A Common Framework for Multitarget Search and Cross-Cueing Optimization

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***Abstract*— Despite their importance in the search theory field, the two problems we introduce in this paper have not been widely studied in the literature. The first problem addresses the optimization of the search for multiple moving targets. The second one addresses the optimization of a cross-cueing problem, i.e. the detection-confirmation problem. In this setup, the goal is to maximize chances to detect a target and confirm the detection, either by considering that two means of search are available at a given time period or by a unique mean of search at two consecutive time periods. In all the cases, the search optimization is extended to a multiperiod search for moving targets. In order to solve these two problems in a common framework, we present an effective resolution method based on constraints dualization and on a Forward and Backward algorithm.**

Keywords: Search theory, Convex optimization, Multitarget search, Cross-Cueing.

I. Introduction

This paper addresses two problems of importance in the search theory field. The first one is the problem of maximizing the detection probability for multiple targets, while the second one consists in maximizing the detection probability of detecting and confirming a target (cross-cueing). It is worth stressing that they don't have been the object of great attention so far. First, search theory is usually restricted to the search for a unique target, the reference [3] excepted. Noticeable exceptions are papers devoted to search for a unique target hidden among multiple false alarms [7] [8]. The problem is then to decide if a clue is a target or a false alarm, and to optimize the search for this target. There are also few papers on multiple detections, though the aim is to search for track initialization [10].

Here, we introduce optimization of multitarget search and cross-cueing problems in monoperiod and multiperiod cases (one or more time steps). Both monoperiod problems are solved in the same way: they are dualized. Actually, the key is that the objective functionals we try to optimize are sufficiently separable so that the dual functional can be "easily" calculated. Once the dual functional have been calculated, the optimal dual parameters (related) are easily obtained via any standard optimization algorithm. The solution of the primal problem is then straightforwardly deduced. It is also worth to stress that we are able to have explicit expressions of the (sub)-gradient of the dual functional. So, even if the mathematical background can appear a bit impressive, the actual solution is very simple.

Extensions to multiperiod problems are also solved in the same manner, using a Forward And Backward (FAB) algorithm [1] [11] for both problems. Consequently, we choose to present these two kinds of search problems in the same article. This paper is organized as follows. We first introduce the optimization framework. Two distinct parts are then devoted to multitarget optimization and to cross-cueing optimization. In each part, after presenting the optimization problem, we introduce how to solve monoperiod search and, then, how to solve multiperiod search. The final part of the paper is devoted to results.

II. Framework

This part is devoted to presentation of the optimization framework.

1) The time: Notations are presented here in the multi-period framework, i.e. there is a time index t which represents the time-period where the search problem is considered. When time index is omitted, it means that we consider notations and/or problems in a monoperiod context.

2) The space of search: The search is conducted in a discrete space, namely E. Each element of this set is called a cell, denoted c and indexed by i. It represents the smallest area for which search parameters are constant within. We consider that the number of cells in the space of search is n. Thus $E = (c_1, \ \dots \ , c_i, \ \dots \ , c_n)$.

3) The targets: We want to detect m targets hidden into the space of search. Targets are referred via an index k. The only available information concerning targets is a prior knowledge on their location. We denote α_i^k the probability of target k to hide in cell c_i.

$$\forall k, \ \sum_{i=1}^{n} \alpha_i^k = 1. \tag{1}$$

4) The resources: A limited number of sensors, S, is available in order to detect the targets. These sensors are indexed by s. Each one has a resource amount $\Phi^{s,t}$ (or capacity) available in order to carry the search out at each time period t. Resources are continuous, indefinitely divisible and must be allotted to cells of E in an optimal manner. Of course, resources are limited i.e. :

$$\forall s, \forall t, \ \sum_{i=1}^{n} x_i^{s,t} \leq \Phi^{s,t}. \tag{2}$$

where $x_i^{s,t}$ is the amount of resource allotted to cell c_i at time t for sensor s. The search vector associated with the search space for sensor s at time t is:

$$\mathbf{X}^{s,t} = (x_1^{s,t}, \ \ldots \ , x_i^{s,t}, \ \ldots \ , x_n^{s,t}). \tag{3}$$

If the search is monoperiod and concerns a unique search device, then the search vector is simply $\mathbf{X} = (x_1, \ \ldots \ , x_i, \ \ldots \ , x_n)$.

5) Detection functional: We introduce here the detection functional for the monoperiod case. We will see later how to extend this functional in a multiperiod framework.
Effectiveness of the search is characterized by the elementary conditional non-detection probability $\bar{p}^{k,s}(x_i^s)$ which represents the probability of not detecting a target k with sensor s given that the target is hidden in cell c_i and that we apply an elementary search effort x_i^s on this cell. Some assumptions are made to model $\bar{p}$. For all sensors $x_i^s \longmapsto \bar{p}^{k,s}(x_i^s)$ is convex and non-increasing (law of diminishing return). Under independence hypotheses, a usual model is [9]:

$$\bar{p}^{k,s}(x_i^s) = \exp(-w_i^{k,s} \, x_i^s), \tag{4}$$

where $w_i^{k,s}$ is a visibility coefficient which characterizes the reward for the search effort x_i^s applied in c_i by sensor s in order to detect the target k. An additional assumption to model the non-detection probability is that sensors act independently at cell level, which means that if S sensors are carrying out the search, the probability of not detecting a target k, $\bar{P}_k^S(\mathbf{X}^1, \ \ldots \ , \mathbf{X}^S)$, is simply:

$$\bar{P}_k^S(\mathbf{X}^1, \ \ldots \ , \mathbf{X}^S) = \sum_{i=1}^{n} \alpha_i^k \prod_{s=1}^{S} \bar{p}^{k,s}(x_i^s). \tag{5}$$

The general framework having been defined, we will now turn toward the two specific search problems we want to investigate. Though they widely differ in their formulation, we shall see that we use common tools to solve the related optimization problems. In both cases, it is the separability (both in space and time) which plays the major role.

III. Optimizing the Search for Multiple Targets

The problem here is to optimize multitarget search as a min-max problem (see $\mathcal{P}_{\mathsf{mt}}$). We will present the optimization in the case of a monoperiod search, and then the optimization in the case of a multiperiod search.

