A comparison of discrete versus continuous adjoint states to estimate parameters of groundwater flow in dual porosity systems

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Abstract

In this investigation, we compare the discrete and continuous adjoint states regarding their efficiency to assist descent direction techniques of inversion. We focus the study on groundwater flow in fractured media at the scale of regional aquifers modeled as dual porosity systems [1]. The parameterization of the inverse problem employs an adaptive multiscale triangulation technique already tested in its ability to represent heterogeneous parameter field without any prior guess on their structure [2]. The multiscale triangulation spreads local parameter values over the calculation grid of the forward problem, these local values being sought by inversion and then interpolated to form the parameter fields of the forward problem. As seeds of interpolation can become numerous, descent methods as quasi-Newton algorithms (e.g., the BFGS technique, in [3]) reveal the best optimization technique given their ability to manipulate high-dimensional problems. Computation cost is dramatically reduced if the quasi-Newton algorithm is flanked with adjoint state calculations that evaluate all the entries of the gradient of the objective function in a single step very similar to the calculation of the forward problem.

One can distinguish between the discrete and continuous forms of the adjoint state. The discrete adjoint state (DAS) is derived under the assumption that the calculation structure and algorithm of the forward problem are completely known, DAS then being a copy of the discrete equations of the forward problem, with the exception of their right hand side. For its part, the continuous adjoint state (CAS) is derived from the continuous equations that rule the forward problem. These continuous equations usually require discretization for solving them but this discretization can be handled by any means independent of the techniques employed to solve the forward problem. Stated differently, CAS is non-intrusive which is convenient, for example, regarding its implementation on non-proprietary codes

It has often been conjectured that CAS was less precise than DAS in calculating the gradient components of the objective function, but there exists no trace in the literature proving this feature. We developed the DAS [4] and CAS [5] associated with flow in dual porosity systems before employing both in comparison exercises aimed at retrieving highly heterogeneous structures of hydraulic conductivity and specific storage capacity. In these exercises, we benefit from the non-intrusive character of the CAS to calculate it with discretization techniques and grids different from that of the forward problem (and the DAS). We also coarsened the calculation grids of the CAS in the hope of accelerating the inversion by reducing the calculation times of the adjoint state.

Our results show that CAS and DAS perform similarly when discretized at the same grid scale. They retrieve parameter fields close to the references employed for generating synthetic head data feeding the inverse problem (Fig. 1). Increasing the mesh size of CAS does not denature the sought solutions, even though they may sometimes appear slightly smoother than references. That said, increasing the mesh size employed for solving the CAS renders less precise entries of the gradient of the objective function. As a consequence, the quasi-Newton algorithm is less guided in the parameter space and several inverse solutions do not converge correctly. Finally, we advise to rely upon CAS for inversion of high-dimensional problems, mainly because the technique is appropriate for building a tool easily plugged into any numerical model. However, there is no substantial gain in coarsening the calculation grids of the CAS

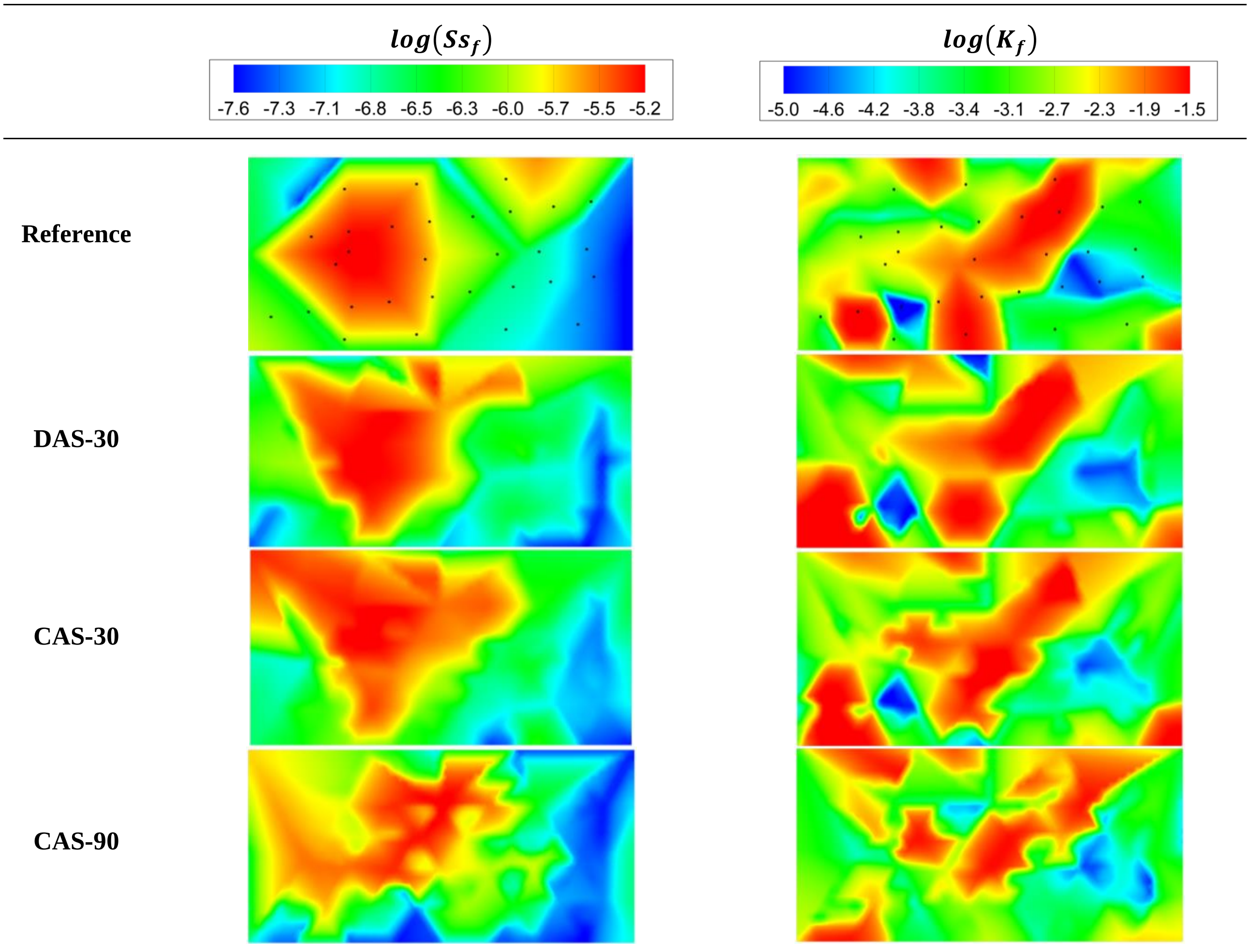

Figure 1: References and examples of parameter fields sought by inversion of flow in a dual porosity system. DAS-30 stands for inversions carried out with discrete adjoint states calculated over grids with 30 m element size, CAS-30 and CAS-90 denote calculations via continuous adjoint states evaluated (after discretization) over elementary mesh sizes of 30 and 90 m. Parameter fields reported correspond to the decimal logarithm of the fracture specific storage capacity $\boldsymbol{log}(\boldsymbol{Ssf})$, and the fracture hydraulic conductivity $\boldsymbol{log}(\boldsymbol{Kf})$. Dots on the reference fields correspond to the location of transient head data conditioning the inversion exercises.

References

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