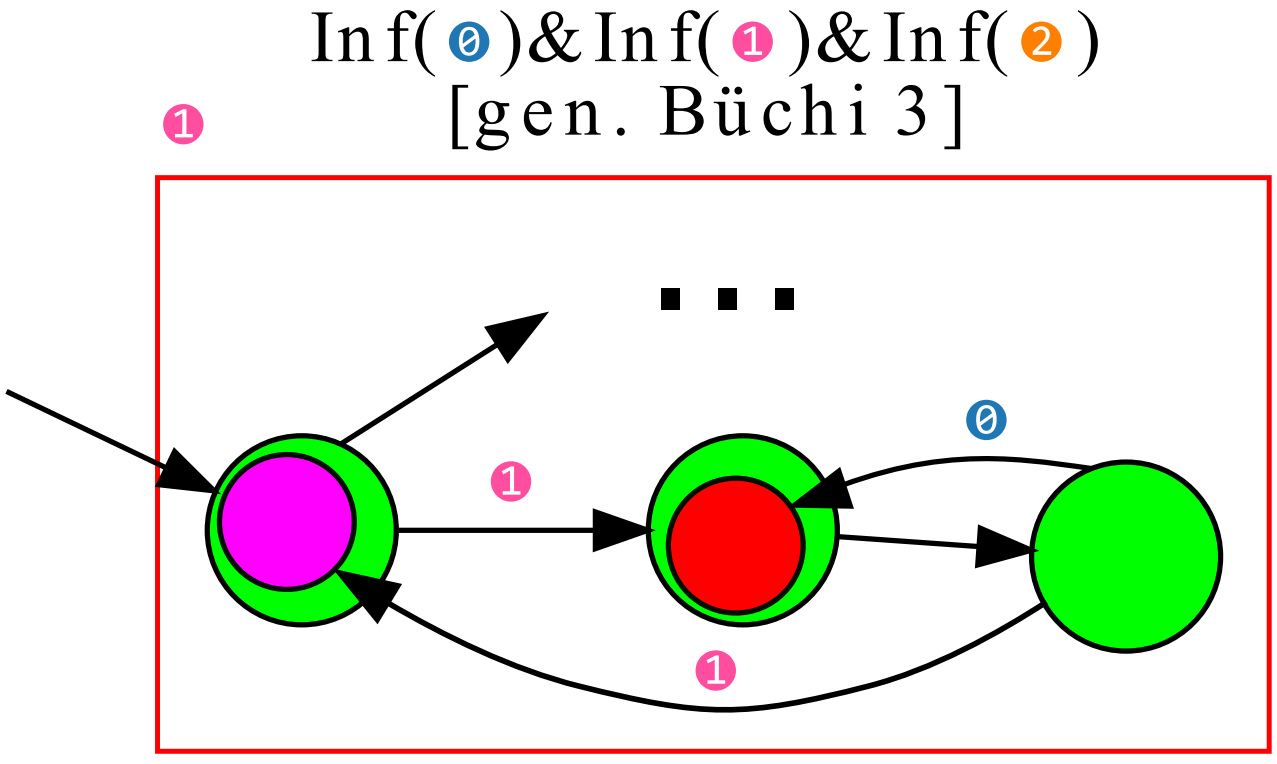


Further optimizations 1



Pruning based on last SCC

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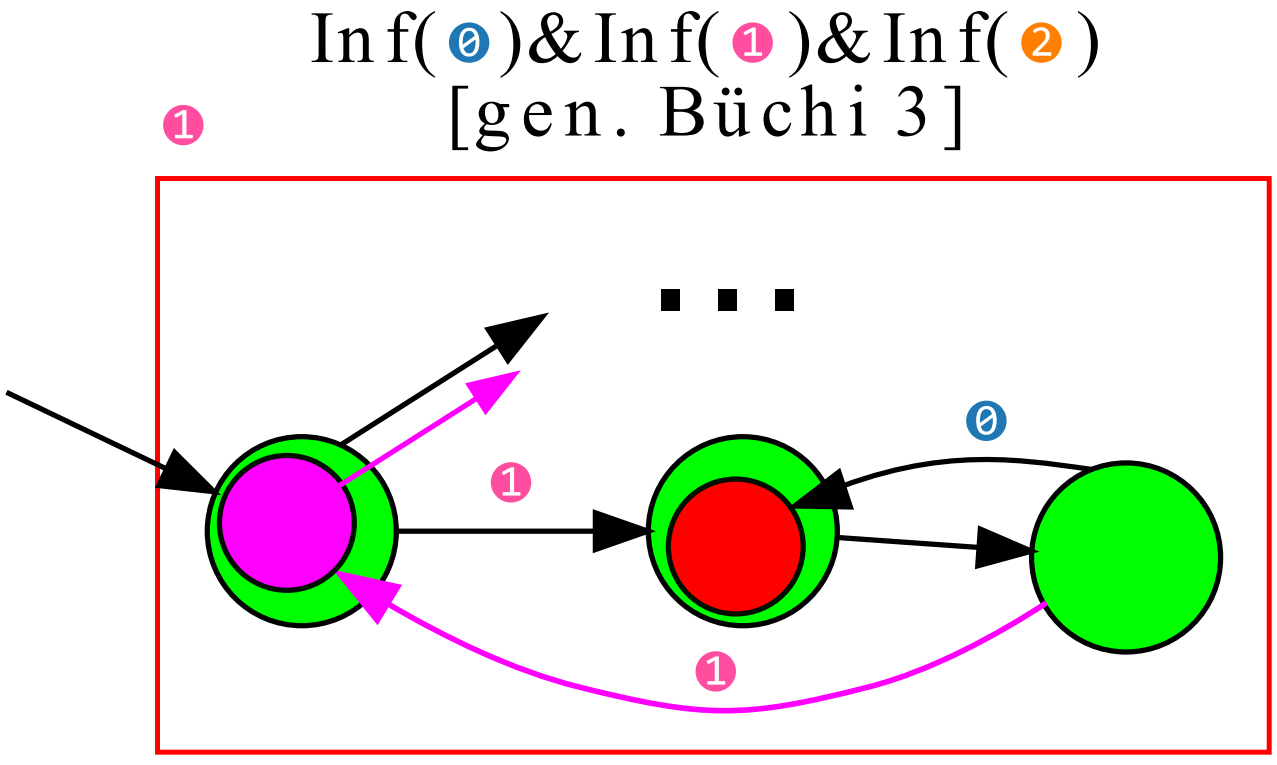

Pruning based on last SCC

We can skip the successor if covered by some node in the scc and the transition has no new color.

