On the Effect of Data Contamination for Multitarget Tracking, Part II

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***Abstract*—This paper deals with the probabilistic data association issue in the context of multiple target tracking. In the continuation of the part I framework, we focus here on scenarios where multiple false measurements may occur. In particular, the influence of various critical parameters on the multi-tracking efficiency, i.e. the probability of correct association, is analyzed. Besides, we study the impact of the tracking scenario, including a large number of misassociations.**

I. INTRODUCTION

In multiple target tracking, a fundamental problem is to evaluate the performance of the association algorithms. Generally speaking, track accuracy is considered without considering the association problem. And yet, tracking and association are completely linked. The association issue is really determining. A remarkable exception is the work of K.C. Chang, C.Y. Chong and S. Mori [2]. However, this work is essentially oriented toward a modelling of misassociations via the effect of permutations.

The work presented here is a natural extension of the first part [1]. In part I, we analyse the problem of multiscan association and focus on the effect of the "contamination" of a target track due to extraneous measurement. The probability of correct association is used as a key performance measure. While the context we considered in Part I is a unique target and a unique false measurement, we will now deal with multiple false measurments. It keeps an analytical point of view to provide accurate closed-form approximations of the probability of correct association. The main contribution of this paper is to show that analytical calculations are still possible. Multiple extensions and applications render it quite attractive.

II. PROBLEM FORMULATION

The problem formulation is similar to the one described in the first part, [1]. In this framework, a target is moving with a rectilinear and uniform motion. Although the hypotheses made in [1] are unchanged, we consider at this stage the section III.D dedicated to multiple false measurements.

Actually, we focus in this part on the situation where multiple false measurements occur. Our aim is again to determine the probability of deciding the right association. Previously, we showed in [1] that the calculation of the probability of correct association ($P(\Delta_{f,c} \geq 0)$ can be extended to the general case. However, we encountered severe difficulties in the derivation of convenient approximations. Consequently, the feasible approaches developed in the current part will rely on the same principles but will certainly require fundamental simplifications.

Roughly, the problems we have to consider here can be split in two parts. In the first part, we focus on a given time period at which **multiple** false measurements can occur. The problem is then to associate the right measurement with the (estimated) track. In the second part, multiple false measurements can occur at various time periods. However, we assume that there is at most one false measurement for each time period. Of course, notice that multiple extensions mixing these two false association modelling can be considered.

III. THE FIRST PROBLEM: SIMULTANEOUS FALSE MEASUREMENTS

A. Problem Scenario

A target is moving on the 2-D plane (rectilinear and uniform motion). Here, we assume that multiple false measurements occur simultaneously at time period t. The number of false measurements at time t is Poisson distributed, depending on known parameters, such as the clutter density (μ) and the size of the validation gate (G). Moreover, it is assumed that each false measurement is uniformly distributed in G (see fig.1).

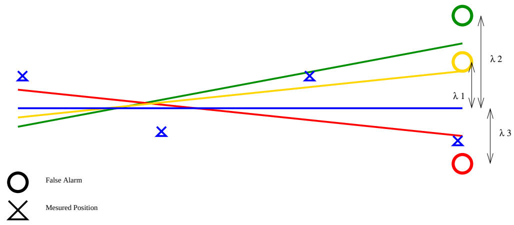

Figure 1. The multiple false measurements (at t) scenario

B. Problem Calculations

Similarly to part I, the association decision process is based on the evaluation of the association cost C_k for each association k. Let denote ca the unknown correct association that the decision process tries to determine. The probability of correct association $P(\mathsf{ca})$ is then the probability that the correct association cost is smaller than the costs of all the other associations, i.e.:

$$P(\mathsf{ca}) \;=\; P\left(C_{\mathsf{ca}} = \min_k\{C_k\}\right) . \tag{1}$$

As the false measurements are independent, we have:

$$\begin{aligned} P(\mathsf{ca}) &= \prod_k P(C_{\mathsf{ca}} \leq C_k) , \\ &= \prod_k P(\Delta_{f,c}(k) \geq 0) . \end{aligned} \tag{2}$$