A. Monoperiod search

1) Statement of the problem: We first consider that the search is conducted by a unique sensor. Assume that we have m targets to detect. Since the *most dangerous* target is the target which is the more difficult to detect, we define the multitarget non-detection functional (say $\bar{P}_{\mathsf{mt}}(\mathbf{X})$) by:

$$\bar{P}_{\mathsf{mt}}(\mathbf{X}) = \max_{k=1,\cdots,m} \left(\bar{P}_1(\mathbf{X}, \cdots, \bar{P}_k(\mathbf{X}), \cdots, \bar{P}_m(\mathbf{X}) \right) \ , \tag{6}$$

where $\mathbf{X} = (x_1, \ \ldots \ , x_i, \ \ldots \ , x_n)$ is the vector representing the search distribution. Our aim is to minimize the $\bar{P}_{\mathsf{mt}}(\mathbf{X})$ functional with respect to resource consumption constraints. We thus have to solve the optimization problem $\mathcal{P}_{\mathsf{mt}}$:

$$\mathcal{P}_{\mathsf{mt}} \left| \begin{array}{l} \min\limits_{\mathbf{X}} \ \bar{P}_{\mathsf{mt}}(\mathbf{X}) \\ \text{s.t.} \ : \\ \sum\limits_{i} \ x_i \leq \Phi; \ \forall i, \ x_i \geq 0. \end{array} \right. \tag{7}$$

Elementary functionals $\left\{ \bar{P}_k\left(\mathbf{X}\right) \right\}_{k=1}^{m}$ are convex and differentiable. The continuity of $\bar{P}_{\mathsf{mt}}$ is preserved by the $\max$ operator, but **not the differentiability**. Indeed, the set of index k for which $\bar{P}_k$ touches $\bar{P}_{\mathsf{mt}}$ at a given point $\mathbf{X}$, i.e. $K(\mathbf{X}) \triangleq \left\{1 \leq k \leq m: \ \bar{P}_k(\mathbf{X}) = \bar{P}_{\mathsf{mt}}(\mathbf{X})\right\}$, plays a major part in the differentiability of $\bar{P}_{\mathsf{mt}}$.
If $K(\mathbf{X})$ is reduced to a unique element then, due to the differentiability of the $\bar{P}_k$ functionals, $\bar{P}_{\mathsf{mt}}$ is differentiable in $\mathbf{X}$. However, things are not as simple when $K(\mathbf{X})$ is not reduced to a single element. Though we may argue that such points are rather rare, the fact is they are natural candidates in order to minimize $\bar{P}_{\mathsf{mt}}\left(\mathbf{X}\right)$ (see Fig. 1). In such cases, the minimum of the $\bar{P}_{\mathsf{mt}}$ functional is achieved in a point for which the functional is not differentiable. So, we have now to turn toward the tools we need for solving our problem.

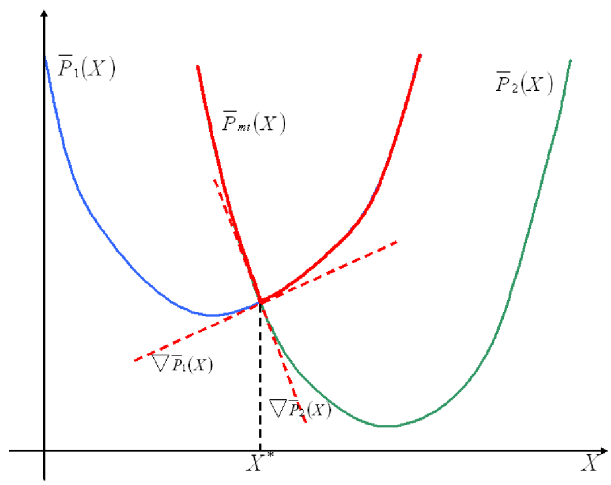

Fig. 1. *The definition of the $\bar{P}_{\mathsf{mt}}$ functional and its consequences.*

2) Elementary sub-differential calculus: When φ is a simple differentiable functional at $\mathbf{X}$, it is well-known that when we move in any direction $\mathbf{d} \in \mathbb{R}^n$, we have ($v$: scalar):

$$\forall\, v > 0\ ,\ \varphi(\mathbf{X} + v\,\mathbf{d}) = \varphi(\mathbf{X}) + v\ \langle \nabla\varphi(\mathbf{X}), \mathbf{d}\rangle + v\ \varepsilon_{\mathbf{d}}(v), \quad \text{where: } \lim_{v \to 0^+} \varepsilon_{\mathbf{d}}(v) = 0. \tag{8}$$

In Eq. 8, $\langle .,. \rangle$ is the standard scalar product in $\mathbb{R}^n$, while $\nabla\varphi(\mathbf{X})$ denotes the gradient of φ at the point $\mathbf{X}$.

When $K(\mathbf{X})$ is not reduced to a singleton, even if we know that Eq. 8 is no longer valid, situation should be much more comfortable if we were able to replace the linear form $\langle \nabla\varphi(\mathbf{X}), \mathbf{d}\rangle$ by another quantity. Actually, the following result holds true:

$$\begin{aligned} \forall\, v > 0\ ,\ \bar{P}_{\mathsf{mt}}(\mathbf{X} + v\mathbf{d}) &= \bar{P}_{\mathsf{mt}}(\mathbf{X}) + v \max_{k \in K(\mathbf{X})} \langle \nabla \bar{P}_k(\mathbf{X}), \mathbf{d}\rangle \\ &\quad + v\ \varepsilon_{\mathbf{d}}(v)\ , \\ \text{where: } \lim_{v \to 0^+} \varepsilon_{\mathbf{d}}(v) &= 0. \end{aligned} \tag{9}$$

This means that the linear form $\langle \nabla \bar{P}_{\mathsf{mt}}(\mathbf{X}), \mathbf{d}\rangle$ is replaced by a maximum of linear forms, i.e. $\max_{k \in K(\mathbf{X})} \langle \nabla \bar{P}_k(\mathbf{X}), \mathbf{d}\rangle$. Moreover, from Eq. 9, we know that if $\mathbf{X}^*$ is a (local) minimum of $\bar{P}_{\mathsf{mt}}$ then we have:

$$\max_{k \in K(\mathbf{X}^*)} \langle \nabla \bar{P}_k(\mathbf{X}^*), \mathbf{d}\rangle \geq 0\ ,\ \forall \mathbf{d} \in \mathbb{R}^n. \tag{10}$$

The usual condition for $\mathbf{X}^*$ to minimize $\bar{P}_{\mathsf{mt}}$ is replaced by the following one.

Proposition 1: Let $\left\{\bar{P}_1, \cdots, \bar{P}_m\right\}$ be m convex and differentiable functionals $\mathbb{R}^n \to \mathbb{R}$, then the following conditions are equivalent:

1) $\mathbf{X}^*$ is a minimum of $\bar{P}_{\mathsf{mt}}$,
2) there exists positive real numbers $\{\rho_k\}_{k \in K(\mathbf{X}^*)}$, summing to 1, such that:

$$\sum_{k \in K(\mathbf{X}^*)} \rho_k\, \nabla \bar{P}_k(\mathbf{X}^*) \quad = \mathbf{0}.$$

The classical gradient vector is replaced by the convex envelop, denoted $\widehat{\nabla} \bar{P}_{\mathsf{mt}}(\mathbf{X}^*)$, which is defined as the convex compact polyhedron spanned by the $\nabla \bar{P}_k(\mathbf{X}^*)$ vectors, i.e. :

$$\widehat{\nabla} \bar{P}(\mathbf{X}^*) = \Big\{ \sum_{k \in K(\mathbf{X}^*)} \rho_k\, \nabla \bar{P}_k(\mathbf{X}^*) \mid \rho_k \geq 0\ ;\ \sum_{k \in K(\mathbf{X}^*)} \rho_k = 1 \Big\}\ . \tag{11}$$

As an example, one can consider Fig. 1. Graphically [1], it is obvious that there exist positive scalars ρ_1 and ρ_2, summing to 1, and such that $\rho_1 \nabla \bar{P}_1(\mathbf{X}^*) + \rho_2 \nabla \bar{P}_2(\mathbf{X}^*) = \mathbf{0}$.

