New advances in the catchment-scale formulation of transport processes

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Introduction

The problem of how catchments store and release solutes at different scales has been tackled using a number of different approaches. Integrated (or "lumped") approaches are naturally suited to deal with large-scale problems as they aim to reproduce the integrated response of the system at an observational point and allow building parsimonious models. A new generation of catchment-scale transport models has been developed in the last years, where transit time distributions (TTDs) result from the formulation of a general age Master Equation (ME) [1]. Among the advantages of this new approach, there is the opportunity to address the transport process through the entire hydrologic balance, i.e., not just in the discharge but also in the catchment storage and in all other outflows like evapotranspiration. Moreover, applications can be extended to any system where the chronology of the inputs plays a role in the output composition. This new theoretical formulation has improved capabilities, but the numerical implementation of the governing equations is more demanding than in traditional lumped methods as the governing equation is usually nonlinear.

Methods

By indicating time as t and age (i.e. the time since the entrance into the catchment) as T, one can define the age distributions of the catchment storage ($p_S(T,t)$) and of its pertaining fluxes ($p_F(T,t)$). The total catchment storage is indicated as $S(t)$ and the fluxes (e.g. precipitation, discharge, evapotranspiration) are termed F_i. A general form of the age ME, relating resident and flux age distributions, can be written as:

$$\frac{d[S(t)\,p_S(T,t)]}{dt} = \sum_i F_i(t)\,p_{F_i}(T,t) \tag{1}$$

with Initial Condition $p_S(T,0) = p_{S_0}$ and Boundary Condition $p_S(0,t) = 0$. Resident and flux distributions can be related by introducing the StorAge Selection (SAS) function $\omega(T,t)$:

$$p_{F_i}(T,t) = p_S(T,t)\,\omega_{F_i}(T,t) \tag{2}$$

By coupling equation (2) to equation (1), one ends up with a non-linear partial differential equation that needs numerical integration. Several parameterizations of the SAS functions can be introduced to simulate characteristic features of the transport process. The age ME solution can then be used to e.g. compute solute concentration at a catchment outlet.

Results

This contribution focuses on the discretization of the age ME and its solution using general SAS functions. Results highlight the opportunity to apply the age ME and the SAS function approach to different contexts, possibly bridging different spatial scales. As a consequence, new questions emerge, in particular: i) how does the "age selection" of the outflows changes across the landscapes (say, from a soil plot to a large catchment)? and ii) is it possible to formulate meaningful equations for reactive solute transport that operate at the catchment scale? Examples from recent applications to catchment [2] and lysimeter transport problems help sheding light on these questions.

References

[1] Botter, G., Bertuzzo, E., and Rinaldo, A.: Catchment residence and travel time distributions: The master equation, Geophysical Research Letters, 38, 2011.

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