A general framework for heterogeneous discretizations in mixed-dimensional single-phase flow

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Fractures are planar or sub-planar discontinuities along which a rock has been broken, and represent important conduits or barrier for fluid flow. Fracture intersections can have a significant impact as well and, because of infilling process, may behave differently from the surrounding fractures. Since fracture aperture is order of magnitude smaller than other characteristic sizes, a mixed-dimensional representation is preferable to the full three-dimensional [1, 2]. We focus on incompressible single-phase flow described by a Darcy-type model.

Realistic fracture networks can have highly complex geometries that pose a challenge to meshing algorithms that aim to resolve the fracture surfaces and their intersections. Depending on geometric details, the resulting grids can have high number of cells, and approaches that can loosen restrictions on meshing can have practical benefits. Similarly, due to heterogeneities in flow characteristics in the fractures and surrounding rock matrix, it may be beneficial to consider different numerical schemes in different parts of the computational domain.

Here we introduce a new framework able to handle in a natural way heterogeneity both in term of numerical schemes (like two and multi-point flux approximation [3] and virtual element methods [4]) and degree of geometrical conformity among the different dimensions. The key ingredient is a conservative mortar-like approach for the intra-dimensional coupling. Advantages of our approach are

1) the accurate choice of the correct numerical scheme in each dimension can substantially increase the accuracy of the solution without escalating its computational cost.
2) The flexibility of the non-conforming geometrical coupling given by the mortar is crucial to solve realistic problems in presence of complex fracture networks, as it allows us to refine and coarsen grids locally, without having to modify grids or change discretization schemes in neighboring geometrical objects.

This approach is of particular importance for geothermal energy extraction, CO_2 sequestration, enhance oil recovery, and nuclear waste disposal.

We present stability estimates for the coupling of numerical approximations in different dimensions. Illustrative bi-dimensional examples and realistic three-dimensional geometries are considered to emphasize the potentialities and a high freedom of the proposed approach.

[1] W.M. Boon, J.M. Nordbotten and I. Yotov: Robust Discretization of Flow in Fractured Porous Media, arXiv:1601.06977.
[2] W.M. Boon, J.M. Nordbotten, J.E. Vatne: Mixed-Dimensional Elliptic Partial Differential Equations, arXiv:1710.00556.

[3] T.H. Sandve, I. Berre, J.M. Nordbotten: An efficient multi-point flux approximation method for Discrete Fracture-Matrix simulations, Journal of Computational Physics 231(9), 3784-3800, 2012.
[4] A. Fumagalli, E. Keilegavlen: Dual Virtual Element Methods for Discrete Fracture Matrix Models, arXiv:1711.01818.