Parameter selection for the inverse problem of EMI surveys with a discontinuous solution

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Key words: focusing inversion, electromagnetic induction, optimal regularization parameters

Introduction

If a magnetic dipole is placed above the surface of the earth, the Electromagnetic Induction (EMI) effect, encoded in Maxwell's equations, causes eddy currents in the soil which, on their turn, induce response electromagnetic fields. The magnetic field can be measured in geophysical surveys to determine the conductivity profile of the ground in a non-destructive manner. The forward model used in the inversion of experimental data usually consists of a set of horizontal homogeneous layers. A frequently used model, proposed by McNeill [1], does not include the interaction between the eddy currents, and therefore fails for larger conductivities. We present a new forward model to estimate the magnetic field caused by a horizontally stratified earth, taking into account the interaction between eddy currents. This makes it valid for a broader range of parameters than the current state of the art. Furthermore, the error with the (numerically obtainable) exact result is substantially decreased [2].

As the forward problem is ill-conditioned, regularization of our problem is necessary. The common methods of regularization result in a continuous solution while one wants a piecewise continuous function for our forward problem. The currently available stabilizers who allow sharp boundaries often depend on an extra parameter. Changing this parameter has a huge effect on the solution, and should therefore be chosen carefully. We compare the effect of the different stabilizers and discuss some methods to automatically determine the optimal value of the parameter.

Our forward and inverse solution combine in a system that is capable of determining the soil conductivity profile. Such profile can, for example, be used to determine the soil salinity, detect anomalies, monitor soil contamination, for non-invasive archeological prospection or for probing salty seawater intrusion into groundwater reservoirs [3, 4, 5]. Due to the chosen stabilizers we are capable of determining the depth of the boundary between two layers. This has the added benefit that, for example, the depth of the saltwater can be determined. We can therefore not only determine if saltwater intrusion is occurring but also the gap between ground- and seawater.

Forward model

McNeill model: The McNeill approach [1] considers a thin sheet at depth h from the magnetic dipole with a conductivity $\sigma(h)$ and an infinitesimal thickness dh floating in air. The actual problem we want to solve consists of a half-space with varying conductivity. Slicing the half-space in an infinite amount of thin sheets on top of each other, the magnetic field can be obtained by integrating from zero to infinity with respect to the depth h.

Damped model: Introducing an interaction between the sheets allows the application of our model for a higher induction number, which will automatically lead to an improvement w.r.t. McNeill. We consider a sheet embedded in a half-space with fixed conductivity. After integration w.r.t. to the depth of the sheet and subtracting the contribution of the background we obtain the magnetic field. Due to this half-space we introduce an interaction and thus dampening, while mostly retaining the linear features of the problem.

Comparison: In Figure 1 the relative error of both the McNeill and the damped model with respect to the result from a Finite Element Method (FEM) is plotted for different intercoil distances. This "exact" result was obtained from a simulation in COMSOL. For both models, a larger intercoil distance causes the low induction number (LIN) requirement to fail which we notice due to an increase in error. However, we also notice that the damped model has a considerably smaller error due to the implemented interaction.

Inverse problem

The inverse problem corresponds to solving the following equation:

$$\left\| K(\vec{\sigma})\vec{\sigma} - \vec{d} \right\|^2 + \alpha S(\vec{\sigma}) = 0. \tag{1}$$

The matrix K is determined by the forward problem while the vector $\vec{d}$ is the measured magnetic field. The function S is a stabilizer (e.g. total variation (TV) [6], minimum support (MS), minimum gradient support (MGS) [7], ...) which depends on an extra parameter β. The problem is therefore separated in two parts: determining the optimal parameters α and β and solving equation (1) for the determined parameters. Due to

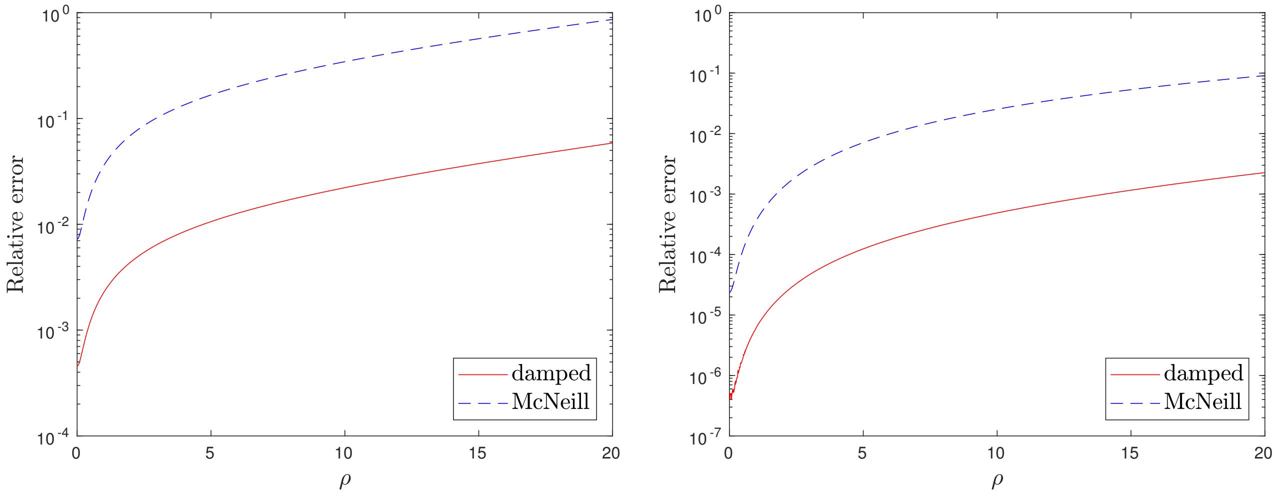

Figure 1: The relative error on the imaginary part of the secondary magnetic field. The soil consists of five layers with conductivity $55\,\mathrm{mS\,m^{-1}}$, $60.5\,\mathrm{mS\,m^{-1}}$, $65.4\,\mathrm{mS\,m^{-1}}$ and $85.9\,\mathrm{mS\,m^{-1}}$ and thickness $2.4116\,\mathrm{m}$, $0.8540\,\mathrm{m}$ and $2.4268\,\mathrm{m}$.

the non-linearity of K and/or S, an iterative method must be used for the latter problem. We use a method based on the Gauss-Newton minimization algorithm to solve equation (1), but with every step the optimal value of α and β is determined [8].

The value of α (the regularization parameter) determines the balance between the minimization of the error and the stabilizer. The value of β determines the smoothness of the solution. A very small value of this parameter results in a very blocky solution. While this is the preferred solution, it has been shown that the optimal value is a non vanishing β [9].

While there exist multiple methods to determine the optimal value of α (e.g. unbiased predictive risk estimator (UPRE), generalized cross validation (GCV), ...), the value of β is often determined a priori, and therefore the inverse problem is solved for a fixed β. We instead propose to optimize the value of β using the same methods used to determine the value of α.

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