Extension and Application of Approximate Commutator Preconditioners to Two-Phase Incompressible Flow

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Introduction

Iterative solution of the linear systems arising from the incompressible Navier-Stokes equations is frequently the most significant expense of multi-physics hydrodynamics models. A common approach to accelerating the solution of this sub-problem uses splitting of the pressure and velocity at the level of the time discretization [6]. While such splitting (or projection) schemes are very effective on dynamic problems, where relatively small time steps can be taken, steady state problems and implicit time integration methods require preconditioning the full pressure-velocity system in order to obtain weak parallel scalability.

Scalable preconditioners for the fully-coupled pressure-velocity problem, based on reduction to the pressure Schur complement and approximate commutator, have been developed [3]. Unfortunately these methods do not perform well when extended directly to two-phase incompressible flows with large density and viscosity jumps across phase interfaces, such as in air/water flows [2].

Rescalings of Approximate Commutator Approaches

Recently, two such approximate commutator approaches, the Least Squares Commutator and Pressure Convection-Diffusion preconditioners [3], were extended to two-phase flow based on novel scalings [2]. The multi-phase versions of the preconditioners draw on an analogy with earlier research on fast solvers for generalized Stokes equations with variable density [4]. In this work we present parallel scaling results for these new preconditioners relative to several simpler but reliable approaches, such as additive Schwarz preconditioning of the full system and a simple-type preconditioner [5].

Boundary Conditions

A significant issue with physics-based preconditioning approaches to the Navier-Stokes equations based on the continuous operators is properly accounting for real-world boundary conditions such as open hydrostatic pressure boundaries. In this work we study both classical strong enforcement of such boundaries and weak enforcement via Nitsche-type penalty methods.

Time Dependence

Another challenge to applying more sophisticated preconditioning techniques to complex problems arises from the variation in time scales due to changing solution dynamics, such as wave slamming on solid boundaries as shown in Figure 1. Since more sophisticated approaches are also typically more expensive on a per-iteration basis, it may be the case that one preconditioner is not effective across the entire range of simulations. In this work we study the preconditioner on a range of problems arising from wave and hydraulic flume tests. These tests were developed initially for verification and validation of the two-phase flow model, but instrumenting preconditioner performance on such complex problems also provides insight into strategies for dynamic preconditioner solution.

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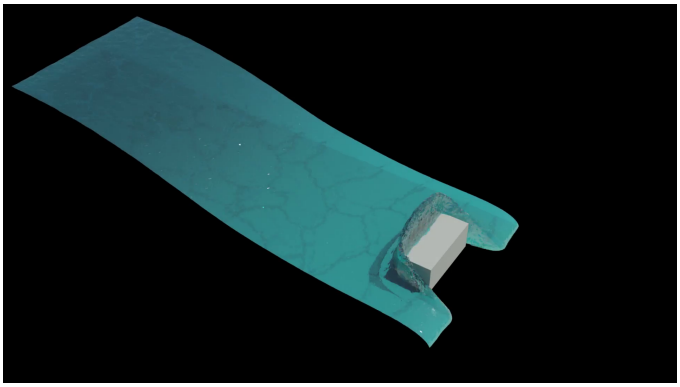

Figure 1: Two-phase flow model of wave slamming into a solid obstacle

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