Let us recall the following final result of [1]:

$$P(\Delta_{f,c}(k) \geq 0) = 1 + (a + b\lambda + c\lambda^2)e^{-\lambda^2/2} ,$$

with:

$$\left|\begin{aligned} a &= -\frac{\left(1+\sum_i \frac{\gamma_i}{i}\frac{\sqrt{\beta_N}}{\alpha_N}\right)+\frac{66\pi}{32n^2}\frac{\beta_N}{\alpha_N^2}\sum_i i^2\,\gamma_i}{2\pi}, \\ b &= \frac{\frac{6}{n}\frac{\sqrt{\beta_N}}{\alpha_N}\sum_i i\;\gamma_i}{2\pi}, \\ c &= \frac{15}{16n^2}\,\frac{\beta_N}{\alpha_N^2}\sum_i i^2\,\gamma_i . \end{aligned}\right. \tag{3}$$

and:

$$\alpha_N = \frac{N(1-N)}{(N+1)(N+2)},$$

$$\beta_N = \frac{4N^3+226N^2-66N+4}{(N+1)^2(N+2)^2} .$$

The γ_i's are just scale parameters for the approximation of a gaussian distribution.
Assuming that the λ_k are strictly different and given that there are K false measurements in the validation gate G, we obtain:

$$P(\mathsf{CA} \mid \mathsf{K}) = \prod_{\mathsf{k}=1}^{\mathsf{K}} \left(1 + (\mathsf{a} + \mathsf{b}\lambda_\mathsf{k} + \mathsf{c}\lambda_\mathsf{k}^2)\mathsf{e}^{-\lambda_\mathsf{k}^2/2}\right) . \tag{4}$$

Integrating w.r.t. the K values, we have:

$$P(\mathsf{CA}) = \sum_{\mathsf{K}=1}^{\infty} \int_{\mathsf{G}^\mathsf{K}} \prod_{\mathsf{k}=1}^{\mathsf{K}} \left[\left(1 + (\mathsf{a} + \mathsf{b}\lambda_\mathsf{k} + \mathsf{c}\lambda_\mathsf{k}^2)\mathsf{e}^{-\lambda_\mathsf{k}^2/2}\right) \mathsf{d}\Lambda\right] \mathsf{e}^{-\mu}\frac{\mu^\mathsf{K}}{\mathsf{K}!}$$

This formula gives the probability of correct association $P(\mathsf{CA})$, in the presence of multiple false measurements. It is naturally adapted to a clutter modelling via the μ (clutter density) parameter and the validation gate G values. Let us now enlarge this approach to a whole track. Henceforth, we must consider the probability of never choosing a false association. This is the aim of the next section. On the other hand, a nice work would be to determine the influence of the density of false alarms, μ. Generally speaking, such a work is very difficult, due to the high dimension of the integration domain, i.e. K. For very small values of K (broadly smaller than 10), it is possible to determine the influence of the false alarm density. This will be presented in the Simulation section.

IV. The second problem: scattered false measurements

A. Problem Scenario

In this section, we assume that false measurements occur at multiple time periods. Our objective is to investigate the effect of these false measurements on the probability of correct association. The scenario is depicted in fig. 2.

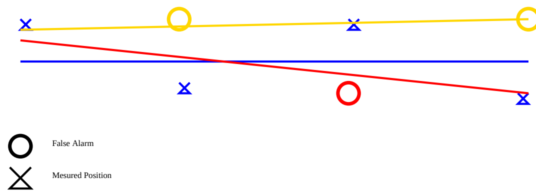

Figure 2. The association scenario number 2.

In the scenario, the following assumption is made: there is at most one false measurement at each time period. Let us now consider the problem modelling and the associated calculations.

B. The multiple false association framework

Let $\mathsf{FA}_K = (l_k)_{k=1}^K$ be the vector composed of the indices l_k of the (possible) false associations. Since the incoming calculations are based on [1], we recall a few results. First of all, let us introduce the functional $\Psi(\mathbf{e}_K)$, where $\mathbf{e}_K$ is the K size vector of errors:

$$\Psi(\mathbf{e}_K) = \frac{(\mathbf{e}_K)^T\mathcal{M}\mathbf{e}_K - \mathsf{FA}_K^T\mathcal{M}\;\mathsf{FA}_K}{2\sqrt{(\mathbf{e}_K - \mathsf{FA}_K)^T\Phi(\mathbf{e}_K - \mathsf{FA}_K)}} , \tag{5}$$

which is going to play a leading part in the analysis of the probability of correct association.

