Hybrid-dimensional two-phase flow in fractured porous media with interface solver

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We consider in this work, the extension to two-phase Darcy flows of the discontinuous pressure model introduced in [9, 3] in which the $(d-1)$-dimensional flow in the fractures is coupled with the d-dimensional flow in the matrix. The model presented in [4, 6] (see also [1]) accounts accurately for gravity effects inside the fractures, for discontinuous capillary pressure curves at the matrix fracture (mf) interfaces and for both drains and barriers. This can be achieved thanks to interfacial pressure and saturation unknowns accounting for the nonlinear two-phase Darcy flux conservation equations at mf interfaces.

Most alternative hybrid-dimensional two-phase Darcy flow models are based on the assumption of continuous phase pressures at the mf interfaces assuming fractures acting as drains [5, 11, 10, 7, 2]. Another alternative approach, accounting for both drains or barriers, amounts to eliminate the interfacial phase pressures using the linear single phase Darcy flux conservation equation at the mf interfaces for each phase. It is usually combined with Two Point [8] or MultiPoint cell centred finite volume schemes for which the interfacial unknowns can be easily eliminated when building the single phase Darcy flux transmissivities. It is important to notice that, in the case of fractures acting as drains, this approach provides basically the same solutions as the ones obtained by the continuous phase pressure assumption at mf interfaces.

To illustrate the importance of keeping the mf interface pressure and saturation unknowns, let us consider the test case exhibited in Figure 1. Figure 2 compares to a reference solution provided by the equi-dimensional model, the solutions obtained using the two-phase hybrid-dimensional mf nonlinear model presented in [4, 6], the two-phase hybrid-dimensional mf linear m upwind model presented in [5, 11, 10, 7, 2] and the two-phase hybrid-dimensional mf linear f upwind model presented in [8].

On the other hand, keeping the mf interface pressure and saturation unknowns and equations increases the difficulty to solve the nonlinear and linear systems at each time step and at each Newton iteration of the simulation. These difficulties come from the highly contrasted permeabilities and capillary pressures between the fracture and the matrix and from the vanishing absent phase mobility in single phase zones combined with the absence of porous volume at the interface leading to singular systems. One possible cure is to add a very small volume at mf interfaces. This avoids the singularity of the Jacobian matrix but still leads to very ill-conditioned systems. Alternatively, we investigate in this work the possibility to eliminate the interfacial unknowns at the nonlinear level through a rigorous mathematical analysis of the local interfacial nonlinear problem obtained from the nonlinear two-phase Darcy flux conservation equations at a given mf interface combined with the jump condition for the saturations.

References

[1] Ahmed, E., M., Jaffré, J., Roberts, J. E., *A reduced fracture model for two-phase flow with different rock types, Mathematics and Computers in Simulation*, 7, pp. 49-70, (2017).

[2] Brenner, K., Groza, M., Guichard, C., Masson, R. *Vertex Approximate Gradient Scheme for Hybrid-Dimensional Two-Phase Darcy Flows in Fractured Porous Media.* ESAIM Mathematical Modelling and Numerical Analysis, 49, pp. 303-330, (2015).

[3] K. Brenner, J. Hennicker, R. Masson, P. Samier, *Gradient Discretization of Hybrid Dimensional Darcy Flows in Fractured Porous Media with discontinuous pressure at matrix fracture interfaces.* IMA Journal of Numerical Analysis, 37, 3, 15511585, (2017).

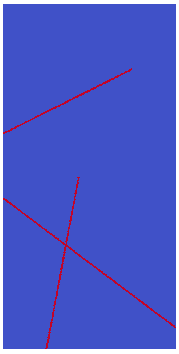

Figure 1: 2D reservoir $\Omega = (0,4) \times (0,8)$ m with 3 fractures of thickness 4 cm. The matrix and fracture permeabilities are isotropic and set to respectively $K_m = 10^{-13}$ m^2 and $K_f = 10^{-10}$ m^2. The reservoir is initially saturated with water and oil is injected from the bottom fracture at fixed flow rate. The capillary pressure is set to $P_{c,m}(s^0) = -10^3 ln(1-s^o)$ in the matrix and to $P_{c,m}(s^0) = 0$ in the fractures.

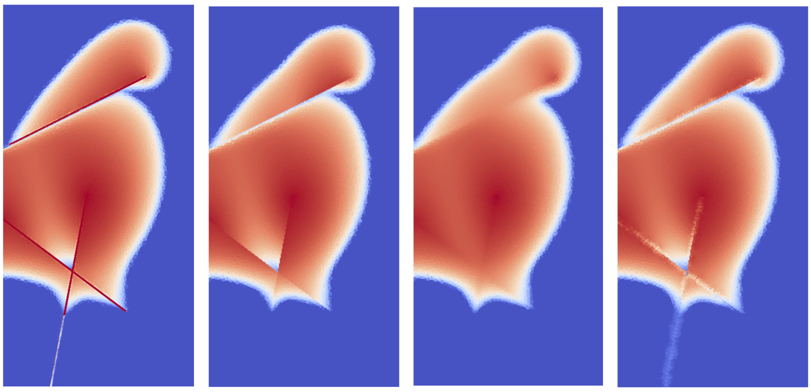

Figure 2: Oil saturation at final time obtained using from left to right: the two-phase equi-dimensional model (reference), the two-phase hybrid-dimensional mf nonlinear model, the two-phase hybrid-dimensional mf linear m upwind model and the two-phase hybrid-dimensional mf linear f upwind model.

[4] K. Brenner, J. Hennicker, R. Masson, P. Samier, *Hybrid-dimensional modelling of two-phase flow through fractured porous media with enhanced matrix fracture transmission conditions*, 2017, `https://hal.archives-ouvertes.fr/hal-01518930`

[5] Bogdanov, I., Mourzenko, V., Thovert, J.-F., Adler, P. M., *Two-phase flow through fractured porous media*, Physical Review E 68, 026703, (2003).

[6] J. Droniou, J. Hennicker, R. Masson, Numerical analysis of a two-phase flow discrete fracture model, 2016, `https://hal.archives-ouvertes.fr/hal-01422477`

[7] Hoteit, H., Firoozabadi, A. *Numerical modeling of two-phase flow in heterogeneous permeable media with different capillarity pressures.* Advanced Water Resources 31, pp. 56-73, (2008).

[8] Karimi-Fard, M., Durlovski, L.J., Aziz, K. *An efficient discrete-fracture model applicable for general-purpose reservoir simulators*, SPE journal, june (2004).

[9] V. Martin, J. Jaffré, J. E. Roberts, *Modeling fractures and barriers as interfaces for flow in porous media*, SIAM J. Sci. Comput. 26, 5, 1667-1691, (2005).

[10] Monteagudo, J.E.P. and Firoozabadi, A. *Control-Volume Model for Simulation of Water Injection in Fractured Media: Incorporating Matrix Heterogeneity and Reservoir Wettability Effects*, SPE Journal 12, 3, (2007).

[11] Reichenberger, V., Jakobs, H., Bastian, P., Helmig, R.: *A mixed-dimensional finite volume method for multiphase flow in fractured porous media.* Adv. Water Resources 29, 7, pp. 1020-1036, (2006).