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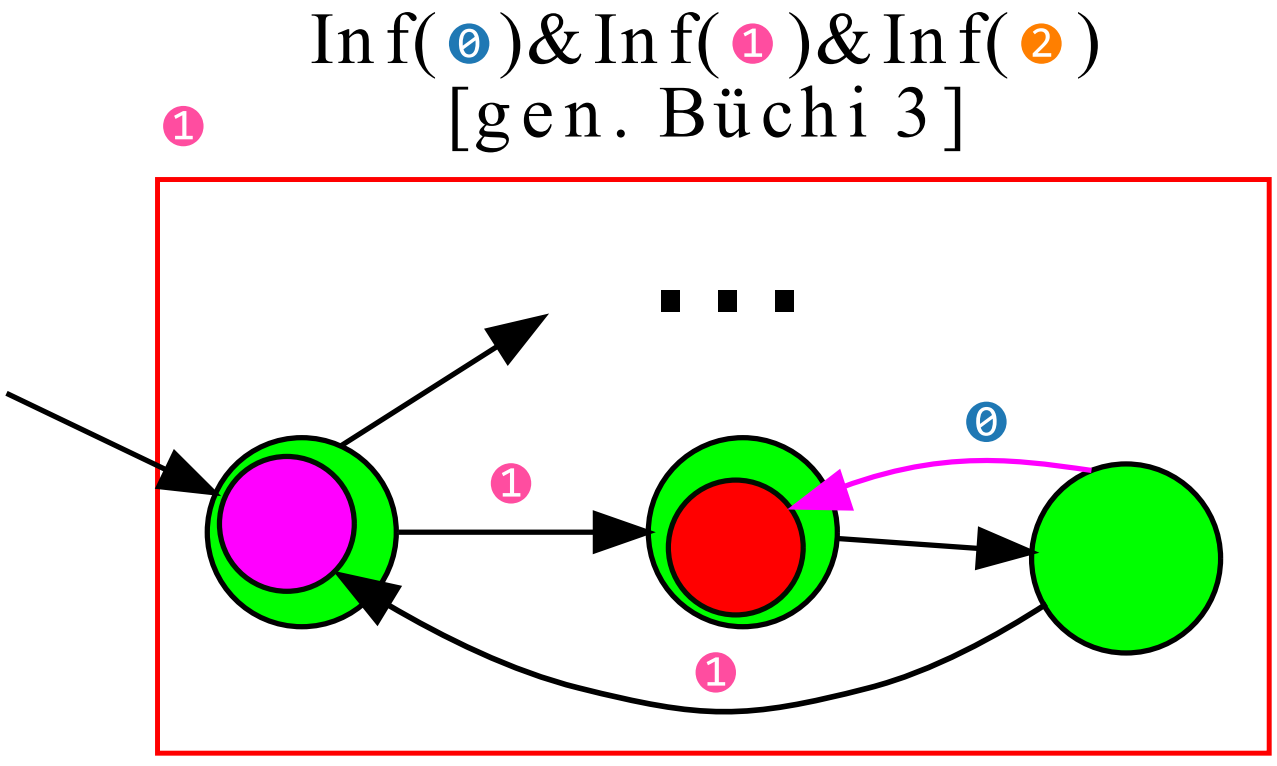

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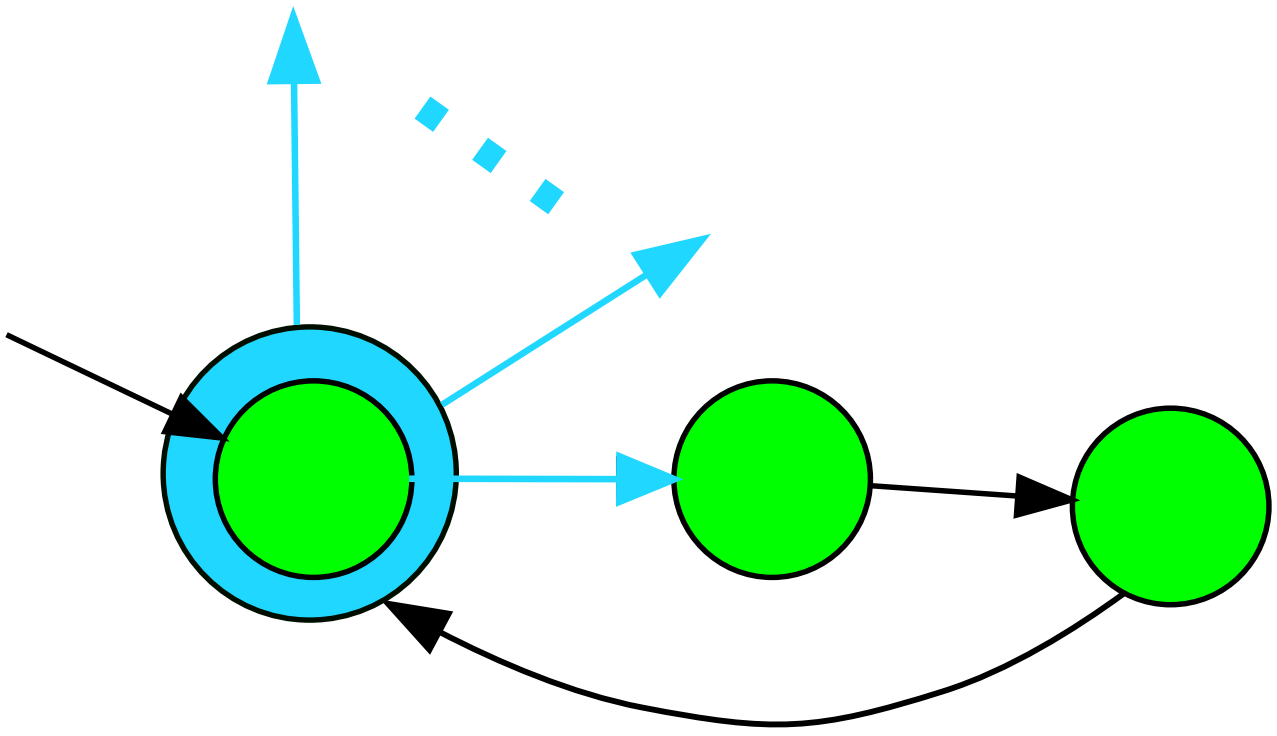
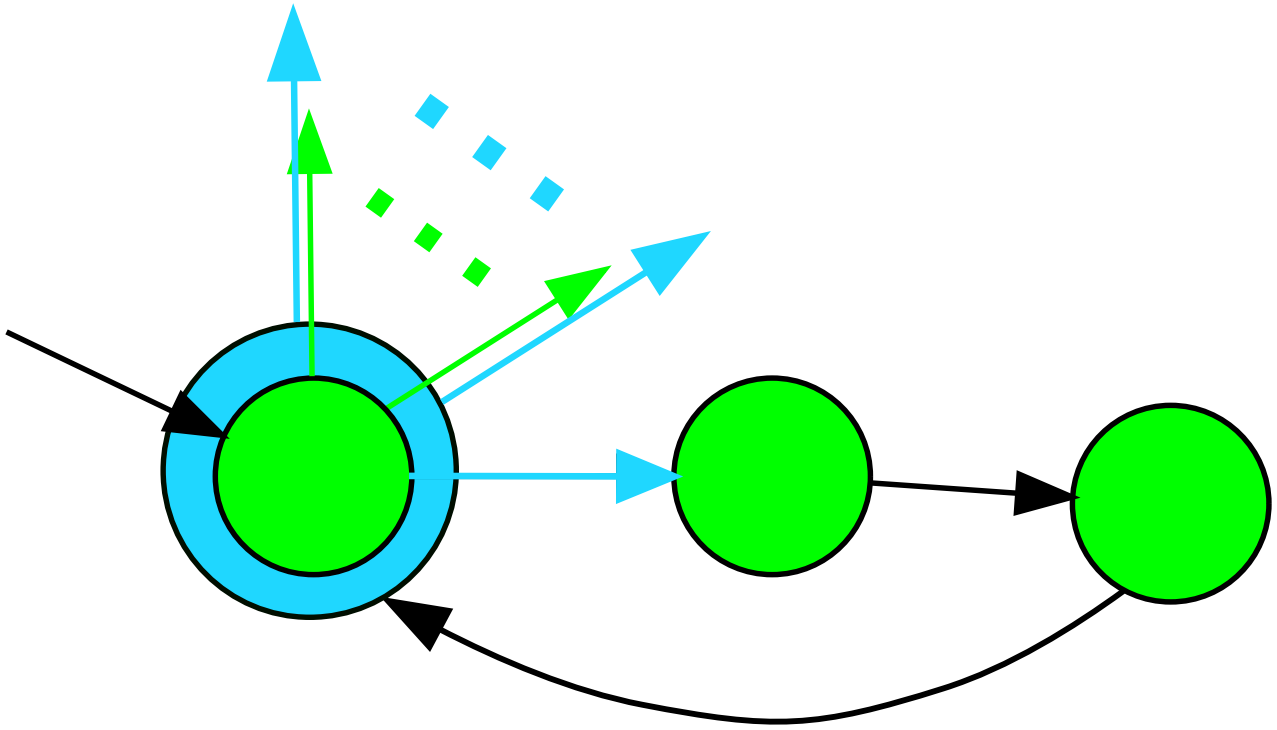
Further optimizations 2



Superseded nodes

Further optimizations 2



Superseded nodes

We can skip the successors of a node if a larger node was already explored.