A closed-form expression of the numerator is:

$$\begin{aligned} &\mathbf{e}_K^T\mathcal{M}\mathbf{e}_K - \mathsf{FA_K}^T\mathcal{M}\mathsf{FA_K} = \\ &\sum_{k=1}^K\sum_{k'=1}^K \left(1_{\{k=k'\}} - \frac{2(2N+1-3\,l_{k'}-3\,l_k+\frac{6\,l_k^2}{N})}{(N+1)(N+2)}\right) \\ &\left(\langle \mathbf{e}_{l_k}, \mathbf{e}_{l'_k}\rangle - \langle \mathsf{fa_k}, \mathsf{fa_{k'}}\rangle\right) \\ &= \sum_{k=1}^K\sum_{k'=1}^K \alpha_N(k,k')\left(\langle \mathbf{e}_{l_k}, \mathbf{e}_{l'_k}\rangle - \langle \mathsf{fa_k}, \mathsf{fa_{k'}}\rangle\right) . \end{aligned} \tag{6}$$

Similarly, for the denominator D_{Ψ_K} of $\Psi(\mathbf{e}_K)$, we have:

$$D_{\Psi_K} = 2\sqrt{\sum_{k=1}^{K}\sum_{k'=1}^{K}\theta(l_k,l_{k'})\,\langle \mathsf{e}_{l_k}-\mathsf{fa}_k, \mathsf{e}_{l_{k'}}-\mathsf{fa}_{k'}\rangle}\ ,$$

with:

$$\begin{aligned}
&(N+1)^2(N+2)^2\,\theta(l_k,l_{k'}) = \\
&[Q_1^*(\mathsf{FA}_K,N) + (l_k+l_{k'})\,Q_2^*(\mathsf{FA}_K,N) + l_k l_{k'}\,Q_3^*(\mathsf{FA}_K,N)] \\
&-\big(\alpha_N(l_k,l_{k'})+\alpha_N(l_k,l_{k'})\big)(N+1)^2(N+2)^2.
\end{aligned} \tag{7}$$

The polynomials Q_1^*, Q_2^* and Q_3^* stand as follows:

$$\begin{aligned}
Q_1^*(\mathsf{FA}_K,N) &= \textstyle\sum_{l=0,l\notin \mathsf{FA_K}}^{N}(4N+2-6l)^2\ , \\
Q_2^*(\mathsf{FA}_K,N) &= -\tfrac{6}{\delta}\left[\textstyle\sum_{l=0,l\notin \mathsf{FA_K}}^{N}(4N+2-6l)(1-\tfrac{2\,l}{N})\right];, \\
Q_3^*(\mathsf{FA}_K,N) &= \tfrac{36}{\delta^2}\left[\textstyle\sum_{l=0,l\notin \mathsf{FA_K}}^{N}(1-\tfrac{2\,l}{N})^2\right]\ .
\end{aligned}$$

Finally, we obtain the following closed-form of $\Psi(\mathbf{e}_K)$:

$$\Psi(\mathbf{e}_K) = \frac{\displaystyle\sum_{k=1}^{K}\sum_{k'=1}^{K}\alpha_N(k,k')\left(\langle \mathsf{e}_{l_k},\mathsf{e}_{l'_k}\rangle - \langle \mathsf{fa_k},\mathsf{fa_{k'}}\rangle\right)}{2\sqrt{\displaystyle\sum_{k=1}^{K}\sum_{k'=1}^{K}\theta(l_k,l_{k'})\,\langle \mathsf{e}_{l_k}-\mathsf{fa}_k,\mathsf{e}_{l_{k'}}-\mathsf{fa}_{k'}\rangle}}\ . \tag{8}$$

In order to investigate the difficulties we have to face, let us restrict to the case of **two** false associations. Then, we have to consider a functional ψ, with [1] :

$$\begin{aligned}
&\psi = n/d \ \text{ with:} \\
&n = \|\mathbf{e}_1\|^2 + \|\mathbf{e}_2\|^2 - 2\,\lambda^2\ , d^2 = \|\mathbf{e}_1-\mathsf{fa}\|^2 - \|\mathbf{e}_2-\mathsf{fa}\|^2\ .
\end{aligned} \tag{9}$$

We are interested in the $(\mathbf{e}_1,\mathbf{e}_2)$ domain for such that $\psi \le \varepsilon$ ($\varepsilon \ll 1$). The trouble is that n can be small without the elementary terms $(\|\mathbf{e}_1\|^2-\lambda^2)$ and $(\|\mathbf{e}_2\|^2-\lambda^2)$ being small too. So, we have to "conditionate", e.g. with respect to $\mathbf{e}_2$.

