Well-Balanced, Positivity Preserving, Second-Order Finite Element Approximation of the Shallow Water Equations with Friction

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Abstract

The Shallow Water Equations (SWEs) are popular for modeling non-dispersive incompressible water waves where the horizontal wavelength is much larger than the vertical scales. They can be derived from the incompressible Navier-Stokes equations assuming a constant vertical velocity. The SWEs are important in Geophysical Fluid Dynamics for modeling surface gravity waves in shallow regimes; e.g., in the deep ocean. Some common geophysical applications are the evolution of tsunamis, river flooding and dam breaks, storm surge simulations, atmospheric flows and others.

This work is concerned with the approximation of the time-dependent Shallow Water Equations with friction. We use a high-order explicit Strong Stability Preserving time stepping and continuous Galerkin finite elements. We present a method that is second-order accurate in space and third-order accurate in time, positivity preserving, well-balanced with respect to rest states, well-balanced with respect to steady sliding solutions on inclined planes and robust with respect to dry states. Although common in the finite volume literature, to the best of our knowledge, schemes with the above properties are not well developed in the context of continuous finite elements.

The method is based on a finite element method, by [1], that is positivity preserving and well-balanced with respect to rest states. We extend it by:

1. Incorporating high-order artificial viscosity via the entropy viscosity method. By doing this we improve the accuracy and deal with loss of accuracy around local extrema.

2. Introducing a singular Manning friction term handled via an explicit discretization under the usual CFL condition.

3. Introducing a water height regularization consistent with the polynomial approximation.

4. Reducing dispersive errors introduced by lumping the mass matrix.

These modifications, however, introduce loss of positivity. This deficiency is handled via the Flux Corrected Transport methodology by [2] and [5] using either a global bound on the water height or local bounds, proposed in [3], based on the low-order solution. We present the details and results of both approaches.

After presenting the details of the method we show numerical tests that demonstrate the well-balanced nature of the scheme and its convergence properties. We conclude with well-known benchmark problems including the Malpasset dam break (see figure 1). All numerical experiments are performed and are available in the Proteus toolkit 1.4.2 [4], which is an open source python/C++ package for modeling continuum mechanical processes and fluid flow.

References

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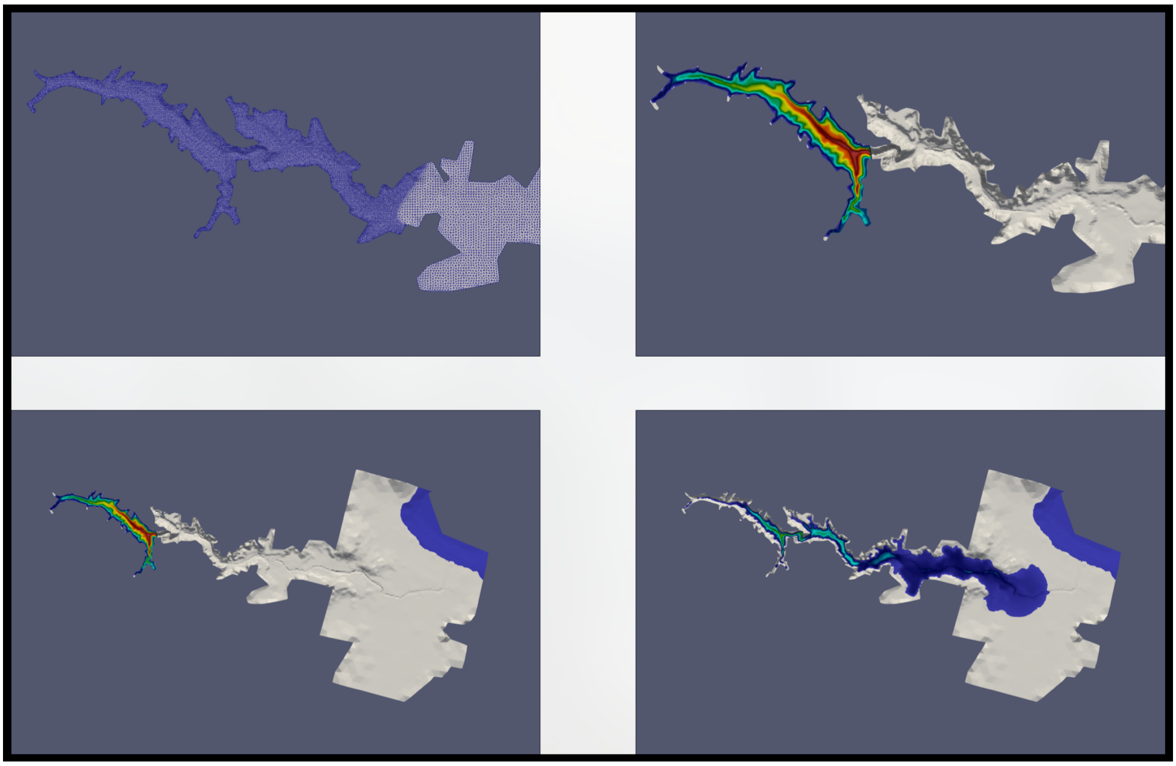

Figure 1: Malpasset dam break problem.

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