Further optimizations 3

The set of dead states Red (NDFS/Couvreur) are typically implemented as hash maps.
This means we have to check the states against a sub-set sharing the same hash value.

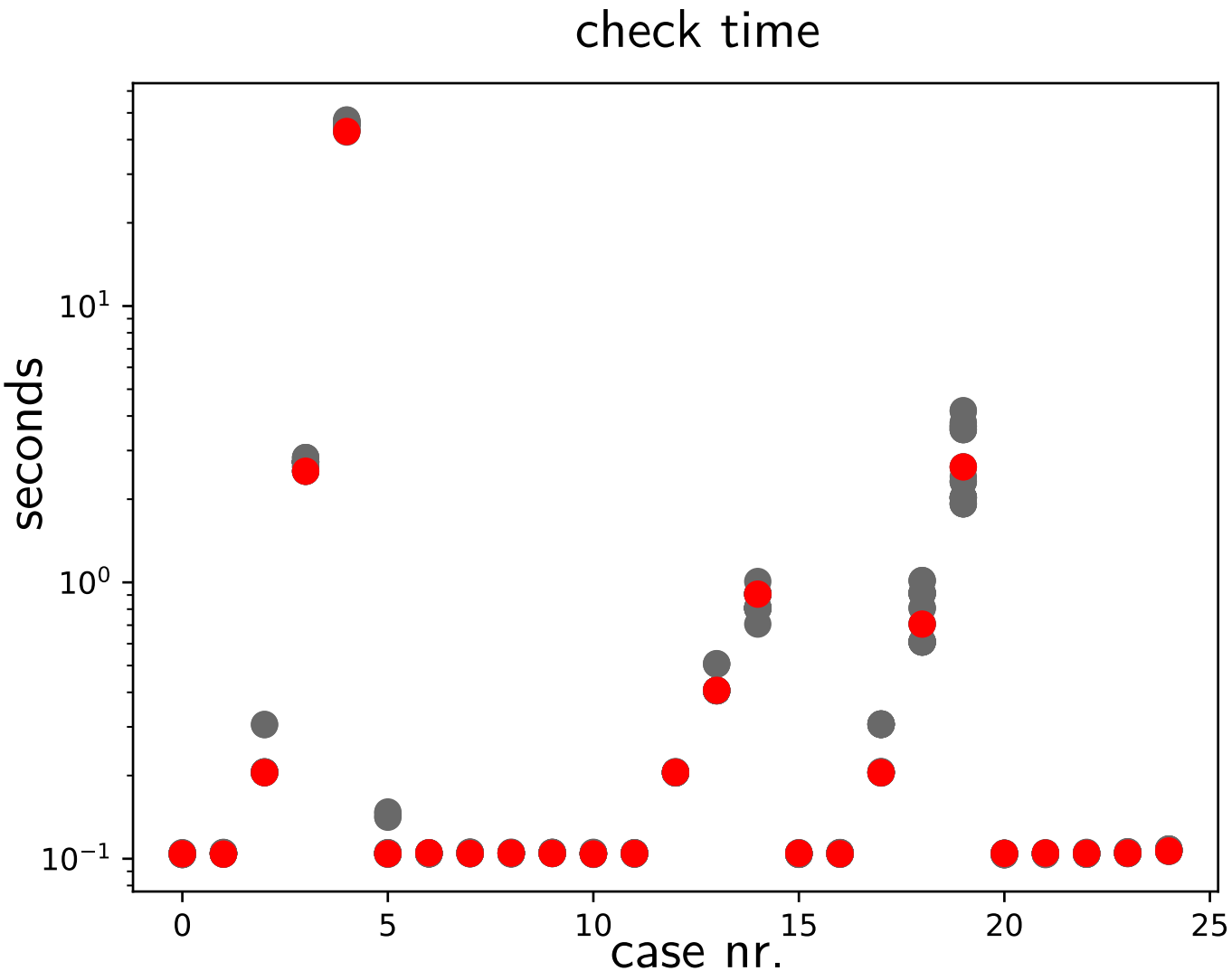
In order to decrease the (average) number of comparisons, we sort the set of candidate states.

$$\left[s_0 \quad s_1 \quad s_2 \quad s_3 \quad s_4 \right]$$

$$\downarrow \quad succ \preceq s_2$$

$$\left[s_2 \quad s_0 \quad s_1 \quad s_3 \quad s_4 \right]$$

Benchmarks 1 - Optimization impact



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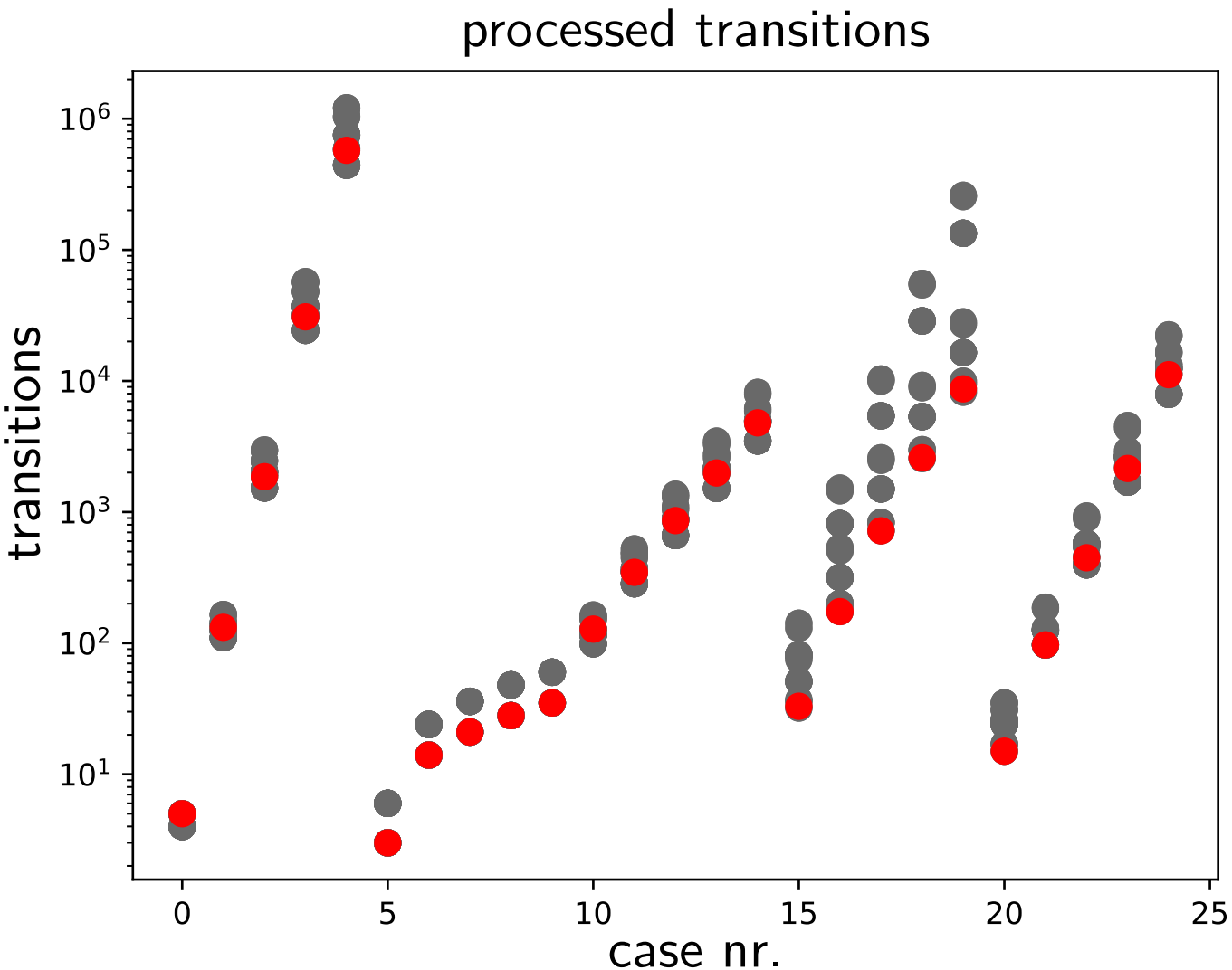

