Dissipation-based Homotopy Continuation Method for Nonlinear Multiphase Flow in Porous Media

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Abstract

In reservoir simulation, solving highly nonlinear algebraic equations arising from a fully-implicit discretization is challenging. There is generally no acceptable initial iterate available for multiphase flow and transport problems. Hence, a globalization strategy is usually needed when applying Newton's method to the nonlinear system.

Homotopy continuation, which has been widely adopted in numerical algebraic geometry and bifurcation analysis, provides a promising way for globalizing Newton's method. For a given nonlinear system to be solved, a homotopy between the given system and a new system (which is easier to solve) is constructed. Then the new system is gradually deformed into the original one along the homotopy through a path-tracking algorithm, thereby obtaining the solution to the original system.

In this paper, we develop a homotopy continuation method based on dissipation operator for solving multiphase flow and transport in porous media with combined viscous, gravitational and capillary forces. The homotopy is constructed by adding numerical dissipation to the discrete flow equations, with a continuation parameter controlling the magnitude of the dissipation. Numerical evidences and detailed analysis are provided through single-cell and 1D examples to illuminate why the dissipation operator can significantly improve the nonlinear convergence of hyperbolic PDE problem. In addition, an adaptive strategy to determine optimum dissipation coefficient is proposed. The adaptive dissipation is computed in a face-wise manner and is applicable to coupled flow and transport.

We demonstrate the efficacy of the new nonlinear solver using several numerical examples, including 1D scalar transport and 2D heterogeneous problems with fully-coupled flow and transport. The dissipation operator is found to be highly effective in resolving the specific convergence difficulty associated with the low wave speed around saturation front. Overall, the new solver exhibits superior convergence properties compared to the standard Newton's method. Moreover, the new solver works robustly for a wide range of flow conditions without parameter tuning in the dissipation operator.