Let us now consider an extension to the previous analysis in the case where we want to minimize the $\bar{P}_{\mathsf{mt}}$ functional under constraints. To that aim, let us define the subset of constraints Ω by:

$$\Omega = \left\{ \mathbf{X} \middle| \max_{j=1,\cdots,n_1} h_j(\mathbf{X}) \leq 0 \right\}\ , \tag{12}$$

where the h_j functionals ($\mathbb{R}^n \to \mathbb{R}$) are assumed to be convex and differentiable. The following proposition holds true [2].

[1]Note that $\nabla \bar{P}_1(\mathbf{X}^*)$ and $\nabla \bar{P}_2(\mathbf{X}^*)$ are positive and negative scalars (the slopes) for this figure

Proposition 2: A necessary condition for the functional $\bar{P}_{\mathsf{mt}}(\mathbf{X})$ to be minimum on Ω at a point $\mathbf{X}^* \in \Omega$ is that there exits a vector $\mathbf{\Lambda}^*$ of (Lagrange) multipliers:

$$\lambda^* = \left(\lambda_1^*, \cdots, \lambda_m^*\ ;\ \mu_1^*, \cdots, \bar{\mu}_{n_1}^*\right)\ ,$$

such that:

$$\begin{aligned} &\bullet \sum_{k=1}^{m} \lambda_k^*\, \nabla \bar{P}_k(\mathbf{X}^*) + \sum_{j=1}^{n_1} \mu_j^{*\,h}\, \nabla h_j(\mathbf{X}^*) = \mathbf{0}\ . \\ &\bullet\ \forall k,\ \ \lambda_k^* \geq 0\ ;\ \lambda_k^* = 0 \text{ iff } \bar{P}_k(\mathbf{X}^*) < \bar{P}_{\mathsf{mt}}(\mathbf{X}^*)\ . \\ &\bullet\ \forall j,\ \ \mu_j^{*\,h} \geq 0\ ;\ \lambda_j^{*\,h}\, h_j(\mathbf{X}^*) = 0. \end{aligned} \tag{13}$$

The following Lagrange functional is then defined:

$$\mathcal{L}\left(\mathbf{X}, \mathbf{\Lambda}\right) \triangleq \sum_{k=1}^{m} \lambda_k\, \bar{P}_k(\mathbf{X}) + \sum_{j=1}^{n_1} \mu_j\, h_j(\mathbf{X})\ ,$$

and, as usually, a saddle point $(\mathbf{X}^*, \mathbf{\Lambda}^*)$ is:

$$\mathcal{L}\left(\mathbf{X}^*, \mathbf{\Lambda}\right) \leq \mathcal{L}\left(\mathbf{X}^*, \mathbf{\Lambda}^*\right) \leq \mathcal{L}\left(\mathbf{X}, \mathbf{\Lambda}^*\right)\ . \tag{14}$$

Then we have:

Proposition 3: Let the functionals $\bar{P}_k$ and h_j be convex and continuously differentiable. Then, the functional $\bar{P}_{\mathsf{mt}}(\mathbf{X})$ achieves its minimum at the point $\mathbf{X}^* \in \Omega$ if and only if there exits a vector of Lagrange multipliers $\mathbf{\Lambda}^*$ such that $(\mathbf{X}^*, \mathbf{\Lambda}^*)$ is a saddle point of $\mathcal{L}\left(\mathbf{X}, \mathbf{\Lambda}\right)$.

For the sequel, we shall make a constant use of Prop. 2 and 3.

3) Solving the elementary monoperiod problem: For the sake of simplicity, we first restrict to two targets. The problem we have to solve is the following minimax problem:

$$\left| \begin{array}{l} \min\limits_{\mathbf{X}} \left[\max\limits_{1,2} \left(\bar{P}_1(\mathbf{X}),\ \bar{P}_2(\mathbf{X}) \right) \right] \\ \text{s.t. }\ : \\ \sum\limits_i x_i \leq \Phi;\ \forall\, i,\ x_i \geq 0. \end{array} \right. \tag{15}$$

Considering the results of the preceding section, we have to consider the following Lagrange functional:

$$\begin{aligned} \mathcal{L}\left(\mathbf{X}, \lambda, \mu\right) &= \lambda\ \bar{P}_1(\mathbf{X}) + (1 - \lambda)\ \bar{P}_2(\mathbf{X}) \\ &\quad + \mu \left(\sum_i x_i - \Phi \right) - \sum_i \nu_i x_i, \\ &\text{with: } 1 \geq \lambda \geq 0;\ \mu \geq 0\ ;\ \forall i,\ \nu_i \geq 0. \end{aligned} \tag{16}$$

Then by Prop. 2, the following condition is necessarily satisfied at a minimum $\mathbf{X}^*$:

$$\begin{aligned} &\forall i,\ \frac{\partial \mathcal{L}}{\partial x_i} \left(\mathbf{X}^*, \lambda, \mu\right) = 0 \Longleftrightarrow \forall i,\ \lambda w_i^1 \alpha_i^1 \exp\left(-w_i^1\, x_i\right) + \\ &(1 - \lambda)\, w_i^2 \alpha_i^2 \exp\left(-w_i^2\, x_i\right) = \mu - \nu_i\ . \end{aligned} \tag{17}$$

The assumption $x_i > 0$ imply $\nu_i = 0$ (see Prop. 2). So, under these conditions, the search effort is determined by the following equation:

$$\lambda w_i^1 \alpha_i^1 \exp\left(-w_i^1\, x_i\right) + (1-\lambda)\, w_i^2 \alpha_i^2 \exp\left(-w_i^2\, x_i\right) = \mu \ . \tag{18}$$

Moreover, if $x_i = 0$ then $\nu_i \geq 0$ and the condition Eq. 17 becomes:

$$\lambda w_i^1 \alpha_i^1 + (1-\lambda)\, w_i^2 \alpha_i^2 = \mu - \nu_i \leq \mu. \tag{19}$$

Gathering Eqs. 18 and 19, the following condition has been obtained:

$$\boxed{x_i > 0 \Longleftrightarrow \lambda w_i^1 \alpha_i^1 + (1-\lambda)\, w_i^2 \alpha_i^2 > \mu \ .}$$

Assuming that the Lagrange parameters λ and μ are fixed and that $x_i(\lambda, \mu)$ is strictly positive, then $x_i(\lambda, \mu)$ is the unique positive root of the equation:

$$\begin{aligned} &\lambda w_i^1 \alpha_i^1 \exp\left(-w_i^1\, x_i(\lambda,\mu)\right) + \\ &\quad (1-\lambda)\, w_i^2 \alpha_i^2 \exp\left(-w_i^2\, x_i(\lambda,\mu)\right) = \mu \ . \end{aligned} \tag{20}$$

In general (except for $w_i^1 = w_i^2$, $\forall i$), there does not exist an explicit solution to Eq. 20. However, the solution is easily obtained via any numerical procedure (e.g. 1-D dichotomy). So, $x_i^*(\lambda, \mu)$ is either 0 or the unique (strictly) positive root of Eq. 20.

