Finite Analytic Method for Fluid Flows in Heterogeneous Porous Media

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Natural reservoirs exhibit wide permeability variations. In this case, the nodal flow effects will lead the flow fingering to the high permeability region (see figure 1). It is a challenge problem to numerically describe the nodal fingering effects. With the traditional numerical scheme, refining the coarse grid enough is the only manner to describe the flow pattern accurately.

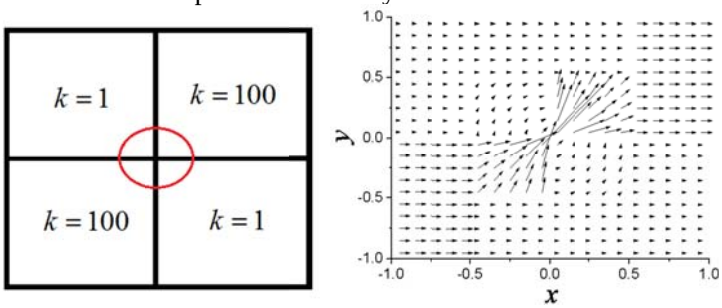

Figure 1: Velocity vector field on the 2×2 checkerboard for permeability ratio $k_2/k_1=100$.

For 2D problem with the heterogeneous scalar permeability distribution, enlightened from the numerical calculations on the flow pattern in the canonical 2×2 block with checkerboard geometry, which show that the pressure and its spatial gradient exhibit the power-law behaviors and especially the pressure gradient will tend to infinity when approaching the node joining different permeability domains, a local analytical nodal solution is derived. Based upon the analytical nodal solution, a finite analytic scheme is formulated for solving the fluid flows in 2D heterogeneous porous media. Numerical examples show that the detailed flow pattern can be reconstructed with the proposed numerical scheme under few grid refinements. Especially, only with 2×2 or 3×3 subdivisions, the proposed numerical scheme can provide rather accurate solutions. The convergent speed of the numerical scheme is independent of the permeability heterogeneity (see figure 2). In contrast, when using the traditional numerical schemes to simulate flow through a strong heterogeneous porous medium, the refinement ratio for the grid cell needs to be increasing dramatically to get an accurate result [1,2].

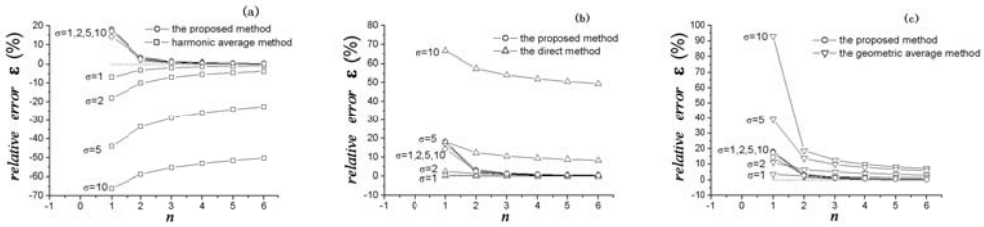

Figure 2: Plot of the relative errors of the equivalent permeabilities as a function of the grid refinement parameter *n*, calculated from different numerical schemes.

For 3D problem with the heterogeneous scalar permeability distribution, it is pointed out that the 3D flow will reduce to the 2D one in the neighborhood around each edge. Based upon the quasi-2D hypothesis, the 3D finite analytical numerical scheme can be constructed. Only with 2×2×2 or 3×3×3 subdivisions, the proposed numerical scheme can provide rather accurate solutions. The convergent speed of the numerical scheme is independent of the permeability heterogeneity (see figure.3). In contrast, when using the traditional numerical schemes to simulate flow through a strong heterogeneous porous medium, the refinement ratio for the grid cell needs to be increasing dramatically to get an accurate result. [3].

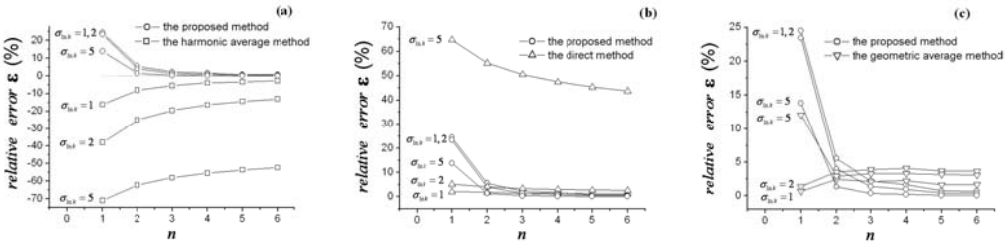

Figure 3: Plot of the relative errors of the equivalent permeabilities as a function of the grid refinement parameter *n*, calculated from different numerical schemes.

The finite analytic method is also developed to solve the 2D and 3D fluid flow in heterogeneous porous media with permeability in tensor form [4,5]. For the unstructured grids, a finite analytic scheme is constructed based upon the power-law analytic nodal solution in the angular domain with arbitrary shape. When approaching the grid node joining the subdomains, three different flow patterns may exist: power-law flow, linear flow or the stagnant flow. Numerical examples show that the proposed numerical scheme makes the convergences much quickly than the traditional methods [6].

For multi-phase flow in porous media, we construct a finite analytic numerical scheme [7]. Numerical examples show that the proposed scheme makes the convergences much faster as the refinement parameter increases, and the accuracy is independent of the heterogeneity. In contrast, when using the traditional numerical schemes to simulate flow through a strong heterogeneous porous medium, the refinement ratio for the grid cell needs to increase dramatically to get an accurate result (See figure. 4).

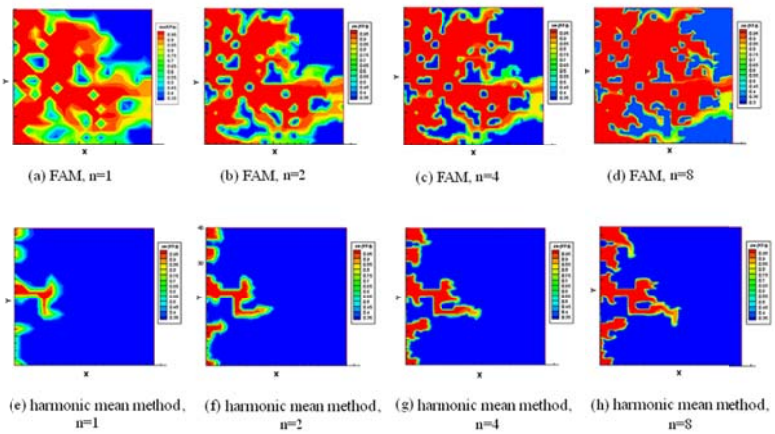

Figure 4: The saturation field calculated from the FAM and traditional IMPES under different grid refinement parameter n.

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