Using a posteriori error estimates to stop the iterations in a space-time domain decomposition method for porous media flow

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Simulations of flow through heterogeneous porous media are widely used in many applications. Among others, these models are used to simulate the migration of radio-nuclides around a nuclear waste repository in the geological underground or in petroleum engineering to predict the motion of oil in the underground. A distinguishing feature of these applications is the wide range of both spatial and temporal scales that make their simulation challenging. Domain Decomposition (DD) is one among several techniques that have been called upon to provide accurate, robust and efficient tools to carry out those simulations.

This contribution develops a posteriori error estimates for both steady state [1] and transient flow [2, 3] simulations using DD methods. The DD methods are of Schwarz type, and are global in time (Optimized Schwarz Waveform Relaxation methods [5]) for the transient problem. They use Robin transmission conditions whose coefficients can be optimized to improve convergence rates. The problem is formulated as a space-time interface problem solved by a Krylov-type method, which enables solving space-time subproblems in parallel with local time stepping, so as to adapt to the heterogeneities often encountered in the subsurface [7]. At the end of each iteration, the Robin boundary data required for the next iteration are exchanged across the space-time interface, using an optimal order projection algorithm [6].

The resulting problem is discretized in space via Raviart-Thomas mixed finite elements (as they have built-in conservation properties and are well suited to handle heterogeneous problems), and in time by a discontinuous Galerkin method of degree 0 (closely related to the backward Euler method).

In order to decide when to stop the DD iterations, we have developed fully computable a posteriori error estimates that allow for the separation of the part of the error due to the space and time discretizations, and that due to the DD iterations. These error estimates are based on reconstruction techniques [4, 8], building a continuous pressure and conforming fluxes. To estimate the DD error, an additional pressure field is introduced. It is continuous in the subdomains, but discontinuous along the interfaces. The fluxes require the solution of local Neumann problems in narrow bands around the interfaces.

The method is illustrated on an example provided by ANDRA, the French agency for nuclear waste management. In that case, the unknown represents the concentration of some tracer. The value of the diffusion coefficient is $2\,10^{-9}\mathrm{m}/^2\mathrm{s}$ in the repository and $5\,10^{-12}\mathrm{m}/\mathrm{s}^2$ elsewhere. A source term is located in the central region, acting as the repository. It is active over 10^5 years, and is zero afterwards. The geometry of the domain, and a snapshot of the concentration after 100 000 years are shown in Figure 1.

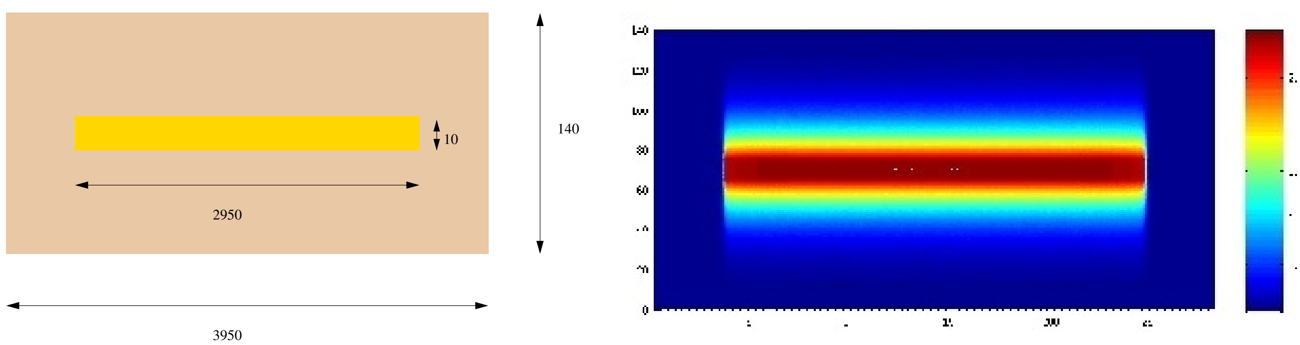

Figure 1: Andra example. Left: geometry, right: snapshot of the solution after 100 000 years

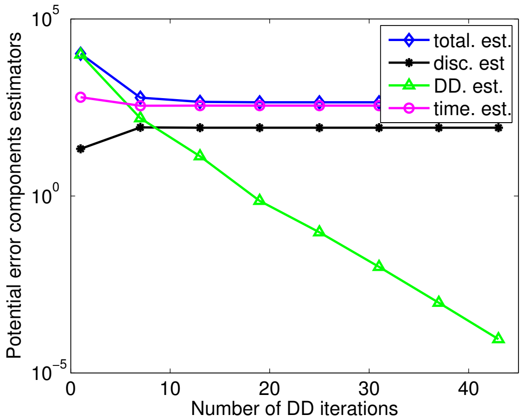

Figure 2: Andra example. Estimators for the total error (in blue), the spatial (in black) and temporal (in pink) discretization errors, and the DD error (in green)

The evolution of the various error estimators as a function of the iterations is shown in Figure 2. As expected, the discretization estimators do not vary after the first few iterations. The DD error estimator (in green) decreases as the DD method converges. Using a common stopping criterion (when the relative residual becomes smaller than some given tolerance, say 10^{-5}), one would need close to 40 iterations. If instead we rely on the error estimates to stop the iterations once the error due to the DD becomes smaller (say by a factor of 10) than the error already incurred because of the discretization, the DD method can be stopped after 12 iterations, a savings of more than two thirds in this example.

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