A Finite Volume Discretization Approach to Solving Flow of Groundwater and Tracer Transport in Karst Aquifers
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Introduction

Modelling flow through karst aquifers is considered to be a challenging task because of the presence of geomorphological features such as fractures, vugs, and caves etc. on a micro and macro scale, which introduce complex flow regimes and non-Darcy flow regions. The presence of these geomorphological features can alter the effective permeability of the system, the lack of knowledge of the exact position of the interface between porous media and vugs/caves would introduce even more difficulties in accurately modelling the flow in such geological features [1]. These features also affect the rate and direction in which the fluid moves in the aquifer [2]. In this paper, we model the flow of nonreactive tracers in karst reservoirs by coupling the continuity equation, the Brinkman’s equation to the Advection-Diffusion-Adsorption equation. The flow of fluid is modelled as single-phase, slightly-compressible flow in a 2D karst aquifer consisting of megakarstic geological terrains such as caves by solving the continuity equation and the Brinkman’s equation simultaneously to obtain the pressure and velocity distribution in the entire reservoir. Using the computed velocity distribution, the Advection-Diffusion-Adsorption equation is then solved numerically to model the transport of the tracers through the reservoir. The cell-centered finite volume discretization approach was adopted in this work in solving the differential equations of flow and tracer transport. The transport of tracer as modelled in this work is then compared with the tracer transport obtained by using Darcy’s equation in place of the Brinkman’s equation. Two examples are presented to study the transport of tracers and to show the difference in results between using the Brinkman’s equation and using the Darcy’s equation. The first example (Figure 1a) consists of a simple linear aquifer model being flooded from one side. The aquifer consists of three regions: two porous regions on each side of a free-flow conduit (cave region). We studied the effect of Peclet number on tracer transport, and the effect it has on the mathematical flow model selection. The second example (Figure 1b) is a more complex heterogeneous geological structure consisting of more realistic caves with randomly placed water producers and injectors.

Mathematical Models

Brinkman (1949) [3], developed a general equation that can be used to model the coupled flow. The Brinkman’s equation given by

$$\nabla p + \mu \kappa^{-1} \bar{u} - \mu_{eff} \nabla^2 \bar{u} = \nabla \cdot (\rho g z)$$

(1)
incorporates the effect of viscous shear in the Darcy’s model. In Eq. 1, \( p \) is the pressure \( (ML^{-1}T^{-2}) \), \( \mu \) is the fluid viscosity \( (ML^{-1}T^{-1}) \), \( K \) is the permeability tensor \( (LT^{-1}) \), \( \vec{u} \) is the velocity vector (darcy or free flow) \( (LT^{-1}) \), \( \rho \) is the fluid density \( (ML^{-3}) \), \( \vec{g} \) is gravitational acceleration \( (LT^{-2}) \), \( z \) is the depth \( (L) \), and \( \mu_{\text{eff}} \) is the effective fluid viscosity. One of the greatest advantages of using the Brinkman’s equation is that it can theoretically interpolate between the Stokes equation and the Darcy’s equation by adjusting the values of \( K \) and \( \mu_{\text{eff}} \). Under the Brinkman’s equation, the following are true.

- When \( \mu_{\text{eff}} = \mu \) and \( \bar{K} = \infty \), Eq. 1 becomes Stokes equation.
- When \( \mu_{\text{eff}} = 0 \) and \( \bar{K} = K \ (\neq \infty) \), Eq. 1 becomes Darcy’s equation.

The Brinkman’s equation for a single phase flow is solved by using Eq. 1 along with a continuity equation given by

\[
\frac{\partial (\rho \phi)}{\partial t} + \nabla \cdot (\rho \vec{u}) = \frac{\rho q_w}{V_b} \tag{2}
\]

where \( q_w \) is the volumetric flow rate \( (LT^{-1}) \), \( V_b \) is the block volume \( (L^3) \), and \( t \) is the time \( (T) \).

The advection-diffusion-adsorption equation is used to model the flow of tracers within a karst medium. The equation is given by:

\[
\nabla (s c) - \nabla \cdot (\phi \vec{u}) + \dot{c}_s = \frac{\partial (\phi c)}{\partial t} + (1-\phi) \rho_s \frac{\partial c_a}{\partial t} \tag{3}
\]

where, \( D \) is the dispersion coefficient tensor \( (LT^{-1}) \), \( c \) is the concentration of the tracer \( (ML^{-3}) \), \( c_s \) is the sink/source term of the tracer in terms of specific mass rate \( (ML^{-3}T^{-1}) \), \( \rho_s \) is the density of the rock \( (ML^{-3}) \), and \( c_a \) is the adsorption of tracer on the surface of the rock (dimensionless).

**Results and Conclusions**

Results from these examples show that the difference between the concentration profiles obtained from Brinkman’s model and those obtained from the Darcy’s model could be appreciable in some cases. Specifically, Example 1 showed that at high Peclet number, the results obtained from the two models are significantly different while at low Peclet number, the difference between the two models are not significant. The effect of adsorption on tracer transport was also studied. The effect of adsorption is more prominent in the porous region and near to the walls of the cave than at the center of the cave. Also, in Example 2, faster transport of tracers in the free-flow region was observed in the Brinkman’s model than in the Darcy’s model.

**References**

