Comparison of POD-based iterative solvers for the solution of porous media flow problems

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The equations describing e.g. reservoir simulation or groundwater flow problems, after discretization, can be transformed into systems of linear equations. If the problem is large or ill-conditioned, i.e. the resulting matrix has a large condition number, the solution of the system, through preconditioned iterative solvers, is time consuming. Therefore, it is necessary to find possible ways to accelerate this process.

Recently, Proper Orthogonal Decomposition (POD) [1–4], based on information obtained from the system, has been studied to obtain an acceleration of iterative methods for large systems of equations. The capture of a series of snapshots, solutions of the system with slightly different characteristics, is necessary to construct the POD basis. Astrid et al. [2] propose the capture of system information with POD in an offline phase. After the information is obtained, it is reused for the acceleration of problems with slightly modified parameters. This approach is particularly useful for optimization or history matching problems.

Other acceleration approaches are studied by Markovinovic et al. [1], who propose the use of the solution computed with POD methods to find a more accurate initial guess and by Pasetto et al. [3], who propose the construction of a preconditioner based on the reduced model for the acceleration of a Krylov-subspace iterative method. When the system is ill-conditioned, an acceleration with deflation techniques is also possible [5–7]. For an optimal performance of the deflation techniques, it is necessary to find good deflation vectors. If a good selection of these vectors is made, only a small increase in the required computing time per iteration and an important decrease in the number of iterations is achieved.

Diaz Cortes et al. [7] use a POD basis as deflation vectors and they prove that, if the system is completely described by the snapshots, the solution is achieved in one iteration of the deflation method, for an incompressible single-phase problem. If the linear system is changing, as is the case for a compressible problem, a considerable decrease in the number of iterations is obtained.
In the compressible case, the information acquired with the POD method is used to construct the deflation-subspace matrix. In this work, we explore the use of a POD basis as deflation-subspace matrix for the acceleration of a Krylov subspace method. The deflation-subspace matrix is used in two different approaches. The first approach is the use of this matrix as preconditioner, following the methodology proposed by Passeto et al. [3]. For the second approach, we use the deflation-subspace matrix for a standard deflation procedure [5–7]. We study the differences and similarities for these two approaches from a theoretical point of view and on a series of computational examples. The two approaches are compared in terms of number of iterations and computational cost for the solution of the linear systems resulting from simulation of flow through porous media with a large number of unknowns and high condition number.

References


