Upscaling of pore scale transport: ergodicity and stationarity of Lagrangian velocities and their representation as a Markov process

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Abstract

The sound understanding of pore-scale particle statistics is key for the upscaling of dispersion, mixing and reaction processes from the pore to the Darcy scale. We present a detailed study of particle transport in the flow through a pore-structure obtained from x-ray microtomography of a Berea sandstone sample [1], see Figure 1. Flow is solved numerically using Openfoam, transport by particle tracking. We focus on the statistics of the magnitude of Lagrangian velocity measured equidistantly and isochronically along streamlines. As observed by other authors [2], the particles’ temporal velocity series exhibit intermittency which makes difficult its modeling in time. This intermittency can be traced back to the fact that particle velocities change over a characteristic length scale of the order of a pore length.

In order to characterize the transport we then define the Lagrangian and Eulerian velocity PDFs. We first define the streamwise velocity in time and in space which respectively sample the velocity along a streamline isochronically or equidistantly. Then we define the ensemble spatial and temporal Lagrangian velocity PDFs which samples the velocity of all the streamlines after a distance $s$ has been traveled or a time $t$ has passed. The spatial and temporal Eulerian PDFs, in turn, are respectively defined by the Eulerian velocities observed in the domain occupied by the particles after distance $s$ or time $t$. We investigate the relationships between all these PDFs and find out that the temporal and spatial ones are linked through a flux weighted relation.

In order to gain insight into the stochastic dynamics of Lagrangian velocities, we study the ergodicity and the stationarity of the velocity statistics through the PDFs previously obtained and find out that our velocity process is ergodic and becomes stationary after a few pores, see Figure 2. These statistics allow us to use a simplified upscaled model since their are characterizing the system for all time $t$ or spatial position $s$.

The quick convergence of these velocity distributions in space leads us to propose a new definition of an REV: a sample is considered an REV if the velocity statistics inside are ergodic and if the sample is large enough for the velocity PDF in space to become stationary before the end of the sample. Note that this definition does not imply that the transport properties inside a REV are Fickian and the flow can therefore exhibits an anomalous behavior. We then propose two distinct 1D Markov models [3], the first one based on the transition velocity matrices computed for the 3D direct simulations which will select the velocity at a step in function of the one at the previous step; and a simplified version based on the use of the characteristic distance $l_c$ and on an accurate computation of the point Eulerian velocity distribution. The two models produce comparable results, are able to reproduce pore-scale results and to predict higher scales transport and velocity evolution.

References


Figure 1: Twenty particles passing through the 1mm$^3$ 3D Berea sandstone used. The black color represents the pore space while the grey color represents the solid parts. The change of color indicates the velocity changes. Two compute accurate velocity statistics, the simulations are usually performed with millions of particles.

Figure 2: Evolution of the velocity probability density function in space. The distribution evolves from $s = 0$ (green crosses curve) to $s = 5l_c$ (Black triangles curve). A correlation length $l_c$ represents the average pore length and the average distance needed for the velocity to decorrelate. The sample is about $10l_c$ long. Intermediate spatial steps are respectively $0.5l_c$, $1l_c$, $1.5l_c$, $2l_c$, $2.5l_c$, $3l_c$ and $4l_c$, from bottom to top. The PDF evolves from the initial distribution (green crosses) to the Eulerian flux weighted stationary distribution. Once that distribution reached, convergence is achieved and the PDF does not evolve anymore.