Plant root dynamics via Optimal Transport

Enrico Facca, Mario Putti, Franco Cardin.
Università degli Studi di Padova

Key words: Plant root dynamics, Branched Transport problems, Energy Minimization

Abstract

The theory of Optimal Transport (OT) is an expanding area of mathematics that looks for the least-cost resource movement. Within this theory, the Branched Transport Problem (BTP) studies mass reallocation strategies where the functional to be minimizes encourages mass concentration along the transport paths. These problems lead to the formation of complex networks typically emerging in a number of natural environments, such as plant branches and roots, river networks, blood vessels, etc. Recent research efforts in the field of complex network often concentrated on conceiving appropriate functionals minimized by the observations. In BTP, on the other hand, networks naturally arise as solution of the problem of minimizing the BTP action principle. Application of the theory of OTP or BTP has achieved little success mainly for the lack of efficient numerical approaches.

In we proposed an infinite-dimensional extension of biologically inspired model introduced in which can be efficiently solved numerically. The model couples a diffusion equation with ODE imposing a transient dynamics to the diffusion coefficient. Numerical evidence and partial mathematical results suggest that these structures are solutions of a BTP problem, possibly non-standard. Again formal and non-rigorous mathematical developments allow us to identify the action-principle functional that is minimized by the long-time equilibrium solution of our model problem. The functional is able to capture the leading energy that shapes these optimal structures and its minimization is a trade off between the energy dissipated during the diffusive transport and the cost of building the transport network, assumed to be non-linearly proportional to the transport intensity (i.e. the diffusion coefficient). Moreover, the model is able to describe the network evolution using a quite simple dynamics. One of the most important advantages of the proposed formulation is that its numerical solution is very efficient and well-defined using simple discretization schemes.

In this work we apply our BTP approach to develop a plant root evolution model within a coupled Soil-Plant-Atmosphere simulator. Starting from these premises, we implement the above BTP model to simulate the plant root dynamics in response of water availability in the soil, assumed to be the resource to be transported from the soil to the xylem. It is believed that a fractal-like structure acting simultaneously in a large range of spatial scales will be able to simulate the behavior of large roots as well as root hairs, adapting dynamically to the soil water content as driven by external forcing. This model is coupled to the Richards-based CATHY simulator, which already contains an advanced optimization-based plant functioning model tested in several applications. For a given root density distribution, the vegetation model dynamically calculates the plant transpiration flux adapted to the current atmospheric forcings and the soil water/nutrient constraints and drives the photosynthesis process that transforms atmospheric carbon into plant biomass. This biomass, appropriately partitioned between above and below ground, contributes to the augmentation/reduction of the total root mass. The BTP-Richards model then calculates the optimal water/nutrient transport network as constrained by the soil water/nutrient availability and the total root biomass. More precisely, we identify the masses to be transported with the soil water availability (soil water content) as dynamically calculated by the Richards equation solver and the plant transpirative demand as calculated by the plant model, respectively. Starting from the previously calculated network structure, the BTP-Richards model calculates the new optimal root distribution using the additional constraint on the total root biomass calculated by the photosynthesis model. This nonlinear process requires tight iterations between the Richards equation solver and the BTP plant model to accurately capture the feedback between the biological and hydrological compartments.

Numerical results on synthetic test cases will complement the more theoretical results.

References


