An ecohydrological model to explore topographic and rainfall variability effects on vegetation self-organisation

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Introduction

Water-limited ecosystems (WLE) often exhibit vegetation self-organisation. This self-organisation manifests as characteristic arrangements of vegetation in space sometimes with very clear patterns [1], well documented and studied through field observations [2, 3] and numerical modelling [4]. Vegetation self-organisation in WLEs is explained as the result of scale-dependent feedbacks between water and vegetation [1]: vegetation locally increases infiltration capacity, thus inducing a heterogeneous water redistribution which facilitates vegetation growth in clusters or patches and characteristic spacing between them. This hypothesis has been extensively tested by modellers and is well supported by observations and ecohydrological theory. Modelling has indeed been key in studying the relevance of these mechanisms as drivers of the ecohydrological phenomena in WLEs.

Ever since the introduction of simple PDE-based mathematical models [4] the modelling effort has focused on studying the properties and behaviours of converged ecohydrological steady states, i.e., stable vegetation patterns responding to a steady or quasi-steady water input in the form of rainfall [e.g. 5]. The result has been a thorough understanding of the relevance and sensitivity of a variety of environmental factors on the converged states and their stability [e.g. 6, 7]. In contrast, the transient dynamics of these systems are only recently receiving much needed attention [8]. The reason for this can be argued to be multi-causal. The first reason is methodological: it is reasonable to first attempt to test hypothesis and collect insights on converged steady states, where the dynamics are somewhat simpler than full transients. The second is that quantitative assessment of model results against quantitative field data remains, at best, experimental [9]. Thirdly –and most relevant to this work– is that most numerical models that have been proposed and used in this context, in part for the sake of mathematical and numerical simplicity, have greatly simplified hydrodynamic and hydrological (as well as ecological and biological) processes, thus making transient simulations unreliable.

This work addresses the third issue, proposing an approach to improve the hydrodynamic models in the context of WLEs vegetation self-organisation, to enable considering the effects of complex topography and rainfall intermittency. A modelling upgrade to cope with hydrodynamics by making use of a process-based modelling *Ansatz* typical of many hydrological and hydraulic models [e.g. 10] is proposed, which both requires a more complex, rigorous and robust mathematical/numerical formulation and a better computational implementation, and opens up a plethora of new opportunities to study WLE vegetation self-organisation.

Numerical model

The proposed model is an improvement on the well-established HilleRisLambers–Rietkerk (HRLR) [1] model for vegetation self-organisation. From this model we keep unchanged the reaction-diffusion equation describing vegetation dynamics in terms of biomass density $b(\mathbf{x}, t)\,[g\,m^{-2}]$

$$\frac{\partial b}{\partial t} = c_b g_b \mathcal{U} - d_b b + D_b \nabla^2 b \qquad \mathcal{U} = \frac{W}{W + k_b} b \tag{1}$$

where $\mathcal{U}(\mathbf{x}, t)\,[L]$ is the water uptake rate, c_b is water to biomass conversion efficiency, g_b is the maximum water uptake per unit biomass, k_b is a half-saturation water uptake constant, D_b is the vegetative biomass diffusion coefficient and d_b is the biomass death rate coefficient. Finally, $W(\mathbf{x}, t)\,[m]$ is the soil water volume per unit area available for biomass to uptake, and is controlled by the water redistribution equations for surface and subsurface water. Subsurface water redistribution is governed by a linear diffusion equation and interacts with vegetation through uptake, and with surface water through the infiltration rate $i\,[m/s]$, which is affected by the infiltration enhancement function γ which represents the increase in infiltration rate due to vegetation on the bare soil infiltration rate i_*.

$$\frac{\partial W}{\partial t} - D_w \nabla^2 W = i - \mathcal{U} \qquad i(\mathbf{x}, t, b) = i_*(\mathbf{x}, t)\gamma(b) \tag{2}$$

We couple equations (1) and (2) to the Zero-Inertia (ZI) approximation to the shallow-water equations, instead of the originally proposed naïve linear-diffusion equation [1] for surface flow

$$\frac{\partial h}{\partial t} + \nabla\left(\frac{h^{5/3}}{n\sqrt{||\mathbf{Z}||}}\mathbf{Z}\right) = i \qquad\qquad \mathbf{Z} = \nabla(h+z) \tag{3}$$

where $h\,[m]$ is water-depth, n is the Manning's roughness coefficient, $z\,[m]$ is topographical elevation and $\mathbf{Z}$ is the water surface gradient. The inclusion of the ZI equation instead of a diffusion equation explicitly introduces the topography field z and the roughness field. As a by-product, wetting and drying phenomena may arise together with the complex topography. This upgrade has significant implications in the complexity of the numerical solution as compared to the original HRLR model. To name a few, equation (3) is non-linear, has stricter stability restrictions, and thus is more expensive to solve and requires careful positivity-preserving control. The system is solved on 2D domains by means of a parallelize, explicit, first-order finite volume scheme [11].

Exploratory study and preliminary results

The new opportunities opened by the proposed model allow to explore previously untested behaviours and dependencies. Clearly, the introduction of explicit topography allows to study the effects of geometrical surface properties and their impact on vegetation self-organisation and the relative importance of ecohydrological feedbacks in such context. Furthermore, the importance of rainfall variability and intermittency is highlighted, by the fact that wetting and drying fronts over complex topography can strongly affect water redistribution, and because the threshold-nature of rainfall-runoff-infiltration partitioning can –under the proposed model– be better captured.

We designed a set of simulations as a pilot study to assess the new possibilities opened by the model. We perform simulations over a hillslope with linear, concave and convex topologies, with different average slopes, and forced by different rainfall patterns and explore and analyse the resulting vegetation distributions in both their quasi-steady and transient properties and behaviours, as well as the feedbacks and interactions with water. The preliminary results suggest that (i) topography affects vegetation self-organisation in various degrees as it can govern surface water redistribution, (ii) rainfall variability is a driver of ecohydrological complexity and should not be neglected and (iii) spatial heterogeneity –such as that introduced by topography (but also by other factors)– affects the response of the ecohydrological system to rainfall variability, and vice-versa, both in terms of spatial water and vegetation distributions, but also in the temporal evolution of water and vegetation. Interestingly, the thresholded-nature of the interacting processes becomes clear in presence of spatiotemporal heterogeneity. An example observed in our simulation is that even under homogeneous soil properties, wetting and drying fronts –an interaction between topography and rainfall intermittency– induce soil moisture heterogeneities. Simulating these behaviours requires a model at least as sophisticated as the one presented here. These computations and results suggest that more complex dependencies may arise when exploring more realistic forcing and when further introducing process complexity in the subsurface domain.

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