Obviously, for $K = 1$, the approach from Part I [1] is relevant. It provides accurate and simple closed-form approximations of the $P(\mathsf{CA})$ probability. Yet, for $K = 2$, the approach becomes hardly feasible. For larger values of K, it involves to turn towards a **radically** different approach based on normal density approximations. A key feature of normal densities is that they are exhaustively represented by their two first moments. Further, we will explain how these moments can be easily calculated.

In order to give the general scheme, let us repeat the general (linear regression) result from [1]:

$$\mathcal{L}\left(\Delta_{\mathsf{FA}_K}\,|\,\tilde{\varepsilon}_K = \mathbf{e}_K\right) = \mathcal{N}\left[\underbrace{\mathsf{FA}_K^T\mathcal{M}\mathsf{FA}_K - (\mathbf{e}_K)^T\mathcal{M}\mathbf{e}_K}_{m_1}, \underbrace{4(\mathbf{e}_K-\mathsf{FA}_K)^T\Phi(\mathbf{e}_K-\mathsf{FA}_K)}_{v_1}\right]\ . \tag{10}$$

Assuming that the mean (m_1) and the variance (v_1) of Δ_{FA_K} are both random but with determined law, it is possible to deduce the expression of the posterior law of the Δ_{FA_K} random variable. More precisely, assume that they follow the laws $\mathcal{L}_1$ and $\mathcal{L}_2$:

$$m_1 \sim \mathcal{L}_1(\theta_1) \quad \text{and} \quad v_1 \sim \mathcal{L}_2(\theta_2)\ ,$$

with θ_1 and θ_2 being deterministic parameters. Furthermore, suppose that the density functions for $\mathcal{L}_1$ and $\mathcal{L}_2$ are respectively g_1 with support S_1 and g_2 with support S_2. Then, $h(\Delta_{\mathsf{FA}_K})$, the posterior density of Δ_{FA_K}, can be expressed simply as:

$$h(\Delta_{\mathsf{FA}_K}) = \int_{S_1}\int_{S_2} f(\Delta \mid m_1,v_1)g_2(v_1)g_1(m_1)dv_1dm_1 \tag{11}$$

At the moment, the strong point is is that, even if we do not have yet the right expression of the posterior density, we are able to consider the double integration. So, the problem we have to address now can be summed up by the following question: are there convenient approximations for these two densities whatever the number of false associations? The answer is developed in next section.

V. Approximations

A. General Case

First, let us start up with the approximation of the mean m_1 law by a normal distribution. For a high number of random variables, the Central Limit theorem allows us to make this approximation. Thus, it is possible to assume that $m_1 \sim \mathcal{N}(m_0,\sigma_0^2)$. Notice that the distribution of v_1 will be discussed **further**.

As Δ_{FA_K} and m_1 are normally distributed, the posterior density of Δ_{FA_K} can be formulated more precisely (see Appendix A for the proof):

$$\begin{aligned}
h(\Delta_{\mathsf{FA}_K}) &= \int_{S_1}\int_{S_2} f(\Delta \mid m_1,v_1)g_2(v_1)g_1(m_1)dv_1dm_1 \\
&= \int_{S_2} f_{\mathcal{N}(m_0,\sigma_0^2+v_1)}(\Delta_{\mathsf{FA}_K})g_2(v_1)dv_1
\end{aligned} \tag{12}$$

Thus, we have:

$$P(\Delta_{\mathsf{FA}_K} \ge 0) = \int_{S_2} \mathsf{erfc}\Big(\frac{\mathsf{m_0}}{\sqrt{\sigma_0^2+\mathsf{v_1}}}\Big)\mathsf{g_2}(\mathsf{v_1})\mathsf{dv_1}\ . \tag{13}$$

This expression is quite simple. The main difference with the previous one of Part I [1] is that the complexity is now reduced. Indeed, assuming that the number of misassociations is equal to K, we just have to simulate a one-dimensional Markov chain, versus a K-dimensional one. Moreover, in this setup, the precision of the approximation increases with K, thanks to the Central Limit theorem.