Once $x_i^*(\lambda, \mu)$ has been obtained, we are able to calculate the dual functional $\psi(\lambda, \mu)$:

$$\begin{aligned} \psi(\lambda,\mu) &= \mathcal{L}\left(\mathbf{X}^*_{\lambda,\mu}, \lambda, \mu\right) \ , \\ &= \lambda\ \bar{P}_1\left(\mathbf{X}^*_{\lambda,\mu}, \lambda, \mu\right) + (1-\lambda)\ \bar{P}_2\left(\mathbf{X}^*_{\lambda,\mu}, \lambda, \mu\right) \\ &\quad + \mu\left(\sum_i x_i^*(\lambda,\mu)\right) . \end{aligned} \tag{21}$$

The problem is now to optimize the dual functional $\psi(\lambda, \mu)$ with respect to the $\{\lambda, \mu\}$ parameters. It is worth stressing that $\psi(\lambda, \mu)$ is concave, and under mild conditions (no duality gap) is differentiable. Assuming that $x_i^*(\lambda, \mu) > 0$, then thanks to the implicit function theorem, the partial derivatives $\frac{\partial}{\partial \mu} x_i^*(\lambda, \mu)$ and $\frac{\partial}{\partial \lambda} x_i^*(\lambda, \mu)$ are easily calculated, yielding:

$$\begin{aligned} &\Big[-\lambda (w_i^1)^2 \alpha_i^1 \exp\left(-w_i^1\, x_i^*(\lambda,\mu)\right) - (1-\lambda) \\ &(w_i^2)^2 \alpha_i^2 \exp\left(-w_i^2\, x_i^*(\lambda,\mu)\right)\Big] \frac{\partial}{\partial \mu} x_i^*(\lambda,\mu) = 1 \ ; \\ \\ &\Big[(w_i^1)^2 \alpha_i^1 \exp\left(-w_i^1\, x_i^*(\lambda,\mu)\right) - (w_i^2)^2 \alpha_i^2 \exp\left(-w_i^2\, x_i^*(\lambda,\mu)\right)\Big] \\ &+ \Big[-\lambda (w_i^1)^2 \alpha_i^1 \exp\left(-w_i^1\, x_i^*(\lambda,\mu)\right) - (1-\lambda) \\ &(w_i^2)^2 \alpha_i^2 \exp\left(-w_i^2\, x_i^*(\lambda,\mu)\right)\Big] \frac{\partial}{\partial \lambda} x_i^*(\lambda,\mu) = 0 \ . \end{aligned} \tag{22}$$

It can be shown that the partial derivatives $\frac{\partial}{\partial \lambda}\psi(\lambda, \mu)$ and $\frac{\partial}{\partial \mu}\psi(\lambda, \mu)$ can be deduced from Eqs. 22.

In order to optimize ψ we simply employ the BFGS method with the gradient described above (partial derivatives), which gives the optimal couple (λ^*, μ^*) and then $\mathbf{X}^*$ via Eq. 20. We stress that the (apparently) difficult primal problem has been reduced to the the maximization of a concave, **bi-dimensional** concave functional. Of course, it is the separability properties of the functional which render feasible this approach.

B. The multiperiod search

Assume now that we consider a multiperiod search. First, let us define the multiperiod multitarget non-detection functional for a Markovian target, with time horizon T. $\pi^{k,t}(j)$ is a (row) vector representing the probability that the target k has attained a cell j at time t, having remained undetected up to time t. Then, we have:

$$\forall k,\ \pi^{k,t}(j) = \sum_i \pi^{k,(t-1)}(i)\ \bar{P}^{k,t}(i,j) \ , \tag{23}$$

In Eq. 23, $\bar{P}^{k,t}(i,j)$ is the probability that target k goes from cell i (time period $(t-1)$) to cell j (time period t), being undetected by the search effort put on the cell j at time period t. We thus have:

$$\forall k,\ \bar{P}^{k,t}(i,j) = P^k_{i,j}\ \exp\left(-w_j^k\ x_j^t\right) ,$$

where P^k is a standard transition matrix for the target k. The above equation can also be written in matrix form:

$$\begin{aligned} &\forall k,\ \bar{P}^{k,t} = \bar{\Delta}^{k,t}_{\mathsf{mt}}\, P^k \ , \\ &\text{with:} \\ &\bar{\Delta}^{k,t}_{\mathsf{mt}} = \mathsf{diag}\left(\exp\left(-w_j^k\ x_j^t\right)\right) \ . \end{aligned} \tag{24}$$

Gathering Eqs. 23 and 24, we obtain the following recursion:

$$k = 1, \cdots, m,\ \pi^{k,t} = \pi^{k,t-1}\ \bar{\Delta}^{k,t}_{\mathsf{mt}}\, P^k \ . \tag{25}$$

Considering a T time-period search, we deduce that the probability that the target k remains undetected within the multiperiod search is:

$$\begin{aligned} \bar{P}^{k,T} \quad = \quad \Big[&\underbrace{\left(\pi^{k,1}\ \bar{\Delta}^{k,1}_{\mathsf{mt}} P^k\ \cdots\ \bar{\Delta}^{k,(t-1)}_{\mathsf{mt}} P^k\right)}_{U^{k,t}_{\mathsf{mt}}}\ \bar{\Delta}^{k,t}_{\mathsf{mt}} \\ &\underbrace{\left(P^k \bar{\Delta}^{k,(t+1)}_{\mathsf{mt}}\ \cdots\ P^k \bar{\Delta}^{k,T}_{\mathsf{mt}} \mathbf{1}\right)}_{D^{k,t}_{\mathsf{mt}}} \Big] \ , \end{aligned} \tag{26}$$

where $\pi^{k,1}$ represents the prior knowledge on the location of the target k, i.e. $\pi^{k,1} = \alpha^k$. We thus have:

$$k = 1, \cdots, m,\ t = 1, \cdots, T\ \bar{P}^{k,T} = U^{k,t}_{\mathsf{mt}}\ \bar{\Delta}^{k,t}_{\mathsf{mt}}\ D^{k,t}_{\mathsf{mt}}, \tag{27}$$

where the $U^{k,t}_{\mathsf{mt}}$ (row) and $D^{k,t}_{\mathsf{mt}}$ (column) are propagated via the following induction:

$$\begin{cases} D^{k,t}_{\mathsf{mt}} = P^k \bar{\Delta}^{k,(t+1)}_{\mathsf{mt}}\ D^{k,(t+1)}_{\mathsf{mt}} \ , \\ U^{k,t}_{\mathsf{mt}} = U^{k,(t-1)}_{\mathsf{mt}}\ \bar{\Delta}^{k,(t-1)}_{\mathsf{mt}} P^k \ . \end{cases} \tag{28}$$