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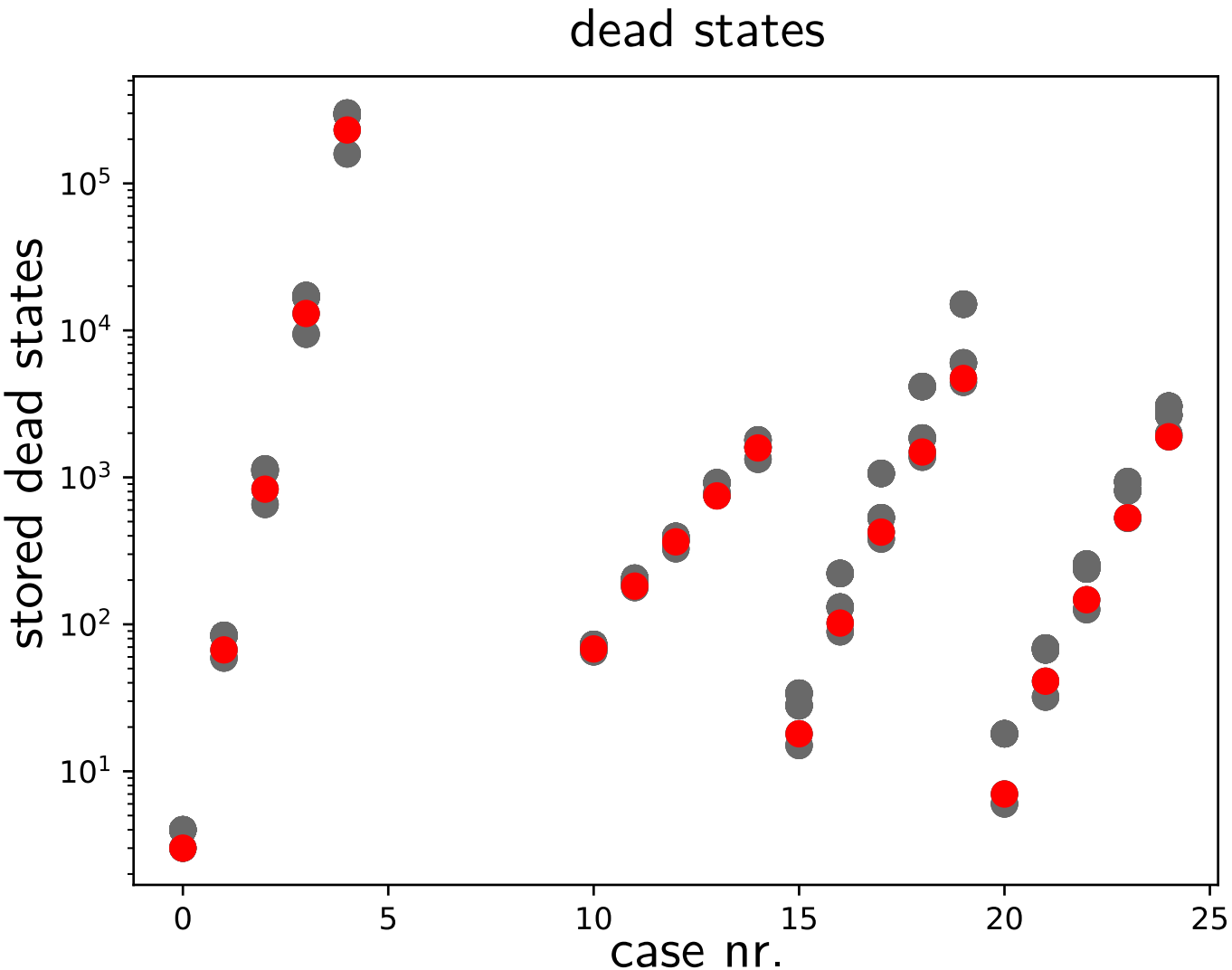
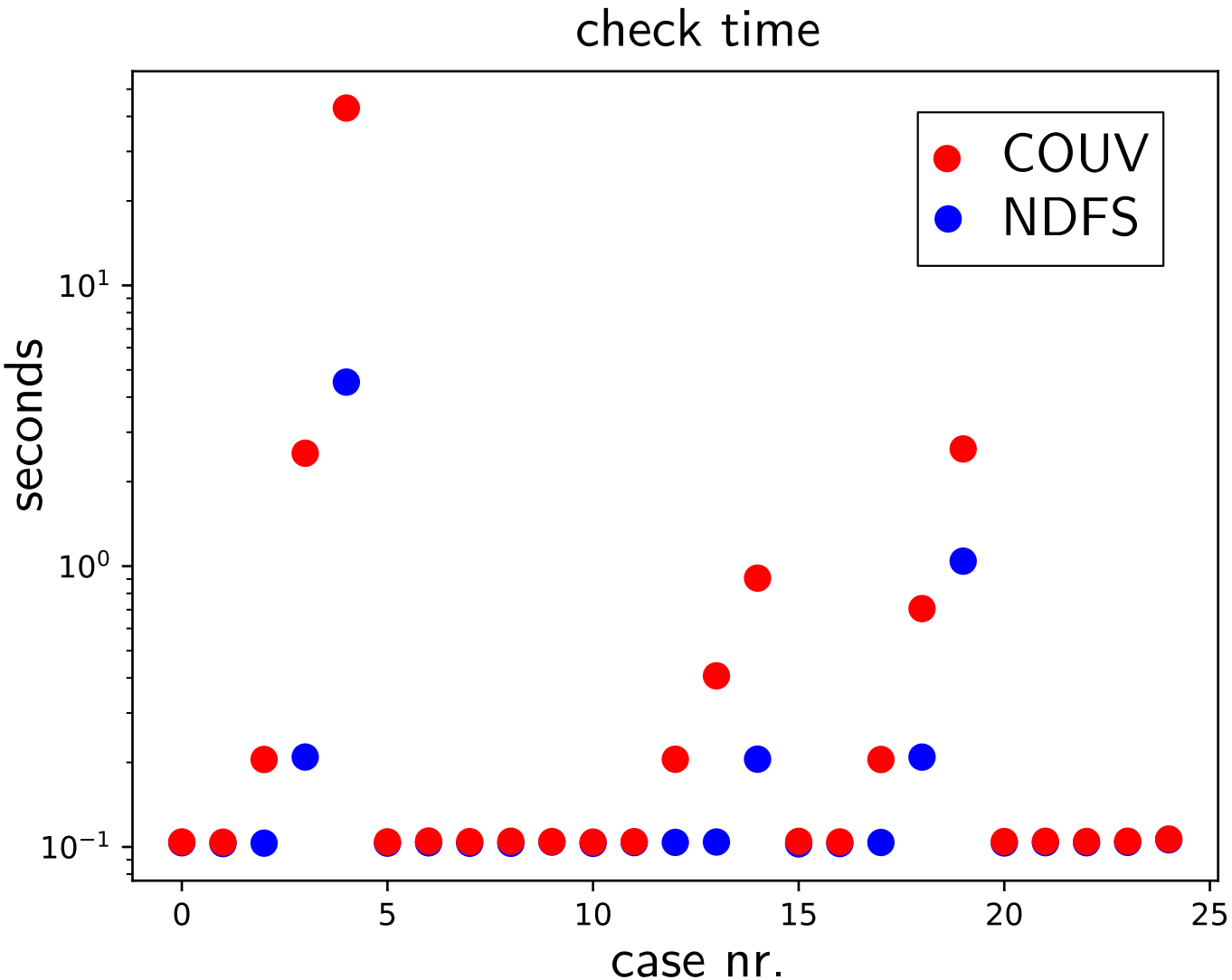


