A spectral approach to the Shallow Water Equations, and its implication for At-Many-stations Hydraulic Geometry

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Introduction

Solving the Shallow Water Equations (SWE) in 1-D or 2-D geometry requires inputs such as along-stream channel and floodplain bathymetry, friction parameters, and reach boundary conditions (typically, upstream inflow hydrograph and downstream rating curve). Bathymetry and friction data are not always easily available at a sufficient spatial resolution, e.g. for discharge-stage modeling at the scale of a full hydrographic network.

On the other side, from a geomorphological point of view it is unsatisfactory to have to fall back on crude asumptions such as uniform geometry without a longitudinal variability, regardless of the particular shape chosen (be it rectangular or more complex). Indeed, natural rivers do exhibit such variability as one moves downstream, an observation coined *downstream hydraulic geometry* (DHG ; see e.g. [1]). Most noticeably, channel slope decreases and channel width increases for a given, reference discharge quantile along-stream. However, a much shorter-scale (higher spatial frequency) variability is superimposed on this large-scale trend, with alternating *facies* or morphological units: shallow, fast-flowing stretches (riffles, or crossings between two meanders) alternate with deeper, low-velocity areas (pools, e.g. in the outsides of the bends of meanders). Such sequences typically occur at intervals of 5 to 10 times the bankfull width. For a given discharge, considered a constant at the reach scale, the variability in river cross-sections yields a variability in hydraulic variables (width, depth, hydraulic radius, velocities, etc.) but also in the energy slope. Consequently, any geometric variability "smoothed out" by a uniform approach will come back in the form of a bias in effective friction parameters, thus reducing the robustness of the model.

The parameterization of this variability of rating curves at the scale of a few morphological sequences, coined *at-many-stations hydraulic geometry* (AMHG), is a new research topic (see e.g. [2]). However, very few approaches based on the simple, 1-D steady-state flow equation (backwater curve) have tried to even qualitatively explain the emergence of AMHG and the longitudinal variability of rating curves at the scale of a few times the bankfull width. It is nonetheless likely that AMHG is the result of a coevolution of geometrical bed parameters [3], and of an emerging covariance structure between these parameters at the reach scale. We can take advantage of this covariance structure to reduce the dimensionality of the bathymetric model, without smoothing out the high frequency variability as in the uniform approach. At the extreme end, it can be represented by strictly periodic functions, allowing at the same time a quasi-analytical resolution of the backwater curve equation (Fourier transform, resolution in the frequency domain, and inverse transform), sending upstream and downstream boundary conditions to $\pm\infty$.

Outline of the study

We test this spectral treatment of AMHG on a dataset of 17 reaches of small French rivers extensively studied in [4]. In our approach, a reach is modeled as a succession of trapezoidal cross-sections, with periodic series for talweg depth anomaly, channel bank and floodplain lateral slope, banfkfull depth and bankfull width. The mean spatial period of the sequences is obtained using wavelet analysis. The approach allows us to reproduce a pattern of variability in rating curves at successive cross-sections, with only a handful of parameters describing the along-stream covariance structure of the main cross-sectional parameters. This parcimonious model is well-suited for inverse problems in which the inflow discharge has to be estimated using the sole measurement of water level, without a knowledge of low-flow bathymetry and friction parameters (e.g. in remotely-sensed discharge estimation).

We propose perspectives to extend the results obtained in 1-D in the case of a 2-D steady-state flow, using a conformal mapping of a periodic half-floodplain into a rectangular, computational domain where a spectral solution is to be calculated.

References

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