From now on, the goal is to express $h(\Delta_{\mathsf{FA}_K})$ (see eq. 12).So, we have to perform integration w.r.t. the variance v_1. To that aim, we have to choose a law for the variance v_1. We shall consider two solutions. The first one consists in using the Central Limit theorem, as previously,

[1] $\|\mathsf{fa}_1\|^2 = \|\mathsf{fa}_2\|^2 = \lambda^2$

and in modelling v_1 via a gaussian distribution[2]. The second solution is to calculate the right law of v_1, that is expected to be a kind of Chi-2.

Notwithstanding, whatever the chosen solution, the brute force of calculation applied to the integral (see eq. 12) is unfeasible. That is why -quite similarly to the first part- we consider again an approximation of this density by a sum of indicator functions.

B. Chi-2 and Gaussian v_1 modellings and their implications

Considering the expression of v_1, we notice (see eqs. 7,10) that it is a weighted sum of elementary quadratic forms of normal vectors ($\|\mathsf{e}_{l_k} - \mathsf{fa}_k\|^2$), with weights $\Phi_K(l_k)$. Each elementary quadratic form is Chi-square distributed. However, when the weights are different, a tractable distribution of the weighted sum is not available (see [9]). So, a first simplification is required; it consists in supposing that the weights are approximately equal [3]. In this setup, we consider that v_1 is Chi-square distributed with $2K$ degrees of freedom. Then, we obtain:

$$\begin{aligned} P(\Delta_{\mathsf{FA}_K} \geq 0) &= \int_{\mathbb{R}_+} \mathsf{erfc}(\frac{\mathsf{m}_0}{\sqrt{\sigma_0^2 + \mathsf{v}_1}}) \mathsf{f}_{\chi^2(2\mathsf{K})}(\mathsf{v}_1)\mathsf{dv}_1 \\ &= \frac{1}{2^K\Gamma(K)} \int_{\mathbb{R}_+} \mathsf{erfc}(\frac{\mathsf{m}_0}{\sqrt{\sigma_0^2+\mathsf{v}_1}}) \mathsf{v}_1^{\mathsf{K}-1} \mathsf{e}^{-\mathsf{v}_1/2}\mathsf{dv}_1 \end{aligned} \quad (14)$$

We turn now toward the second solution, i.e. the normal approximation of v_1. That yields to:

$$\begin{aligned} P(\Delta_{\mathsf{FA}_K} \geq 0) &= \int_{\mathbb{R}_+} \mathsf{erfc}(\frac{\mathsf{m}_0}{\sqrt{\sigma_0^2 + \mathsf{v}_1}}) \mathsf{f}_{\mathcal{N}(\mathsf{v}_0,\mathsf{s}_0^2)}(\mathsf{v}_1)\mathsf{dv}_1 \;, \\ &= \frac{1}{s_0\sqrt{2\pi}} \int_{\mathbb{R}} \mathsf{erfc}(\frac{\mathsf{m}_0}{\sqrt{\sigma_0^2 + \mathsf{v}_1}}) \mathsf{e}^{-\frac{(\mathsf{v}_1-\mathsf{v}_0)^2}{2\mathsf{s}_0^2}} \mathsf{dv}_1 \;, \end{aligned} \quad (15)$$

where the parameters m_0, σ_0^2, v_0 and s_0^2 are given by:

$$\begin{aligned} m_0 &= \mathbb{E}_{XY} \sum_{k=1}^{K}\sum_{k'=1}^{K} \alpha_N(k,k')(x_k.x_{k'} + y_k.y_{k'} \;, \\ &\quad -\lambda_k.\lambda_{k'}) \;, \\ \sigma_0^2 &= \mathbb{V}_{XY} \sum_{k=1}^{K}\sum_{k'=1}^{K} \alpha_N(k,k')(x_k.x_{k'} + y_k.y_{k'} \;, \\ &\quad -\lambda_k.\lambda_{k'}) \;, \\ v_0 &= 4\mathbb{E}_{XY} \sum_{k=1}^{K}\sum_{k'=1}^{K} \theta(k,k')((x_k - \lambda_k).(x_{k'} - \lambda_{k'}) \;, \\ &\quad +y_k.y_{k'}) \;, \\ s_0^2 &= 16\mathbb{V}_{XY} \sum_{k=1}^{K}\sum_{k'=1}^{K} \theta(k,k')((x_k - \lambda_k).(x_{k'} - \lambda_{k'}) \;, \\ &\quad +y_k.y_{k'}) \;. \end{aligned} \quad (16)$$