Considering the optimization of the $\bar{P}^{k,T}$, we see that the complexity increases rapidly. However, this problem is drastically simplified by considering that optimization has been split throughout the various time-periods. More precisely, we consider the following problem:

$$\left\{ \begin{array}{l} \min_{\mathbf{X}^t} \left\{ \max_{1,\cdots,m} \left[\bar{P}^{1,T}, \cdots, \bar{P}^{m,T}\right] \right\} \\ \text{s.t. : } \sum_i x_i^t = \Phi^t \; t = 1, \cdots, T \; . \end{array} \right. \tag{29}$$

We denote $\mathbf{X}^t = (x_1^t, \dots, x_n^t)$ the search effort vector at time-period t. The quantities Φ^t represent the amount of resource available in order to carry out the search at the epoch t. Then the FAB algorithm, introduced by Brown [1] [11], simply consists in finding (recursively in time) the vector $\mathbf{X}^{t,l^*}$ defined by :

$$\left\{ \begin{array}{l} \mathbf{X}^{t,l^*} = \arg\min_{\mathbf{X}^{t,l}} \Big(\max \Big[U_{\mathsf{mt}}^{1,t,l} \, \bar{\Delta}_{\mathsf{mt}}^t \, D_{\mathsf{mt}}^{1,t,l-1}, \cdots, \\ \qquad\qquad\qquad\qquad\qquad U_{\mathsf{mt}}^{m,t,l} \, \bar{\Delta}_{\mathsf{mt}}^t \, D_{\mathsf{mt}}^{m,t,l-1} \Big] \Big) \\ \text{s.t. : } \sum_i x_i^{t,l} = \Phi^t \; . \end{array} \right. \tag{30}$$

In Eq. 30, the index l refers to the iteration number. It worth also stressing that for the first iteration of the algorithm ($l = 1$), the $D_{\mathsf{mt}}^{k,t,l-1}$ are not available and simply replaced by $\mathbf{1}$ vectors, which means that the algorithm works in the myopic mode. The global convexity of the $\max_k [\bar{P}_{\mathsf{mt}}^{k,T}]$ functional ensures convergence of this iterative multiperiod optimization algorithm.
Let us now study the cross-cueing problem.

IV. Optimizing the cross-cueing problem

The problem here is to optimize the cross-cueing search for a unique target, $\mathcal{P}_{\mathsf{cc}}$. We will first present the optimization in the case of a monoperiod search, and then the optimization in the multiperiod case.

A. Statement of the problem

We want to optimize the probability to detect a unique target twice, with two different sensors. We thus optimize the following cross-cueing functional, $P_{\mathsf{cc}}(\mathbf{X}^1, \mathbf{X}^2)$:

$$P_{\mathsf{cc}}(\mathbf{X}^1, \mathbf{X}^2) = P^2(\mathbf{X}^1, \mathbf{X}^2) = \sum_i \alpha_i \, p^1(x_i^1) \, p^2(x_i^2), \tag{31}$$

where $p^1(x_i^1) = 1 - \bar{p}^1(x_i^1)$ is the conditional detection probability for sensor 1 and $p^2(x_i^2) = 1 - \bar{p}^2(x_i^2)$ is the conditional detection probability for sensor 2. We thus aim to solve:

$$\mathcal{P}_{\mathsf{cc}} \quad \left| \begin{array}{l} \min_{\mathbf{X}^1, \mathbf{X}^2} \; -P_{\mathsf{cc}}(\mathbf{X}^1, \mathbf{X}^2) \\ \text{s.t. :} \\ \sum_i x_i^1 \leq \Phi^1 \; ; \; \sum_i x_i^2 \leq \Phi^2 \; ; \\ \forall i, \; x_i^1 \geq 0 \; ; \quad x_i^2 \geq 0 \; . \end{array} \right. \tag{32}$$

In most cases, P_{cc} is not concave everywhere. However, it has the great advantage to be separable. Considering the primal problem ($\mathcal{P}_{\mathsf{cc}}$) leads to consider the following Lagrangian functional.

$$\begin{array}{l} \mathcal{L}(\lambda, \mu, \mathbf{X}^1, \mathbf{X}^2) = -\sum_i \alpha_i \; p(x_i^1) \; p(x_i^2) \\ \quad +\lambda \left(\sum_i x_i^1 - \Phi^1 \right) + \mu \left(\sum_i x_i^2 - \Phi^2 \right) \; . \end{array} \tag{33}$$

The Lagrange multipliers λ and μ are positive, and the dual functional $\psi(\lambda, \mu)$ is defined by:

$$\begin{array}{l} \psi(\lambda, \mu) = \min_{\mathbf{X}^1, \mathbf{X}^2 \in \mathbb{R}_+^{2n}} \mathcal{L}(\lambda, \mu, \mathbf{X}^1, \mathbf{X}^2) \\ \text{where: } \mathbf{X}^1 = \left(x_1^1, \cdots, x_n^1\right) \text{ and } \mathbf{X}^2 = \left(x_1^2, \cdots, x_n^2\right) \; . \end{array} \tag{34}$$

B. Solving the elementary monoperiod problem

For given values of λ and μ, we denote $\mathbf{X}^{*1}$ and $\mathbf{X}^{*2}$ the search vectors which minimize $\mathcal{L}(\lambda, \mu, \mathbf{X}^1, \mathbf{X}^2)$. Considering the minimization of $\mathcal{L}(\lambda, \mu, \mathbf{X}^1, \mathbf{X}^2)$ on the convex domain $\mathbb{R}_+^{2n} \triangleq W$, a necessary condition for the vector $\mathbf{X}^* \triangleq (\mathbf{X}^{*1}, \mathbf{X}^{*2})$ to be a (local) minimum of $\mathcal{L}(\lambda, \mu, \mathbf{X})$ is thus:

$$\left\{ \begin{array}{l} -\nabla \mathcal{L}(\lambda, \mu, \mathbf{X}) \, (\mathbf{X}^*) \in \mathcal{N}(W, \mathbf{X}^*) \; , \\ \text{where:} \\ \mathcal{N}(W, \mathbf{X}^*) \triangleq \{ \mathbf{w} \in W | \; \langle \mathbf{w}, \mathbf{w}' - \mathbf{X}^* \rangle \leq 0 \; , \; \forall \mathbf{w}' \in W \} \; . \end{array} \right. \tag{35}$$

