# Beyond Kozeny Carman

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## Abstract

An accurate modeling of the permeability and diffusion coefficient is essential for predictive flow and transport modeling. Well-established relations are proposed by Kozeny-Carman for the permeability or by Buckingham, Penman, or Millington-Quirk for the diffusion, [1-3]. They relate the scalar permeability or diffusion coefficient to the porous medium's porosity. In order to capture the porous medium's structure in more detail, further models include fitting parameters, geometric, or shape factors. Some models additionally account for the tortuosity, e.g. via Archie's law.

Contrarily to these approaches, upscaling methods directly enable to calculate the full, potentially anisotropic, effective diffusion tensor without any fitting parameters. As input only the geometric information in terms of a representative elementary volume is needed. To compute the diffusion-porosity relations, supplementary cell problems must be solved numerically on this volume and their (flux) solutions must be integrated. In [4] we applied this approach to provide easy to use quantitative diffusion-porosity relations that are based on representative single grain, platy, or blocky soil structures. As a discretization method we used discontinuous Galerkin method on structured grids. To make the relations explicit, interpolation of the obtained data was used.

We furthermore compare the obtained diffusion-porosity relations with the well-established relations and also with the well-known Voigt-Reiss or Hashin-Shtrikman bounds. We discuss the ranges of validity and further provide the explicite relations between the diffusion and surface area and comment on role of a tortuosity - porosity relation.

## References

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