Calculating the expectations of eq. 16, we obtain (see Appendix B):

$$\begin{aligned} m_0 &= 2\sum_{k=1}^{K} \alpha_N(k,k) - \lambda^2 \sum_{k=1}^{K}\sum_{k'=1}^{K} \alpha_N(k,k') \;, \\ \sigma_0^2 &= 4\sum_{k=1}^{K} \alpha_N^2(k,k) - 2\sum_{k=1}^{K}\sum_{k'=1}^{K} \alpha_N^2(k,k') \;, \\ v_0 &= 4\sum_{k=1}^{K} \theta(k,k')(2+\lambda_k^2) + 4\sum_{k=1}^{K}\sum_{k'=1}^{K} \theta(k,k')\lambda_k\lambda_{k'} \;, \\ s_0^2 &= 64\sum_{k=1}^{K} \theta^2(k,k)(1+\lambda_k^2) + 32\sum_{k=1}^{K}\sum_{k'=1}^{K} \theta^2(k,k') \;. \end{aligned} \quad (17)$$

However, though the above approximations (see eqs 15, 16) are quite simple, they are not sufficiently explicit. In particular, it is still necessary to perform integration w.r.t. v_1. Next section is dedicated to the simplification of this integration.

C. Approximating v_1 via a sum of indicator functions

Again, we turn toward the method developed in part I[1], i.e. the approximation of v_1 via a sum of indicator functions. We obtain then:

$$P(\Delta_{\mathsf{FA}_K} \geq 0) = \sum_{i=1}^{n} \gamma_i \; \mathsf{erfc}(\frac{\mathsf{m}_0}{\sqrt{\sigma_0^2 + \mathsf{v}_{1_i}}}) \; \mathbf{1}_{\mathsf{i}\in\mathsf{I}_\mathsf{i}} \quad (18)$$

Now, we just have to find values of weights γ_i and intervals I_i. Refer to [1] where this problem and the associated

[2]The limitation of this approach is that the variance then has a none null probability to be negative!

[3]A reasonable assumption in the current case with its associated hypotheses.

solution are detailed. In next Simulations section, all the results of the approximations are displayed.

VI. Simulations

First, we have to examine the validity of the normal approximations of m_1 and v_1. For a value of K as small as 2, this is presented in fig. 3. It corresponds to a χ^2 approximation of v_1 and two false measurements. The result is quite good, even for a very small of K. Notice that we would get a highly better result if the number of false measurements was greater.

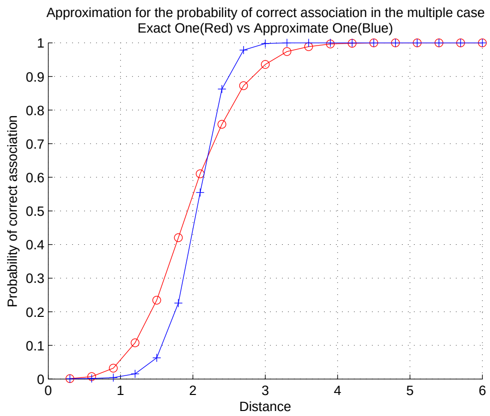

Figure 3. Approximation of the Probability of correct association in multiple successive false alarm case.

The figure 4 displays the difference between four and eight false measurements. That difference looks to be a bit more than a simple translation. The growing stage is not the same, and will be bigger and bigger with increasing number of false alarm as we previously explained. In the case of a very big number of false alarm, we will have a probability that would be equal to 0 or 1, depending on the distance λ. Theoretically, for a small value of λ, an infinite number of false associations leads necessarily to a minimum of one error. And that's what we observe. And the main result is that having eight false alarms, at a constant distance of 3 is equivalent to a double false measurement scenario, with distance 2.5 and only one false alarm, with a distance of 1.8.

Next, let us consider now that the multiple false measurements can appear at the same time. In such a scenario, we would like to know the effect of the dimensioning parameters on the probability of corrct association. In figure 5, the probability of correct association is represented versus the number of false alarms. The $P(\mathsf{CA})$ is 0.18 for one false measurement, but it becomes quite smaller when K increases.

