A precipiton method for solving the shallow water equations with erosion and sediment transfer

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Short abstract: The "precipiton" method consists of routing elementary water volumes on top of topography with erosive and depositional actions. Here we present an original way to calculate both river depth and velocity from a method that remains embedded in the precipiton framework. We have applied the method to different cases including high-resolution LIDAR topography. When coupled with erosion and sediment transport equations, the model is able to reproduce both straight and braided patterns with geometries independent of grid size.

Introduction

The precipiton method is a particle-based approach, which mimics the role of precipitation (precipiton = elementary rainfall volume) on shaping topography [1, 2]. Together with cellular methods, they have been popular for mimicking self-organized emerging properties of geomorphological systems, from high-resolution braided patterns to drainage network organization. Solving hydrodynamics constitutes a major difficulty for cellular automata and precipiton methods, although it is critical to enable water to spread laterally over submerged banks and, for the channel, to maintain a non-vanishing width (i.e. larger than the grid pixel).

In this paper, we resolve this limitation by calculating both river depth and velocity from a method embedded in the precipiton framework, thus maintaining its computing efficiency. To our knowledge, this approach is the first attempt to solve the shallow water equation with precipiton methods in landscape evolution models.

Introducing the shallow water equation in the precipiton framework was carried out in two steps: (i) particles/precipitons move on top of the water surface, as is expected from the shallow water equation without the inertia terms, and (ii) the precipitons interact with the water surface to build up the water depth. For the latter step, each cell is filled up by precipitons when they flow into the cell, with precipiton subsequently leaking out at a rate which is given by the friction law. The water depth evolution is formally described by a stochastic differential equation that is solved each time a precipiton enters a cell. It produces stochastic values of the local water depth, whose average is the one expected. At each cell, the precipiton volume is updated to maintain the water balance.

The method is efficient to manage local minima, lakes or dam reservoir. We test it for straight channels, a dam, and a high-resolution DEM (figure 1).

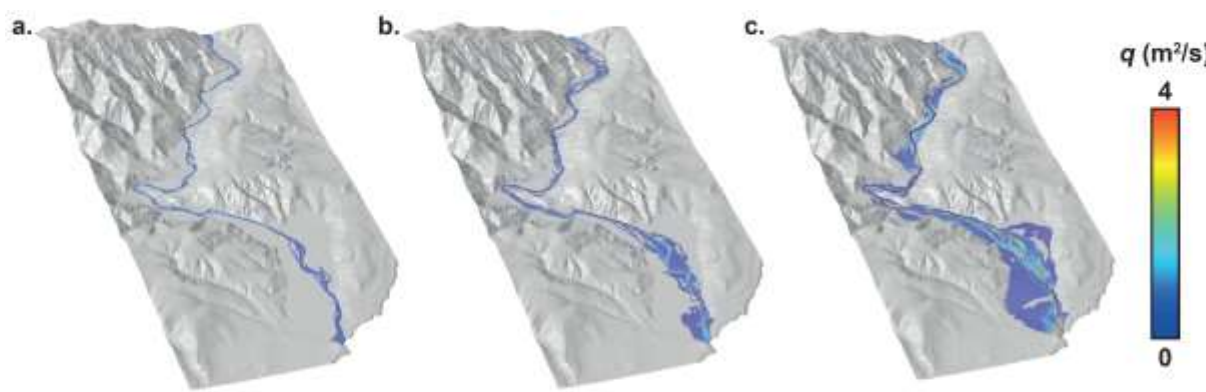

Figure 1: Picture of the predicted flood extent for different discharges in the Cruz DEM. a. Q_{in}=40 m^3s^{-1}. b. Q_{in}=150 m^3s^{-1}. c. Q_{in}=500m^3s^{-1}. *q* is the specific discharge.

Coupling of the hydraulics with erosion/deposition equations is straightforward: in addition to defining the water depth, particles erode the riverbed and banks, transport and deposit sediments [3]. A variety of erosion/transport equations can be implemented in this scheme including lateral erosion, and lateral depositional fluxes.

A few examples of channel formation have been presented to illustrate the method. In tests conducted, the code shows no dependence of the final solution on grid size. The approach also successfully creates both straight and braided channels. The experiments shown in this paper demonstrate some interesting features regarding braiding instability that deserve further investigation, such as conditions for developing braiding patterns, the role of deposition length, the fact that channels are under capacity, and the role of lateral deposition fluxes (Figure 2).

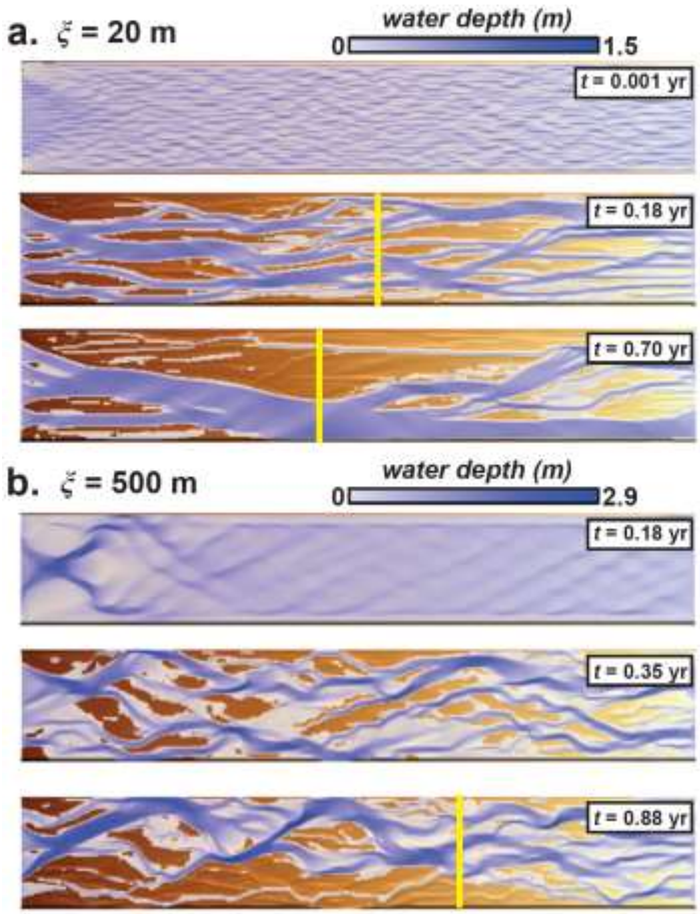

Figure 2. Top views of the channel patterns for both simulations with sediment recirculating conditions (inlet on the left, outlet with constant topography on the right). The blue color scale is related to water depth, and the brown color corresponds to locations with no flow.

References

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