An adaptive domain decomposition method for immiscible two-phase flow in porous media with different rock types

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Abstract

Numerical simulation of two-phase flows in porous formations have been the subject of investigation of many researchers owing to important applications in both the management of petroleum reservoirs and environmental remediation. More recently, modeling two-phase flow received an increasing attention in connection with porous medium with different rock types so that the permeability and the capillary pressure field are changing across the interfaces between the rocks.

Two-phase flows in porous media can be modeled by mass balance laws for each of the fluids [1]. In particular, an equivalent formulation can be obtained for the system of equations governing two-phase immiscible incompressible flows in porous media, by introducing an artificial variable called the global pressure. The dependent variables in such formulation are the global pressure and the wetting phase saturation [1]. Considering these variables, the governing equations consist of a nonlinear elliptic Darcy-type equation for the global pressure and a nonlinear degenerate parabolic equation of advection-diffusion type for the wetting phase saturation. Flow simulation of such systems in porous media with different rock types is very difficult because of the coexistence of different physics in the different rock types that require coupling [2]. Furthermore, due to the variations in permeability and in the capillary forces, the global pressure and the saturation can exhibit strong discontinuities across the interfaces between the rocks

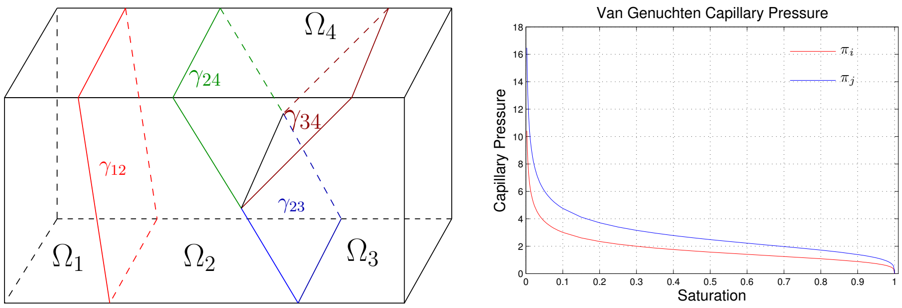

Figure 1: Illustration of a computational domain with dif- ferent rock types (left). Two different capillary pressure curves (right)

Our contribution in this work is on formulating a DD method in the context of operator splitting-based techniques for the two-phase flow between different rock types. Our method is motivated first by the fact that advection and diffusion effects act on different time scales within the same rock type as well as between different rock types, and it is easy to implement this in the DD context. To do so, we use two stage operator splitting techniques together with nonoverlapping domain decomposition methods to transform the original problem into a set of interface equations: First, we decouple the pressure calculation from the saturation calculation, and for the latter we split diffusion and advection, providing three layers of simpler subproblems to be solved sequentially. Then, applying DD techniques in each layer of the calculation procedure, the global problem is reduced to solve sequentially pressure, saturation-diffusion and saturation-diffusion interface problems, by eliminating the interior subdomain variables. The advantage of this DD method, is that allows for using specialized numerical methods for each subproblem. The pressure problem and the saturation-diffusion problem in the subdomains are approximated using hybridized mixed finite element method. Thus, the total velocity as well as the diffusive velocity, considered here as unknowns, are accurately approximated using MFE method. Further, to handle efficiently the advection step, a numerical scheme is presented using hybridized finite volume method based on the Godunov scheme, see [3, 4]. Particularly, the usage of non-matching time grids between different rock types for the advection part is crucial to the flexibility and the efficiency of our domain decomposition approach; see [6].

The advantage of the developed DD method is the possibility to reuse existing codes for different numerical schemes and domain decomposition techniques specialized to each component of the problem. Another advantage distinguishes our method is that it can integrate easily more complex problems involving two–phase flow between different rock types. As an example application, the developed DD approach is extended to address reduced fracture problems for which fractures are modeled as co-dimensional interfaces between different rock types.

Numerical experiments including those with several rock types and fractures illustrate the computational efficiency of the method and highlight the flexibility of the method to handle complex porous media configurations with different rock types; see [5].

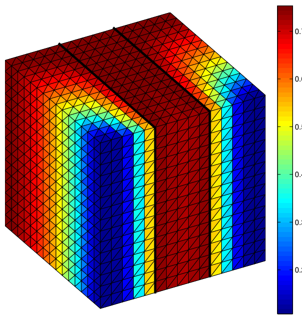

Figure 2: Saturation between two different rock types

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