Calculating elastic properties of rock masses from fracture network models

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Short abstract: We derive the relationships that link the general elastic properties of rock masses to the geometrical properties of fracture networks, with a special emphasis to the case of frictional crack surfaces. We extend the well-known elastic solutions for free-slipping cracks to fractures whose plane resistance is defined by an elastic fracture (shear) stiffness k_s and a stick-slip Coulomb threshold. The results were applied to power-law fracture size distributions, which are likely relevant to geological cases.

Introduction

Assessing the effective mechanical properties of rock masses is a prerequisite to many geotechnical applications, and a still major scientific issue about the way to take account of the heterogeneities of rock mass. Amongst all potential heterogeneities, fractures are those whose impact on rock strength and stiffness is prevalent [1], with the difficulty that the distribution of fractures is complex with a wide range of spatial scales involved, and highly variable densities in space. The quantitative analysis of these expected consequences are well established for simple cases, as a single frictionless disk crack embedded in an homogeneous elastic rock matrix, but the properties of rock masses with a complex set of fractures are still an issue. Expressions have been derived for networks of frictionless cracks with limited size range by neglecting or simplifying stress interactions (see review in [2-4]). The application of these theories to actual rock masses raises two main issues. The first is about the role of frictional stresses in the damaged elastic modulus, since friction is likely prevailing on large geological fractures; the second rocks is the intrinsic complexity of fracture networks, which results in a wide range of fracture sizes from micrometer to kilometer scales [7].

We extend the well-known elastic solutions for free-slipping cracks to fractures whose plane resistance is defined by an elastic fracture (shear) stiffness k_s and a stick-slip Coulomb threshold. Together with the elastic matrix Young's modulus and Poisson's ratio, k_s defines a characteristic fracture size l_M (called the mechanical length), below which the rock mass elastic behavior is dominated by the rock matrix deformation, and above which it is controlled by the resistance on the fracture plane. A complete set of analytical solutions have been derived for the shear displacement in the fracture plane for stresses below the slip threshold and above, including the variations of displacement in the fracture plane and the relationship between stress and average displacement. All the expressions have been checked with numerical simulations. We show that the effective theory (ET) gives a very good approximation of the bulk elastic properties. ET can be calculated analytically by introducing fractures one by one and assuming that the surrounding elastic matrix has the property of the bulk damaged medium.

We also derive a simple expression of the stress partitioning between the resistances of the fracture plane in the one hand, and of the elastic matrix in the other hand. We demonstrate that the stress conditions on the fracture plane define a "stress ellipse", which derives from the remote Mohr's circle. The remote conditions for triggering slip must take into account not only the resistance of the fracture plane but also the mechanical resistance of the surrounding matrix. This has both consequences: *i*) the remote stress threshold is larger than the fracture plane stress threshold by a ratio $(k_s + k_m)/k_s$, where k_m is the matrix-fracture stiffness (i.e. the ratio between stress and displacement for free-slipping fractures); and *ii*) the angle at which fracture can slip must be predicted from the "stress ellipse" rather than from the remote stress Mohr's circle.

The Young's modulus and Poisson's ratio were also derived for a rock mass with a population of fractures, with the intrinsic difficulty to describe properly the fracture interactions. In the case of large k_s values ($k_s \gg k_m$), the bulk elastic modulus is controlled by the total fracture surface, and more precisely by the ratio p_{32}/k_s, where p_{32} is the total fracture surface divided by the sample volume; this is the case of fractures larger than the mechanical length l_M. This result differs from the slipping case ($k_s \ll k_m$), where the elastic moduli is controlled by the percolation parameter, i.e. by the third moment of the fracture size distribution; this is the case for fractures smaller than l_M. For a complete fracture size distribution, the elastic modulus can be efficiently deduced from a combination of these density parameters, providing that they be calculated for the right subset of the fracture size distribution: p_{32}/k_s for large fractures ($> l_M$), and p for small fractures ($< l_M$).

These results were applied to power-law fracture size distributions, which are likely relevant to geological cases. We show that, if the power-law fracture size exponent is in the range -3 to -4, which corresponds to a wide range of geological fracture networks, the elastic properties of the bulk rock are almost exclusively controlled by k_s and l_M, meaning that the fractures of size l_M play a major role in the definition of the elastic properties. In hardrock geological systems (gneissic or granitic), we estimate l_M to range between 1 and 50 m.

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