Formulation of the 3D Shallow Water and 3D/2D Coupled Shallow Water Models in the Adaptive Hydraulics Suite

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Introduction

The Adaptive Hydraulics (AdH) software suite, developed in collaboration between the Coastal and Hydraulics and Information Technology Laboratories, has been the flagship numerical tool for hydrodynamic modeling within the USACE-ERDC community for a number of years now. AdH is a multi-physics suite comprised of 2D and 3D shallow water (SW) models, a 3D Navier-Stokes model, models for sediment and salt transport, etc. Each of the models is solved using a continuous Galerkin finite element engine with streamline upwind Petrov-Galerkin stabilization. The software supports triangular mesh discretization in 2D, tetrahedral and prism meshes in 3D and implements up to second-order implicit time-stepping. As the name implies, AdH is both adaptive in space and time and supports host of features necessary for typical engineering applications, such as hydraulic structures for flow control, friction and turbulence libraries, contaminant transport linkages, sediment process libraries, vessel propagation via pressure lids, etc. The internal finite element engines support typical hydraulic boundary conditions and all external forces required for riverine, estuarine, lake and coastal applications. Lastly, AdH, being extremely portable, can be built serially on local machines or in parallel for either local or HPC applications.

Recently, a 3D SW model was built within AdH that has now been verified and validated extensively over the last few years. In addition to being executed as a stand-alone model, the latest version of AdH allows for coupling between the 3D and 2D SW models. This presentation will focus on the mathematical formulism of the AdH 3D SW model and the motivations and formulations for 2D/3D SW model coupling.

AdH 3D SW Procedure

Traditionally, the 3D baroclinic SW water equations are solved as a three-step procedure, given as follows:

1. In the first step, the 3D momentum equations and the depth-averaged continuity are solved together monolithically as one system of equations. The solutions to these equations are the updated values for the total water depth and the 3D horizontal velocities.

2. After the depth-averaged and momentum equations are solved, a second step is required to calculate the vertical velocity by solving the 3D continuity equation. This is a linear solve.

3. In the final step, all barotropic and baroclinic constituents are solved using the weak linear transport equation. If the constituents are baroclinic, water densities are calculated using the appropriate equation of state, and new hydraulic pressures are obtained before proceeding back to Step 1.

The most notable 3D SW obstacles that can cause failure in the first two (hydrodynamic) steps are as follows:

1.) Free surface tracking: To solve the 3D SW equations, the free surface must be tracked either by statically transforming to a parent coordinate system at each time-step or by assigning Lagrangian, vertical grid speeds to the surface grid points. In the latter, non-bed, sub-surface grid points should also move to ensure conforming 3D elements with reasonable aspect ratios, though they may not be Lagrangian.

2.) Discrete continuity equation consistency: Because the depth-averaged continuity equation is derived/depth-integrated from the 3D continuity, special care is needed to keep the discrete formulations of the two consistent. If they are not, errors in mass conservation can occur. This level of consistency can be difficult,

particularly when dealing with the cross-equation terms introduced by Streamline Upwind Petrov-Galerkin stabilization terms in AdH.

3.) Accuracy of the vertical density integration: Vertical integration of the hydrostatic density to calculate pressure in a 3D SW model can be costly and error-prone for fully unstructured grids. Even small errors in this integration can cause artificially driven density currents.

4.) Accuracy of the horizontal pressure integration: For numerical schemes that place horizontally connected grid points at significantly different pycnoclines, such as SW 3D sigma-transform methods, the SW 3D equations have been shown to be particularly sensitive to precision errors in the pressure gradients.

Errors discussed in Item 3 can be greatly reduced by aligning vertically stacked nodes along a column (i.e. they share the same x,y-coordinates). This eliminates the computational overhead and errors associated with horizontal interpolation before vertical integration, as would be required for fully unstructured grids. This type of quasi-structured, columnar-constrained grid is almost always utilized for 3D SW models, and fully unstructured 3D SW models are rare. Item 2 must be dealt with by carefully formulating the discretized equations in a consistent manner. Lastly, items 1 and 4 are intimately related to the choice of the model's dynamic coordinate system.

AdH Internal Model Coupling

Coupling between the internal models in AdH suite can occur monolithically as one system of equations and/or through flux with the usual operator splitting errors. The AdH framework allows the user ultimate flexibility when deciding how the internal models will interact. All models solved together, as one system of equations, are grouped into a "supermodel". More than one supermodel can be coupled via fluxes. Figure 1 displays a general multi-model AdH layout.

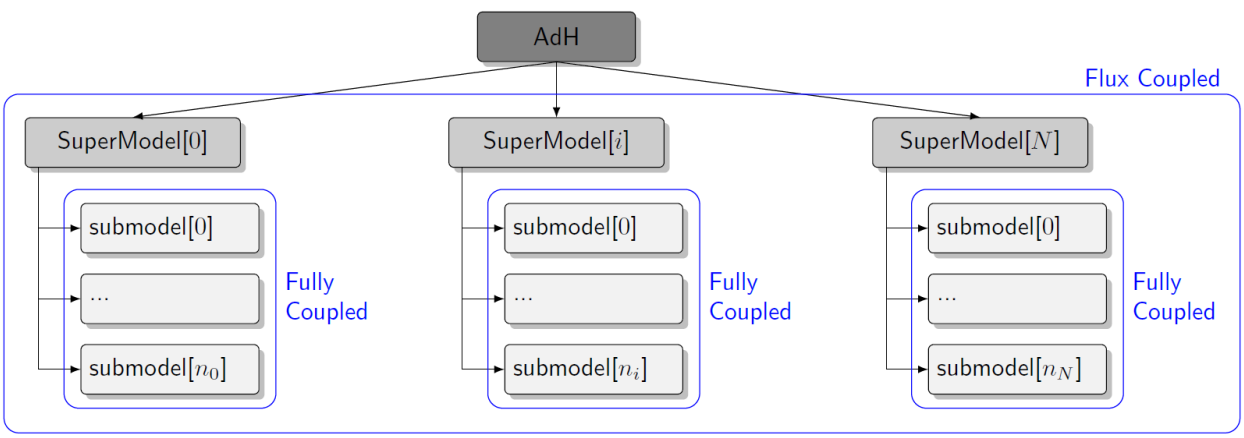

Figure 1: Example of a figure

This presentation will focus on supermodel coupling of the 3D to 2D shallow water models. One motivation for this type of coupling would be to include flood zones, where wetting and drying occurs, in 3D baroclinic estuarine applications.

Presentation Topics

This presentation will first provide the underlying mathematical framework for the discrete 3D AdH shallow water model. The formulations presented will include coordinate descriptions, finite element stabilization methods, adaption schemes, etc., commonly used in AdH 3D SW applications. We then give a brief description of the 2D to 3D monolithic model coupling formulation currently implement for supermodels, including interface constraints, node mappings, residual/equation modification, etc. Lastly, we present verification results for a 3D/2D flume study and recently obtained results from a Galveston Bay validation.