Construction of reduced-order models of discrete fracture networks through physics on graph representations

Shriram Srinivasan, Jeffrey Hyman, Satish Karra, Hari Viswanathan, Gowri Srinivasan
Los Alamos National Laboratory

Key words: Discrete Fracture Networks, Graphs, and Reduced-Order Models

Introduction

In porous media with fractured rocks, it is well-known that most of the sub-surface transport occurs through the fractures, and discrete fracture network modelling has shown itself to be superior to continuum approaches using effective properties [1, 2]. In this approach, the porous media is modelled as a network of intersecting polygons or ellipses and typically the lengths, orientation and other geometrical parameters of it are drawn from a statistical distribution that best reflects geological site data [3]. One challenge with this approach is the trade-off between better accuracy or fidelity that comes with a more detailed network representation and the computational cost of meshing and simulating flow through it. Moreover, multiple realizations of a DFN network are necessary to account for the uncertainty in the parameters due to limited data. This has led to a search for efficient ways to simplify the DFN network and speed up the simulation process.

One obvious method is to construct an invertible mapping of a DFN to a graph network and use graph theory techniques to reduce complexity or prune the graph, and consequently, the DFN. The idea is that simulating transport on the pruned DFN should be relatively inexpensive in comparison to the full DFN. Utilizing purely topological information for graph pruning has been shown to yield accurate first passage times for transport through the network [4], however, for applications such as hydrocarbon extraction or contaminant remediation where the entire distribution of passage times is of interest, one needs to consider the hydrological properties of the network such as permeability and length. In this work we go a step further and use the results of graph-transport to inform the pruning.

Method

The key idea is the mapping that converts a DFN to a graph representation. Specifically, representing intersections as nodes and fractures as a clique of edges allows for geometric and hydrological properties to be inherited by the graph edges. The first step [5] requires solving a flow equation on the graph which produces something analogous to the velocity field in a standard (non-graph based) computational hydrology approach. The flux from vertex A to vertex B is $Q_{AB} = w_{AB}(P_A - P_B)$ where P_A and P_B are the pressures at vertices A and B, respectively, and w_{AB} is a coefficient that describes how readily water can from from the intersection associated with vertex A to the intersection associated with vertex B. Note that the weight w_{AB} is computed *a priori* from properties of the DFN such as the permeability of the relevant fracture and distances between the intersections. Conservation of mass is enforced by summing the fluxes at a vertex and setting the sum equal to zero, resulting in a set of linear equations for the pressures, which can be readily solved using standard techniques from numerical linear algebra.

Here we use the flow solution on the graph (knowing the pressure at every node) and thereby define a flux for each edge. One can now perform a weighted current-flow thresholding on the graph with a given threshold parameter to select a sub-graph such that all of its edges have a flux greater than the threshold. The choice of the threshold parameter determines how aggressive our pruning strategy is. The pruned graph is then inverted to obtain a pruned DFN that ensures computational savings. This is useful for cases where the full breakthrough curve is of interest such as contaminant remediation or hydrocarbon extraction. A sample result that illustrates this for one realization is shown in Figure 1.

Results

The relative error in predicting breakthrough times as a function of mass fractions is shown in Figure 2. The error bars show the results for 30 realizations of the DFN power-law network that was used earlier in [5]. The plot shows that the mean relative error for predicting breakthrough times for larger values of the mass fraction increases, but the spread is almost constant. This and other methods of physics-based pruning are being studied in greater detail.

References

[1] S. Painter and V. Cvetkovic, "Upscaling discrete fracture network simulations: An alternative to continuum transport models," *Water Resources Research*, vol. 41, no. 2, 2005.

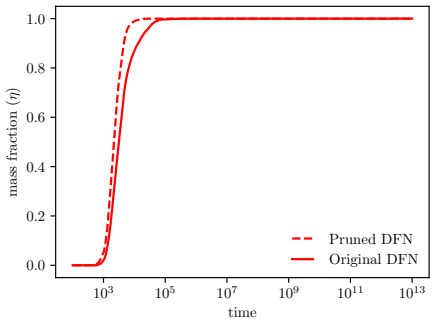

Figure 1: The breakthrough curve for the pruned DFN obtained from physics-based pruning shows close agreement with that of the original DFN (shown for only one instance here). The pruned DFN has less than 50 % of the fractures that made up the original DFN.

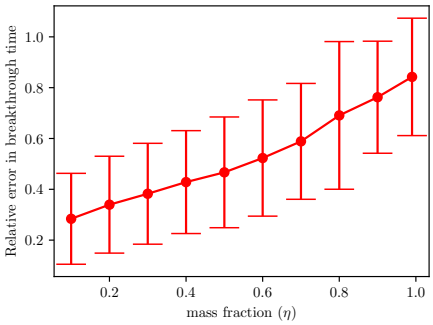

Figure 2: The error between the breakthrough curve for the pruned DFN obtained from physics-based pruning and original DFN for various mass fractions. The results consolidate data from 30 realizations. The pruned DFN has less than 50 % of the fractures that made up the original DFN.

[2] S. Painter, V. Cvetkovic, and J.-O. Selroos, "Power-law velocity distributions in fracture networks: Numerical evidence and implications for tracer transport," *Geophys. Res. Lett.*, vol. 29, no. 14, pp. 20–1–20–4, 2002.

[3] J. D. Hyman, S. Karra, N. Makedonska, C. W. Gable, S. L. Painter, and H. S. Viswanathan, "dfnworks: A discrete fracture network framework for modeling subsurface flow and transport," *Computers & Geosciences*, vol. 84, pp. 10–19, 2015.

[4] J. D. Hyman, A. Hagberg, G. Srinivasan, J. Mohd-Yusof, and H. Viswanathan, "Predictions of first passage times in sparse discrete fracture networks using graph-based reductions," *Phys. Rev. E*, vol. 96, p. 013304, Jul 2017.

[5] S. Karra, D. O'Malley, J. Hyman, H. Viswanathan, and G. Srinivasan, "Modeling flow and transport in fracture networks using graphs," *arXiv preprint arXiv:1708.08556*, 2017.