$\mathcal{N}(W, \mathbf{X}^*)$ is the normal cone to W, at $\mathbf{X}^*$. Considering Eq. 35, the following (necessary) condition holds (assuming that $x_i^{*1} > 0$):

$$\begin{array}{l} \frac{\partial}{\partial x_i^1} \mathcal{L}(\lambda, \mu, \mathbf{X}^*) = 0 \; , \\ \Longleftrightarrow \\ \alpha_i w_i^1 \; \left(1 - p(x_i^{*1}) \right) \; p(x_i^{*2}) = \lambda \; . \end{array} \tag{36}$$

The multiplier λ being non negative, the following implications result straightforwardly from Eq. 35:

$$\left\{ \begin{array}{l} x_i^{*1} > 0 \Rightarrow x_i^{*2} > 0 \; , \\ \alpha_i \, w_i^1 > 0 \; . \end{array} \right. \tag{37}$$

Moreover, since μ is also positive, we have $x_i^{*1} > 0 \iff x_i^{*2} > 0$. Denoting $\mathcal{X}_i^1 \triangleq p(x_i^{*1})$ and $\mathcal{X}_i^2 \triangleq p(x_i^{*2})$, if $x_i^{*1} > 0$, then $\mathcal{X}_i^1$ and $\mathcal{X}_i^2$ are the solutions of the following non-linear system:

$$\left\{ \begin{array}{l} \alpha_i \, w_i^1 \; (1 - \mathcal{X}_i^1) \, \mathcal{X}_i^2 = \lambda \; , \\ \alpha_i \, w_i^2 \; (1 - \mathcal{X}_i^2) \, \mathcal{X}_i^1 = \mu \; , \\ \mathcal{X}_i^1, \; \mathcal{X}_i^2 \in [0, 1] \; . \end{array} \right. \tag{38}$$

Subtracting the second row of Eq. 38 to the first row of Eq. 38, we obtain:

$$\begin{array}{l} \mathcal{X}_i^2 = \mathcal{X}_i^1 + \left(\frac{\lambda}{\alpha_i w_i^1} - \frac{\mu}{\alpha_i w_i^2} \right) \; , \\ \text{and, finally, the second order equation:} \\ -\alpha_i w_i^1 \; (\mathcal{X}_i^1)^2 + \left(\alpha_i w_i^1 - \lambda + \mu \frac{w_i^1}{w_i^2} \right) \; \mathcal{X}_i^1 - \mu \frac{w_i^1}{w_i^2} = 0 \; . \end{array} \tag{39}$$

The problem is then reduced to searching the roots of the (above) second order equation lying in the $[0, 1]$ interval. It is easily shown that there is at most one root in this interval. It simply remains to test which couple (x_i^1, x_i^2) or $(0,0)$ provides the lower value of $\mathcal{L}(\lambda, \mu, \mathbf{X}^1, \mathbf{X}^2)$, for every cell c_i. We can point out that the separability of the cross-cueing functional P_{cc} greatly simplifies the optimization problem.

The dual functional $\psi(\lambda, \mu)$ can now be calculated via Eq. 39. This is simply a two-dimensional functional, which is furthermore concave. In order to optimize, we employ the BFGS method along with the following gradient:

$$\begin{pmatrix} \sum_{i\in z} \varphi_1^*(c_{z,i})(\lambda_k, \mu_k) - \Phi_1 \\ \sum_{i\in z} \varphi_2^*(c_{z,i})(\lambda_k, \mu_k) - \Phi_2 \end{pmatrix} \tag{40}$$

This optimization provides the optimal couple (λ^*, μ^*) and then $\mathbf{X}^*$ solution of the primal problem is straightforwardly deduced.

C. The multiperiod search

a) The multiperiod multiresource cross-cueing problem: Assume now that we consider a multiperiod search. Here again the target is Markovian, and the time horizon is T. At a given time-period, we consider the cross-cueing functional P_{cc}. First, let us define the multiperiod **non**-cross-cueing functional for a Markovian target. Let $\bar{\pi}^t_{\mathsf{cc}}(j)$ the (row) vector representing the probability that the target has attained a cell j at time t, having remained not cross-cued (ucc) up to time-period t. Then, we have:

$$\bar{\pi}^t(j) = \sum_i \bar{\pi}^{(t-1)}_{\mathsf{cc}}(i)\ \bar{P}^t_{\mathsf{cc}}(i,j) \ . \tag{41}$$

In Eq. 41, $\bar{P}^t_{\mathsf{cc}}(i,j)$ is the probability that target goes from cell i (time period $(t-1)$) to cell j (time period t), being ucc by the search effort put on the cell j at time-period t. We thus have:

$$\begin{aligned}\bar{P}^t_{\mathsf{cc}}(i,j) = P_{i,j}\ \Big[1 - \Big(1-\exp(-w_j^1\ x_j^{1,t})\Big)\\ \Big(1-\exp(-w_j^2\ x_j^{2,t})\Big)\Big] \ .\end{aligned} \tag{42}$$

where P is a standard transition matrix. Again, the above equation can also be written in matrix form:

$$\begin{aligned}&\bar{P}^t_{\mathsf{cc}} = \bar{\Delta}^t_{\mathsf{cc}} P \ ,\\ &\text{with:}\\ &\bar{\Delta}^t_{\mathsf{cc}} = \mathsf{diag}\Big(1 - \Big(1-\exp(-w_j^1 x_j^{1,t})\Big)\Big(1-\exp(-w_j^2\ x_j^{2,t})\Big)\Big) \ .\end{aligned} \tag{43}$$

Gathering Eqs. 41 and 43, we obtain the following recursion:

$$\bar{\pi}^t = \bar{\pi}^{t-1}\ \bar{\Delta}^t_{\mathsf{cc}} P \ . \tag{44}$$

Considering a T time-period search, we deduce that the probability that the target remains undetected within the multiperiod search is:

$$\begin{aligned}\bar{P}^T_{\mathsf{cc}} \ = \ [\ &\underbrace{\Big(\pi^1_{\mathsf{cc}}\ \bar{\Delta}^1_{\mathsf{cc}} P\ \cdots\ \bar{\Delta}^{(t-1)}_{\mathsf{cc}} P\Big)}_{U^t_{\mathsf{cc}}}\ \bar{\Delta}^t_{\mathsf{cc}} \ ,\\ &\underbrace{\Big(P\bar{\Delta}^{(t+1)}_{\mathsf{cc}}\ \cdots\ P\bar{\Delta}^{(T)}_{\mathsf{cc}}\mathbf{1}\Big)}_{D^t_{\mathsf{cc}}}\ ] \ .\end{aligned} \tag{45}$$

