

# Using graph theory to increase computational efficiency of discrete fracture network models

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**Key words:** Discrete Fracture Networks, Graph Theory, and Solute Transport

## Introduction

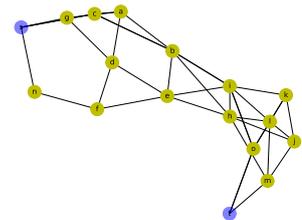
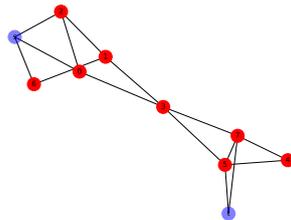
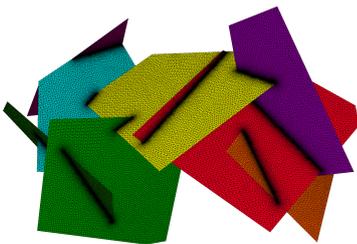
Discrete fracture network (DFN) models have been used to simulate flow and transport through fractured systems, since the structure and topology of the fracture network must be accounted for in these systems. Recent advances in high performance computing have made possible transport simulations in large discrete fracture networks [1, 2, 3, 4, 5, 6].

The increase in model fidelity of large DFNs results in high computational cost due to the number of mesh elements required to accurately represent thousands of fractures with sizes that can span 2-4 orders of magnitude. In addition, multiple realizations of the DFNs model are needed to bound system uncertainty stemming from hydrologic, geometric and topological considerations.

In this talk we utilize graphs, which require far fewer degrees of freedom to capture structural information, to increase the computational efficiency of our DFN work flow. We also explore graph-based reduced order models of a DFN with the goal of producing efficient and accurate DFN emulators.

## Methodology

In order to utilize graphs to reduce computational burden of a DFN or to develop a graph-based reduced order model, the DFN must be mapped to a graph and different mappings are possible. Figure 1 illustrates a simple 8 fracture DFN and two possible graph representations that result from the DFN. We demonstrate that for topological questions that provide information on the connectivity and structure of a fracture network, representing each fracture in a DFN as node and each fracture intersection as an edge is an effective representation (Figure 1 (b) ). For hydrological questions such as relating fractures to parameters like permeability, representing intersections as nodes and fractures as a collection of edges allows for geometric and hydrological properties to be mapped onto the graph edges (Figure 1 (c) ).



(a) Simple DFN with 8 Fractures

(b) Topological Mapping

(c) Hydrological Mapping

*Figure 1:* (a) Illustrative DFN with 8 fractures. (b) Each fracture is represented by a node and each edge is an intersection of fractures. (c) Each fracture intersection is represented by a node and fractures are a clique of edges.

## Results

For DFN transport modeling the breakthrough curve is a typical quantity of interest that describes the solute arrival of the transported species as a function of time as it crosses the outlet of the model. We find that the best mapping from DFN to graph depends on the question being asked.

We demonstrate that graphs can be used to increase the computational efficiency of discrete fracture network (DFN) simulations using techniques such as pruning the DFN, partitioning and bandwidth reduction. In addition, as an alternative to computationally prohibitive DFNs, we develop graph-based reduced order models of a DFN that produce efficient and accurate DFN emulators that can reduce computational cost by 2-4 orders of magnitude.

For problems such as hydraulic fracturing or environmental remediation, the entire breakthrough curve is typically of interest in order to measure production or remediation efficiency, respectively. In addition the tail

of the breakthrough curve is often of particular interest for these applications. For nuclear nonproliferation problems, often it is only of interest to detect early mass arrival of the breakthrough curve to determine when to fly a plane over head to attempt to detect gas seepage from the subsurface. Finally, for many applications where fracture characteristics are only known statistically and numerous realizations are needed to bound flow and transport in the system, a reduced order model of a DFN need not reproduce the breakthrough curve of the DFN exactly but only within a prescribed error bound specified by the subject matter expert. We demonstrate that through the choice of mapping and graph-based technique we can tune our reduced order models to accurately predict specific quantities of interest with orders of magnitude more efficiency than the high-fidelity DFN.

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