Benchmarks 1 - NDFS vs COUVREUR



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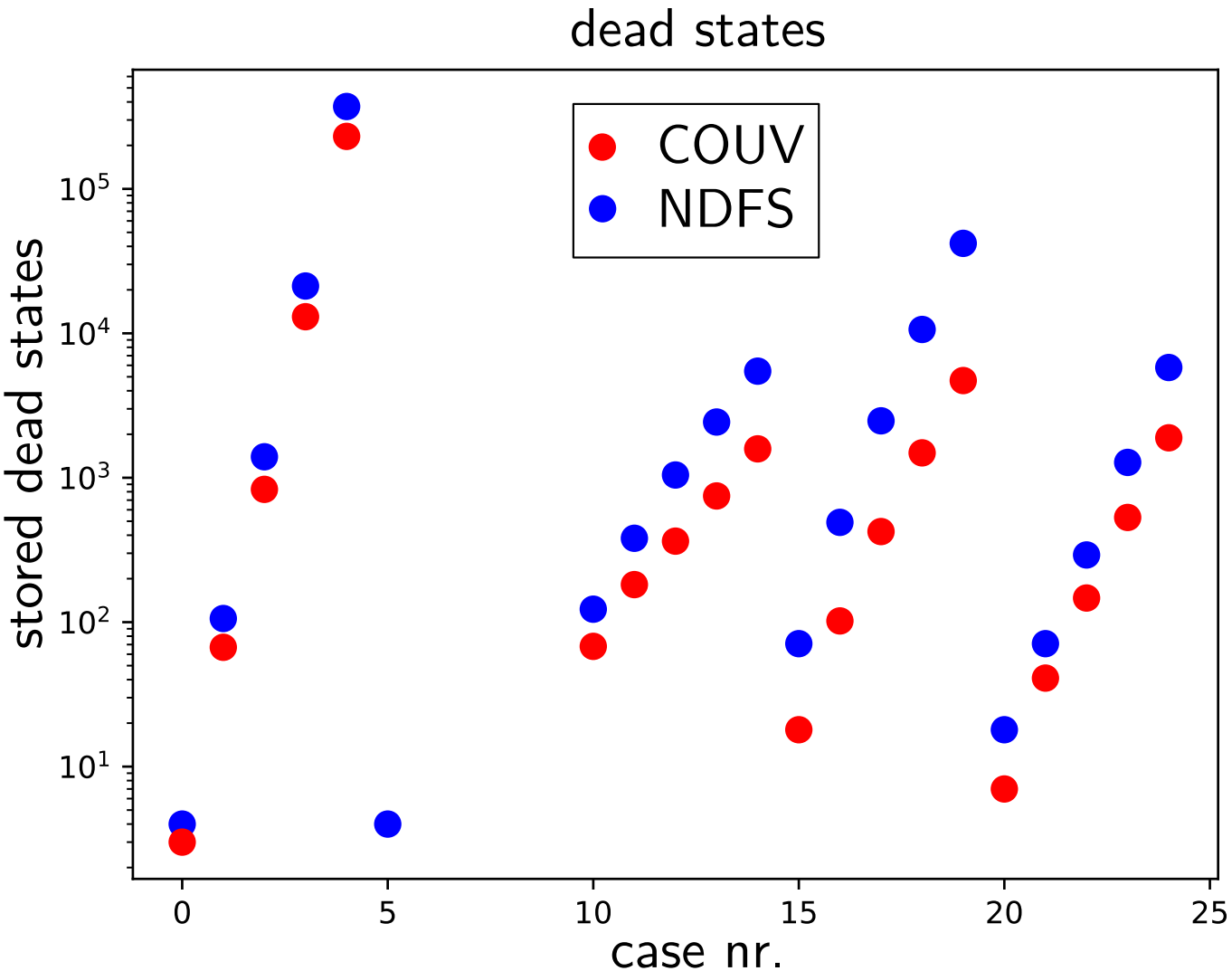

The problem of zeno runs

A run is considered zeno if it makes infinitely many switches in a finite amount of time. That is time gets "frozen".

Such runs are unrealistic and therefore discarded.

- Strong non-zeno construction $\rightarrow$ Adding a clock [2]

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- Guessing zone graph $\rightarrow$ Add information to the nodes [3]

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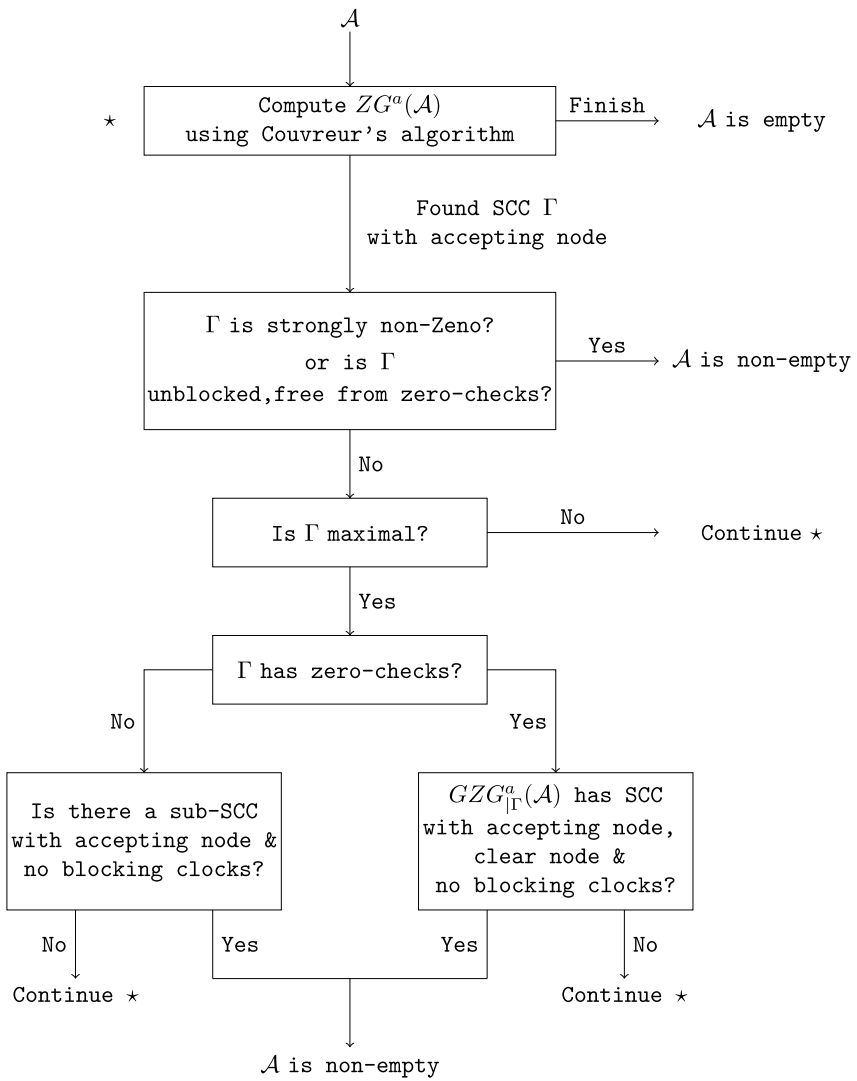
3. Herbreteau, F., Srivathsan, B., & Walukiewicz, I. (2010, July). Efficient emptiness check for timed büchi automata. In International Conference on Computer Aided Verification (pp. 148-161). Springer, Berlin, Heidelberg.