We thus have $\bar{P}^T_{\mathsf{cc}} = U^t_{\mathsf{cc}}\ \bar{\Delta}^t_{\mathsf{cc}}\ D^t_{\mathsf{cc}}$, where the U^t_{cc} (row) and D^t_{cc} (column) are propagated via the following induction:

$$\begin{cases} D^t_{\mathsf{cc}} = P\bar{\Delta}^{(t+1)}_{\mathsf{cc}}\ D^{(t+1)}_{\mathsf{cc}} \ ,\\ U^t_{\mathsf{cc}} = U^{t-1}_{\mathsf{cc}}\ \bar{\Delta}^{(t-1)}_{\mathsf{cc}} P \ . \end{cases} \tag{46}$$

Then, the extension of the FAB algorithm [1] [11] simply consists in:

$$\begin{cases} \mathbf{X}^{t,l^*} = \arg\min_{\mathbf{X}^{t,l}} \big[U^{t,l}_{\mathsf{cc}}\ \bar{\Delta}^t_{\mathsf{cc}}\ D^{t,l-1}_{\mathsf{cc}}\big] \ ,\\ \text{s.t.} : \forall t,\ \forall l,\ \sum_i x_i^{1,t,l} = \Phi^{1,t}\ ;\ \sum_i x_i^{2,t,l} = \Phi^{2,t} \ . \end{cases} \tag{47}$$

In Eq. 30, the index l refers to the iteration index of the FAB algorithm.

b) The multiperiod monoresource cross-cueing problem: We present here a different version of the cross-cueing functional, where our aim is to detect a target at a given time period, say t, and to confirm it a the $t+1$ time period, using a unique search resource. Thus, a target is said detected-confirmed if it has been detected at two consecutive time periods.

Let us define as $\bar{\pi}_t$ the (row) vector whose i-th component is the probability that the target arrives in the cell i at period t without being detected-confirmed by previous search efforts. Conditioning on the elementary events, we have:

$$\bar{\pi}^t \ = \ \bar{\pi}^{t-1}\ \bar{P}^{(t-1)} + \bar{\pi}^{t-2}\ \bar{P}^{(t-2)}\ P^{(t-1)} \ , \tag{48}$$

where P^t is the probability of detecting the target at time t, and $\bar{P}^t$ the probability of not detecting the target at time t. Now, the probabilities of elementary events can be easily calculated:

$$\begin{aligned}&\bar{P}^{(t-1)} = \bar{\Delta}^{(t-1)} P \ \text{ and }\ P^{(t-1)} = \Delta^{t-1} P \ ,\\ &\text{with:}\\ &\bar{\Delta}^t = \mathsf{diag}\left(\exp\left(-w_i^t x_i^t\right)\right)\ \text{ and }\ \Delta^t = \mathsf{diag}\left(1-\exp\left(-w_i^t x_i^t\right)\right) .\end{aligned} \tag{49}$$

As Eq. 48 involves two time periods, we have:

$$\begin{pmatrix}\bar{\pi}^t & \bar{\pi}^{t-1}\end{pmatrix} = \begin{pmatrix}\bar{\pi}^{t-1} & \bar{\pi}^{t-2}\end{pmatrix} \begin{pmatrix} \bar{P}^{t-1} & I \\ \bar{P}^{t-2}P^{t-1} & 0 \end{pmatrix} . \tag{50}$$

Considering a T time-period search, we thus have to minimize

$$\begin{aligned}&\bar{P}^T_{\mathsf{cc}} = \begin{pmatrix}\bar{\pi}^{T+1} & \bar{\pi}^T\end{pmatrix}\begin{pmatrix}\mathbf{1}\\ 0\end{pmatrix},\ \text{i.e.:}\\ &\bar{P}^T_{\mathsf{cc}} = \begin{pmatrix}\bar{\pi}^2 & \bar{\pi}^1\end{pmatrix}\begin{pmatrix}\bar{P}^2 & I\\ \bar{P}^1 P^2 & 0\end{pmatrix}\cdots\begin{pmatrix}\bar{P}^T & I\\ \bar{P}^{T-1}P^T & 0\end{pmatrix}\begin{pmatrix}\mathbf{1}\\ 0\end{pmatrix} .\end{aligned} \tag{51}$$

Once again, $\bar{P}^T_{\mathsf{cc}}$ has been split into three terms:

$$\bar{P}^T_{\mathsf{cc}} = \ U^t_{cc}\underbrace{\begin{pmatrix}\bar{P}^t & I\\ \bar{P}^{t-1}P^t & 0\end{pmatrix}\begin{pmatrix}\bar{P}^{t+1} & I\\ \bar{P}^t P^{t+1} & 0\end{pmatrix}}_{\text{Optimization with respect to } t} D^t_{cc} \ . \tag{52}$$

We set:

$$\begin{aligned} U_{cc}^{12} &= \left(\bar{\pi}^2 \ \bar{\pi}^1\right), \text{ with } \bar{\pi}^1 = \alpha \text{ and } \bar{\pi}^2 = \alpha P; \\ D_{cc}^{T} &= \begin{pmatrix} \mathbf{1} \\ 0 \end{pmatrix}. \end{aligned} \tag{53}$$

The U_{cc}^t (row) and D_{cc}^t (column) vectors are thus propagated via the following induction:

$$\left\{ \begin{array}{l} U_{cc}^t = U_{cc}^{t-1} \begin{pmatrix} \bar{P}^{t-1} & I \\ \bar{P}^{t-2} P^{t-1} & 0 \end{pmatrix}, \\ D_{cc}^t = \begin{pmatrix} \bar{P}^{t+2} & I \\ \bar{P}^{t+1} P^{t+2} & 0 \end{pmatrix} D_{cc}^{t+1}. \end{array} \right. \tag{54}$$

We can note that in the time-splitting (Eq. 52) both U_{cc}^t and D_{cc}^t do not depend on $\bar{\Delta}^t$ (i.e. on the $\{x_i^t\}$). We have again the same sequential structure (FAB) of the optimization problem:

$$\left\{ \begin{array}{l} \text{for } t = 1 \text{ and } t = 2: \\ (\mathbf{X}^{1,l*}, \mathbf{X}^{2,l*}) = \arg \min_{\mathbf{X}^{1,l}, \mathbf{X}^{2,l}} \left[(\bar{\pi}^2 \ \bar{\pi}^1) \begin{pmatrix} \bar{P}^{2,l} & I \\ \bar{P}^{1,l} P^{2,l} & 0 \end{pmatrix} \right. \\ \qquad \left. \begin{pmatrix} \bar{P}^{3,l-1} & I \\ \bar{P}^{2,l} P^{3,l-1} & 0 \end{pmatrix} D_{cc}^{12,l-1} \right], \\ \text{for } t = \{3, \dots, T-1\}: \\ \mathbf{X}^{t,l*} = \arg \min_{\mathbf{X}^{t,l}} \left[U_{cc}^{t,l} \begin{pmatrix} \bar{P}^{t,l} & I \\ \bar{P}^{t-1,l} P^{t,l} & 0 \end{pmatrix} \right. \\ \qquad \left. \begin{pmatrix} \bar{P}^{t+1,l-1} & I \\ \bar{P}^{t,l} P^{t+1,l-1} & 0 \end{pmatrix} D_{cc}^{t,l-1} \right], \\ \text{for } t = T: \\ \mathbf{X}^{T,l*} = \arg \min_{\mathbf{X}^{T,l}} \left[U_{cc}^{T,l} \begin{pmatrix} \bar{P}^{T,l} & I \\ \bar{P}^{T-1,l} P^{T,l} & 0 \end{pmatrix} \right. \\ \qquad \left. \begin{pmatrix} \mathbf{1} \\ 0 \end{pmatrix} . \right], \\ \text{s.t.:} \\ \forall t,\ \forall l,\ \sum_i x_i^{t,l} = \Phi^t, \\ \forall t,\ \forall l,\ x_i^{t,l} \geq 0. \end{array} \right. \tag{55}$$

