Flow of non-Newtonian fluids through porous media: Impact of structure and rheology on the apparent permeability

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Introduction

Although the flows of complex fluids through porous media are found in a wide variety of applications, *e.g.* enhanced oil recovery [1], many aspects of the fundamental physics of such flows are still poorly understood. In case of polymer solutions, there are mainly two modeling possibilities used at macroscale, which are both derived from Darcy's law. The first one uses an apparent permeability tensor [2, 3],

$$\langle \mathbf{v} \rangle = -\frac{\mathsf{K}_{\mathrm{app}}}{\mu_0} \cdot \left(\nabla \langle p \rangle^{\beta} - \rho \mathbf{g} \right), \tag{1}$$

with $\langle \mathbf{v} \rangle$ the average superficial velocity, μ_0 the fluid Newtonian viscosity, $\nabla \langle p \rangle^{\beta}$ the gradient of intrinsic pressure and $\rho \mathbf{g}$ the gravity term. The second one consists in replacing the ratio $\mathsf{K}_{\mathrm{app}}/\mu_0$ in Eq. 1 by $\mathsf{K}_0/\mu_{\mathrm{app}}$ with μ_{app} an apparent viscosity and K_0 the intrinsic permeability tensor [4, 5].

The difference between these two corrections is that the first one is tensorial whereas the second one is purely scalar. We note that for a fixed source term $\nabla \langle p \rangle^{\beta} - \rho \mathbf{g}$, there is *a priori* no reason for the orientation of the non-Newtonian average velocity and the Newtonian average velocity to be identical. The first approach offers the possibility to describe the case where these orientations are different while the second approach does not. Therefore, the tensorial approach is more general.

The model with the apparent viscosity, although the simplest, is the one that is used in petroleum engineering [6]. In these, an heuristic formulation based on the bulk viscosity law is used to define μ_{app}. Besides the fact that this approach is unable to represent a potential anisotropy induced by the non-Newtonian effects (as discussed above), it also presents several problems. The aim of this study is to understand these problems by linking the pore-scale physics and the continuum macroscale models using an upscaling procedure.

The upscaling procedure

Upscaling procedures are generally used to derive macroscale laws directly from the pore-scale physics. For instance, Whitaker in [7] used the volume averaging method (VAM) to derive the classical Darcy's law [8]. To do so, the creeping and incompressible flow of a Newtonian fluid through a porous medium is theoretically investigated. This procedure yields an intrinsic permeability K_0 that is a function of closure variables and can be calculated for a given porous structure. While, for linear flows, the link between the pore-scale physics and the macroscale models is now well understood, the same is not true in the nonlinear case, such as non-Newtonian flows. The main issue is that VAM usual simplifications rely heavily on local flow linearity, which cannot be used in our case.

In this study, we develop an upscaling procedure based on VAM. This method guides us to postulate the following form for the apparent permeability

$$\mathsf{K}_{\mathrm{app}} = k_{\mathrm{n}} \mathsf{P} \cdot \mathsf{K}_0, \tag{2}$$

with k_{n} a scalar and P a rotation tensor. Under this specific form of Darcy's law, we study the behavior of $k_{\mathrm{n}}\mathsf{P}$ using simulations over a broad range of porous structures. More specifically, we wish to investigate the effects of both the disorder, inherent to the porous structure, and the non-linearity, inherent to the non-Newtonian rheology, on the macroscale behavior, which is characterized by $k_{\mathrm{n}}\mathsf{P}$.

The framework

We limit our study to the steady and creeping flow through a rigid porous medium. The pore-scale equations read

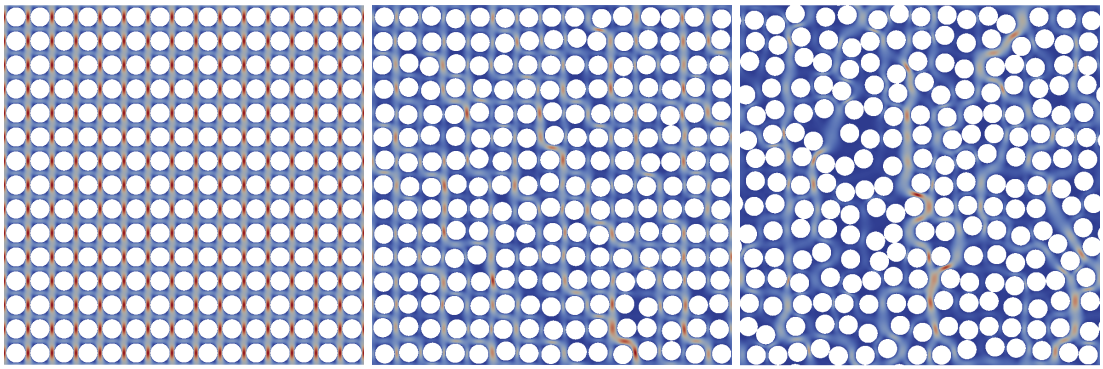
$$\nabla \cdot \left[\mu\left(\dot{\gamma}\right) \left(\nabla \mathbf{v} + \left(\nabla \mathbf{v}\right)^{\mathsf{T}} \right) \right] - \nabla p + \rho \mathbf{f} = 0, \tag{3}$$

and $\nabla \cdot \mathbf{v} = 0$ where $\rho\mathbf{f}$ is the source of the momentum, $\mathbf{v}$ the velocity field and p the pressure field. We choose for $\mu\left(\dot{\gamma}\right)$ the following model,

$$\mu\left(\dot{\gamma}\right) = \begin{cases} \mu_0 & \text{if } \dot{\gamma} < \dot{\gamma}_c, \\ \mu_0 \left(\frac{\dot{\gamma}}{\dot{\gamma}_c} \right)^{n-1} & \text{else,} \end{cases} \tag{4}$$

which is convenient for polymer solutions. Finally, we consider a simple no-slip condition at the solid/liquid interface.

The porous structures that we consider are 2D arrays of cylinders. A parameter, σ, characterizes the disorder. Examples of geometries with various degrees of disorder are presented in Figs. 1a to 1c.



(a) $\sigma = 0$ (b) $\sigma = 0.06$ (c) $\sigma = 0.25$

Figure 1: Newtonian velocity fields in arrays of cylinders for different disorder values.

The results

Results show a competition between disorder and nonlinearity. The main result is that, while the nonlinearity tends to deviate the velocity direction from the associated Newtonian direction, disorder tends to weaken this effect. If the impact of the porous structure dominates over the nonlinearity, which is the case for sandstone, we find that a scalar correction such as μ_{app} to Darcy's law is sufficient. This seems to indicate that an apparent viscosity may be adapted to reservoir simulators in the general case, unless some specific anisotropy features are encountered. Several issues associated to the concept of apparent viscosity remain and should be investigated.

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