Guessing zone graph

Main idea

- For each node, track set of clocks $Y \subseteq X$ which can evaluate to zero : (q, Z, Y)
  - In transitions : add a clock x to Y if it is reset
  - Add the constraint $\forall \boldsymbol{x} \in X \setminus Y : \ x > 0$ for every transition
- Change the acceptance to see $Y = \varnothing$ infinitely often.
- On each transition : Either keep Y or reset it $Y = \varnothing$

Guessing zone graph - Algo 1



Guessing zone graph - Spec. based

Main idea :
Instead of conceiving a dedicated algorithm,
add the non-zeno requirement to the specification.

- Clear node has to appear infinitely often
- If a clock encounters infinitely often an upper bound, it also has to be reset infinitely often.

$$spec_{nz} = spec \wedge \mathsf{GF}\, clear \bigwedge_i (\mathsf{FG}\, !up_i | \mathsf{GF}\, res_i)$$

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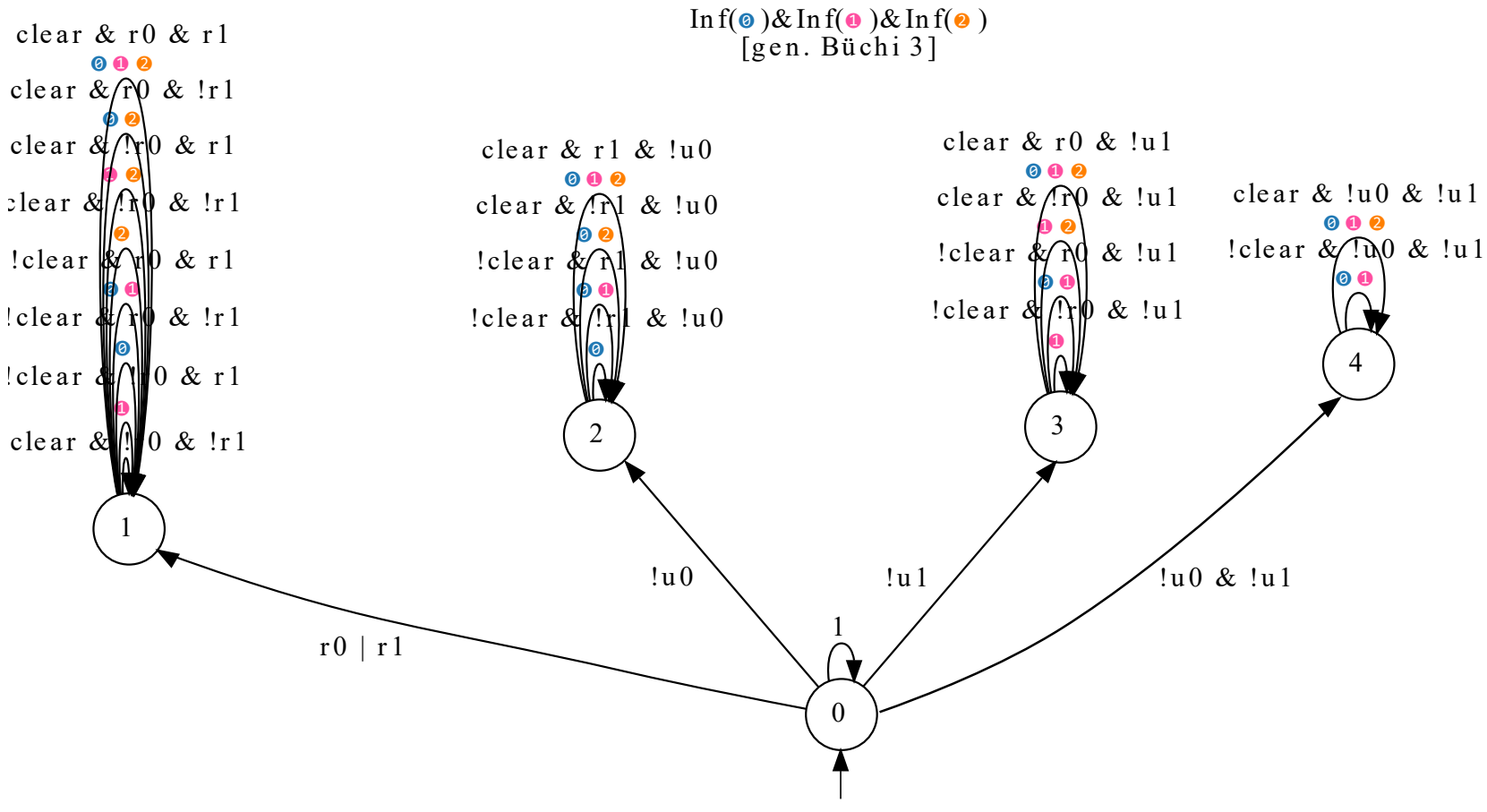
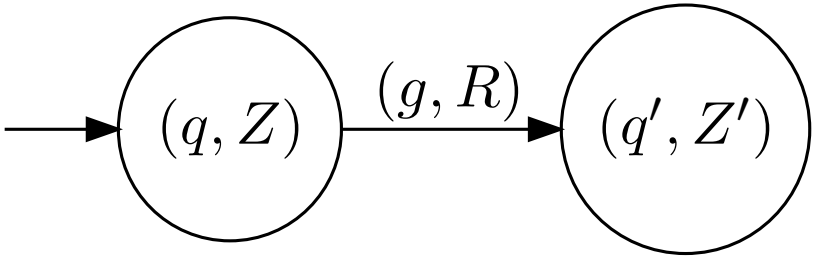
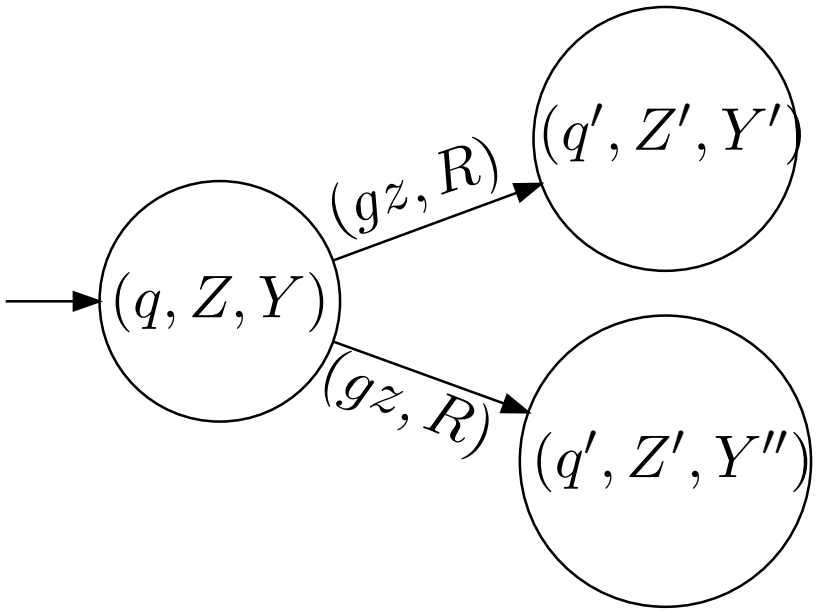