V. Results

In this section we present results of optimization for both multitarget and cross-cueing case studies. We consider the same space of search for these two studies: targets are hidden in the Laouzas lake area, in France. The following figure (Fig. 2) represents a map of this area. In this figure, the different

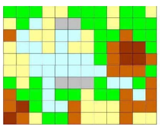

Fig. 2. *A discrete map of the Laouzas lake area.*

colours represents different kinds of grounds. Fields, forests, mountains, high mountains, lakes and towns are represented respectively by the yellow, green, brown, dark brown, blue and gray colours.

A. Multitarget search

Here, we present result of the optimization in the case of the detection of two moving targets. The targets are initially located as in Fig. 3. The first target moves in the north-east

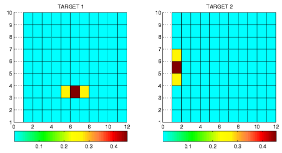

Fig. 3. *Prior on the location of the targets.*

direction, while the second moves in the south-east direction. One sensor is available in order to carry out the search. Its visibility over each of the target domains is represented in Fig. 4. The search is made over a 3 time periods horizon.

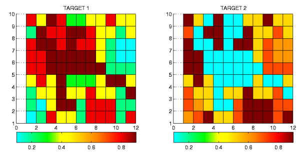

Fig. 4. *Visibility of sensor over the targets.*

Resources available for the sensor at time periods 1, 2, and 3 are respectively 10, 20 and 30.
We now present resources sharing at each time period for the myopic search plan (Fig. 5), i.e. first iteration of the FAB algorithm, and for the optimal search plan (Fig. 6), i.e. last iteration of the FAB algorithm (here $l = 4$). At a first

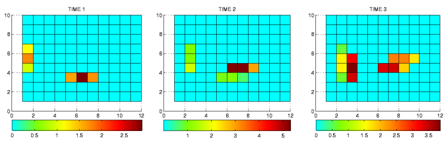

Fig. 5. *Myopic search plan, multitarget search problem.*

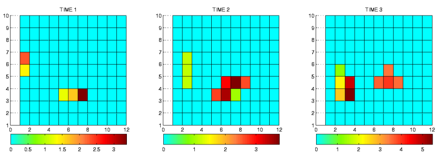

Fig. 6. *Optimal search plan, multitarget search problem.*

glance, the changes in the resource sharing between myopic and optimal search plans provides only a modest decrease of the non-detection probability: the non-detection probability falls from 0.044 to 0.032. However, this small decrease must be considered from the resource consumption point of view. In fact, if we want to obtain the same non-detection probability for a myopic search plan than for this optimal search plan, we must increase resource by 12% at each time period. Moreover, it is worth stressing that the myopic plan is yet a fairly good one.

1) The cross-cueing search: Let us now consider the cross-cueing search for a moving target, in the case where we want to detect and confirm the target at two successive time periods. The target is initially located as shown in Fig. 7. It moves north-east from its initial location. A unique sensor is available in order to perform cross-cueing of the target. Fig. 8 presents visibility of this sensor over the space of search. The search is made over a 6 time periods horizon. Resources available at time periods 1, 2, 3, 4, 5 and 6 are respectively 20, 30, 50, 30, 60 and 40.

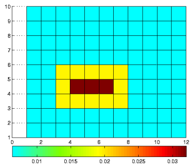

Fig. 7. *Prior on the location of the target.*

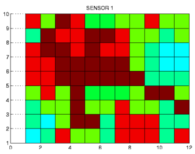

Fig. 8. *Visibility of the sensor over the space of search.*

We now present resources sharing at each time period for the myopic search plan (Fig. 9), i.e. first iteration of the FAB algorithm, and for the optimal search plan (Fig. 10), i.e. last iteration of the FAB algorithm.

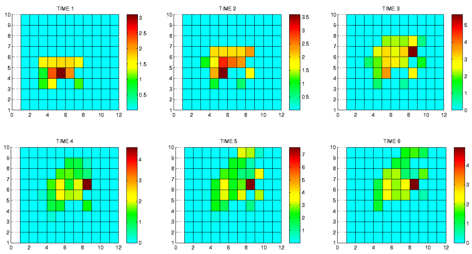

Fig. 9. *Myopic search plan, cross-cueing search problem.*

The changes in the resources sharing between myopic and optimal search plans provide a decrease of the UCC probability, which falls from 0.6961 to 0.5544. Fig. 11 illustrates the evolution of the UCC probability along iterations of the FAB algorithm. The decrease of the UCC probability between myopic and optimal search plans corresponds to a saving in resource consumption around 45% at each time period.

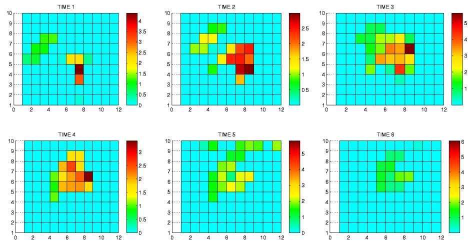

Fig. 10. *Optimal search plan, cross-cueing search problem.*

	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6
Time 2	0.8693	0.6756	0.5908	0.5653	0.5584	0.5550
Time 3	0.8037	0.6674	0.5865	0.5633	0.5574	0.5549
Time 4	0.7537	0.6597	0.5819	0.5620	0.5560	0.5546
Time 5	0.7261	0.6251	0.5735	0.5595	0.5557	0.5545
Time 6	**0.6961**	0.6022	0.5686	0.5585	0.5549	**0.5544**

Fig. 11. *Evolution of the* UCC *probability along FAB iterations*

VI. Conclusion

We have studied there two problems of importance in the search theory field, which had not been much studied before. Both of them aims to optimize detection or cross-cueing for moving targets.

By dualizing the constraints and using the separability properties of the functionals, optimal solutions of the monoperiod problems are quite economically obtained. These algorithms are then easily extended to multiperiod search via the temporal separability induced by the the Markovian hypothesis we made about target motion. The framework we developed here is sufficiently versatile to handle numerous extensions. Results show that the algorithms are quite feasible and reliable.

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