Modified TA

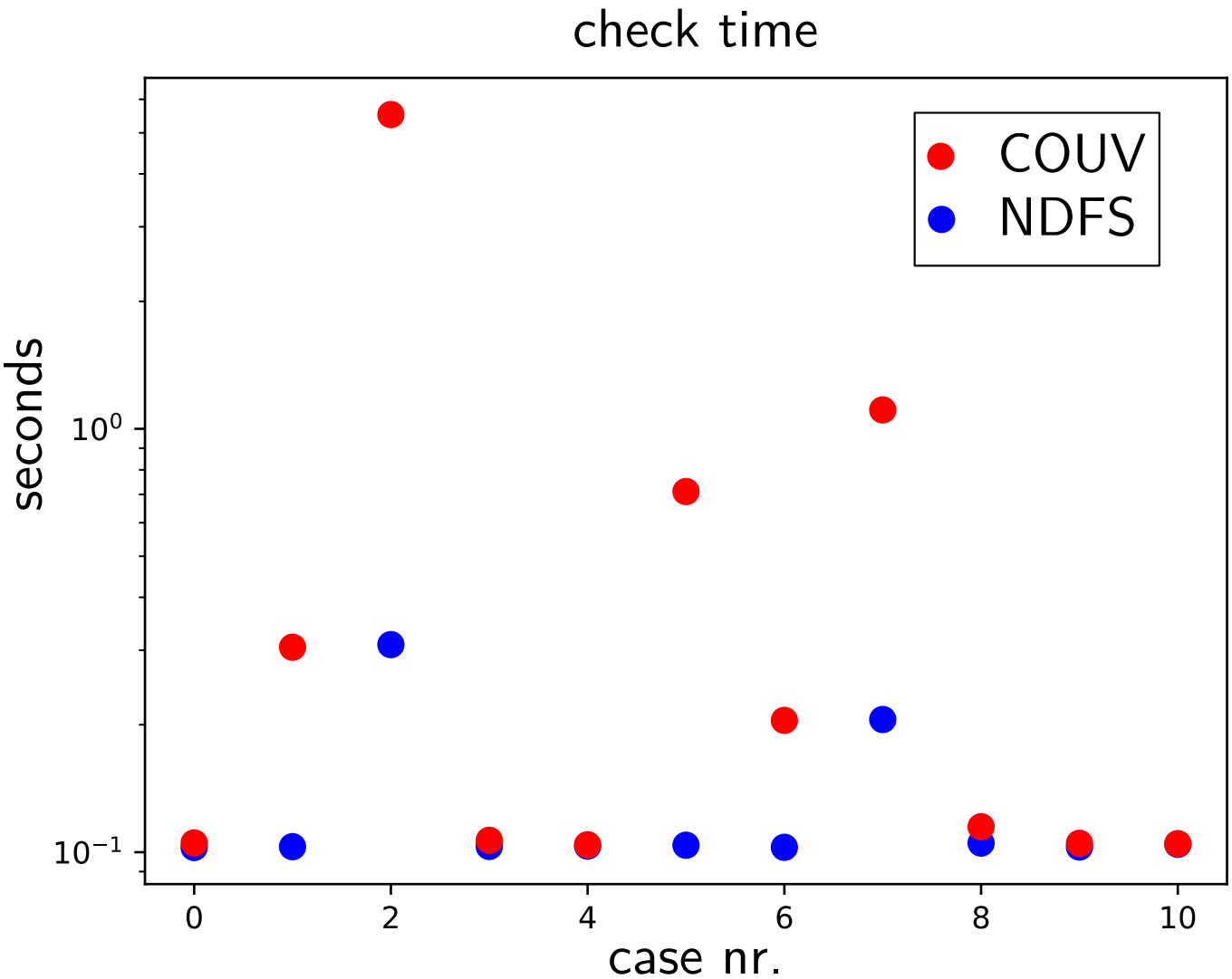


with
$Z' = \overrightarrow{[R](Z \cap g)}$



with
$gz = g \cap \{x_i > 0 | x_i \in X \setminus Y\}$
$Z' = \overrightarrow{[R](Z \cap gz)}$
$Y' = Y \cup R$
$Y'' = \varnothing$

Benchmarks 2 - Non-zeno



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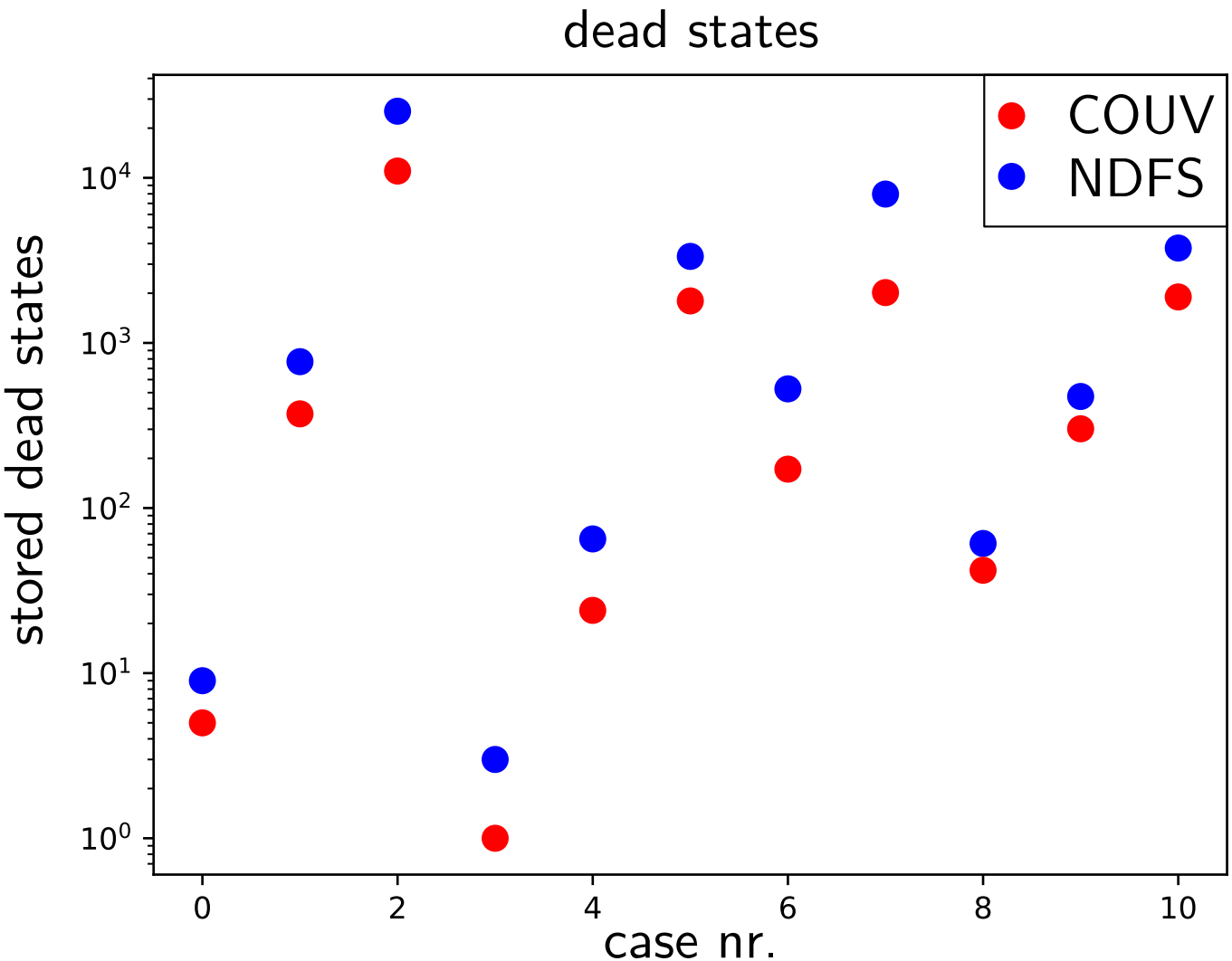


Inclusion relation on Y

- "smaller" $Y \rightarrow$ smaller successor zones
- Y "grows" slowly, but can be reset any time

Inclusion relation

$$(q, Z, Y) \sqsubseteq (q', Z', Y')$$
iff
$$q = q',\ Z \preceq Z' \text{ and } Y \subseteq Y'$$

Inclusion relation on Y

Problem

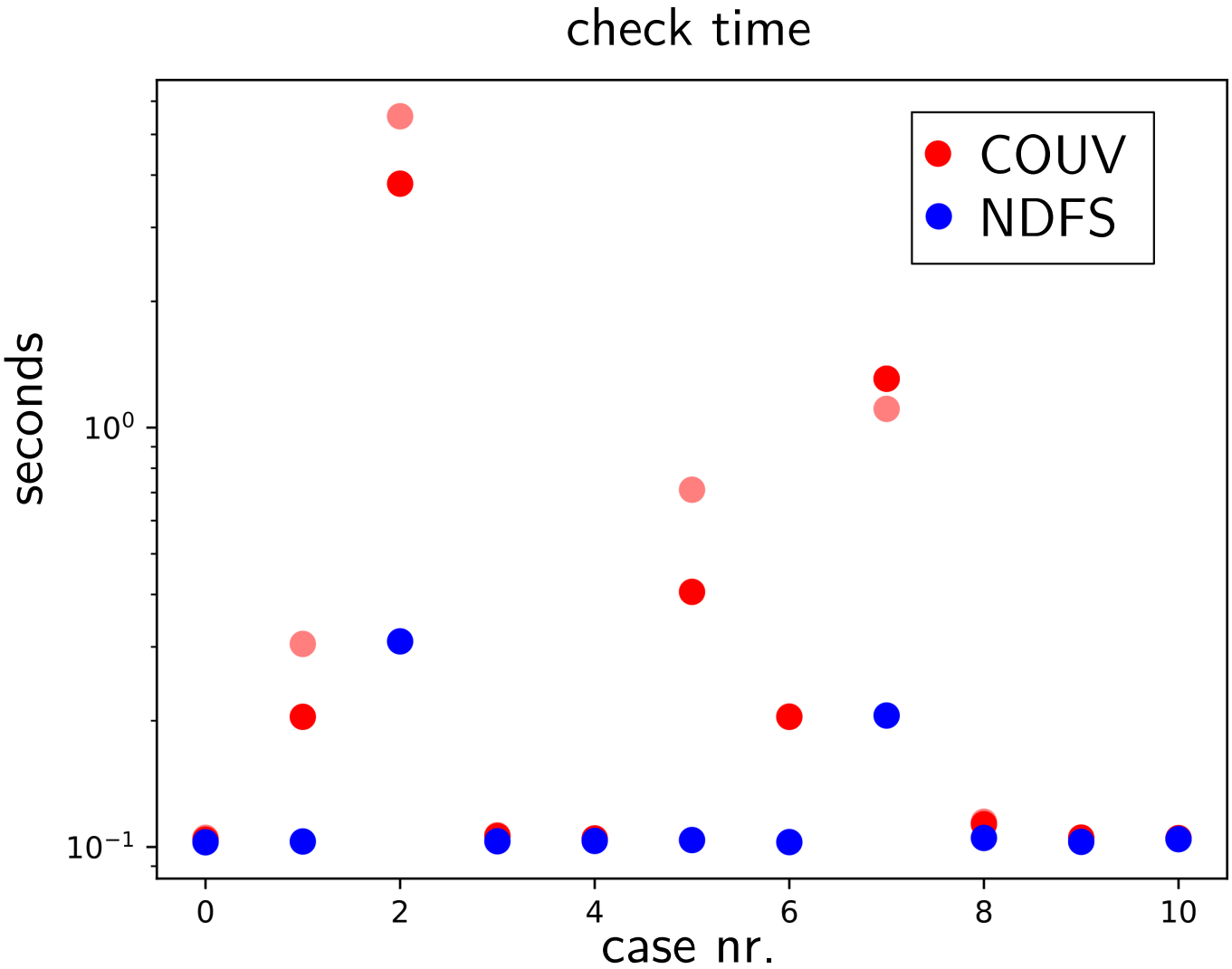
- The inclusion relation disregards the acceptance condition.
- Nodes with $Y = X$ are maximal, but not accepting

For a run to be accepted we have to see the clear node $Y = \varnothing$.

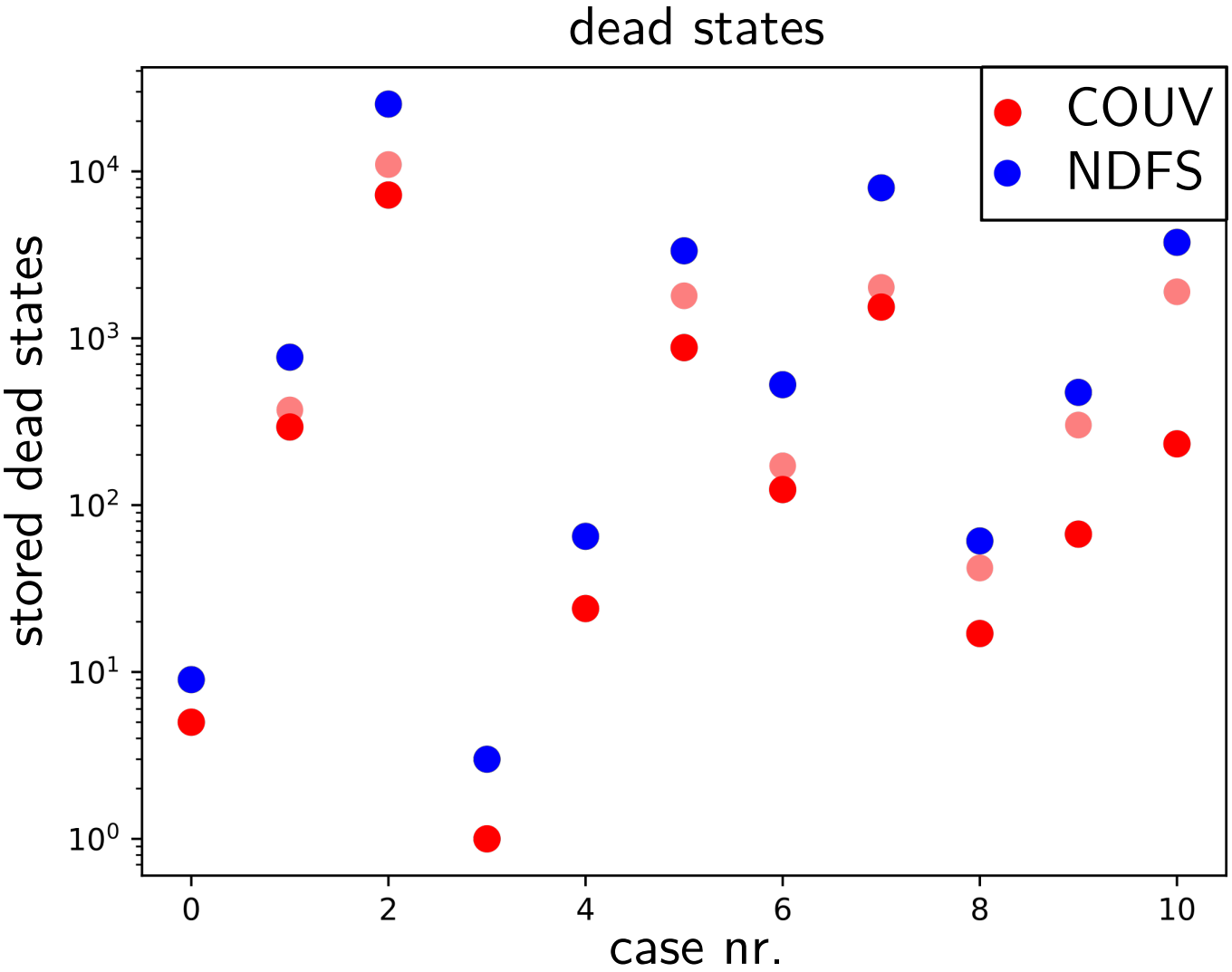
Solution

Always consider the successors with $Y = \varnothing$ first.

Benchmarks 3 - With Inclusion



Benchmarks 3 - With Inclusion



Current work

- Support parallel EC using Bloemen [4]
- Find better formulations for non-zeno condition

4. Bloemen, V., & van de Pol, J. (2016, November). Multi-core SCC-based LTL model checking. In Haifa Verification Conference (pp. 18-33). Springer, Cham.