In the next stage, the probability of correct association is evaluated for the complete tracking scenario, with its clutter density. Not surprisingly, figure 6 shows that the probability of correct association decreases while the clutter density increases. But, what is more surprising

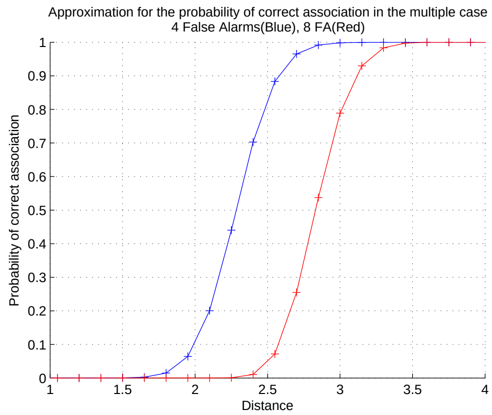

Figure 4. Probability of correct association in multiple successive false alarm case.

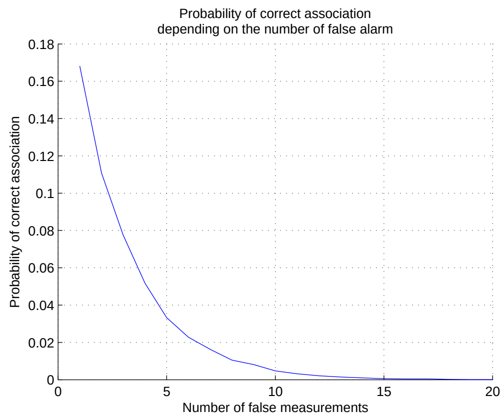

Figure 5. Probability of correct association in multiple simultaneous false alarm case.

is the very low value of the probability of correct association. The reason is that the volume of observation of the target is tiny, involving many false alarms for these values of clutter density. Many false measurements are observed close to the target, leading to an important probability of false association.

There is a last scenario we would like to study. Assume that two targets have crossing trajectories. The scenario is presented by figure 7. We want to know the probability of correct association, knowing the angle θ. This is what is presented in figure 8.

The first point after that computation is to observe the two probabilities. A simple conclusion is that if the number of false measurements increases, but the angle stays the same, the probability decreases. So, to keep the same probability of correct association, you should have, in this case, 5 more degrees if you want to allow 4 more false measurements.
There is something quite important about this simulation. As you can observe on the figure 7, there are no detections when the two targets are crossing. If we had some, then

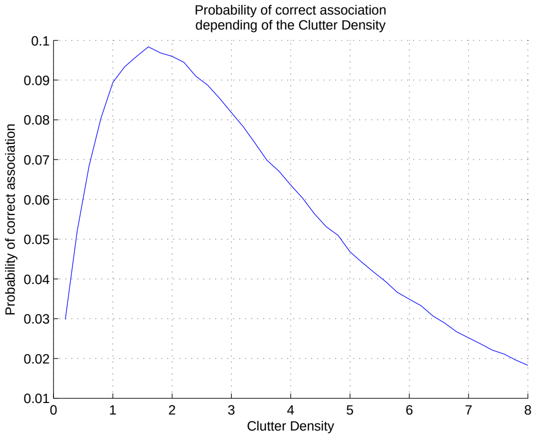

Figure 6. Probability of correct association varying with the clutter density.

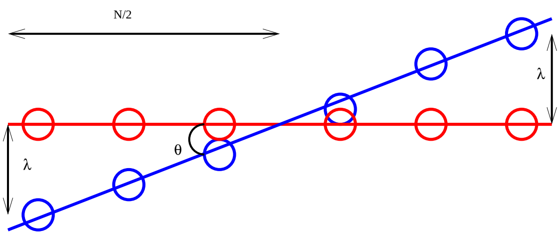

Figure 7. Two crossing targets

the two measurements at the crossing scan would have been the same. And we would have only one measure and then following, only one target remained. And the probability of correct association would have been highly better (Taking one measure or the other as no kind of importance for the tracking of the target). Then, there would have $K-1$ false measurements, and they would be a little bit far from the right ones. That's why correct association probability would have been higher. And why we took the other scenario, the pessimistic one.

There are also many extensions to these results, some of whom are discussed in the conclusion.

VII. Conclusion

In the article, the influence of multiple false measurements is studied. Among the multitude of scenarios that can take place, there are two major categories, which depends on whether the false measurements appear at the same time or successively. Both of them have been treated in the article. Furthermore, a third scenario has been taken into account. It consists in many false associations that occur at each scan. As a matter of fact, it is just a combination of the two previously mentioned categories.

Focusing on the probability of correct association, our work leads to simplified expressions. Moreover, by a great effort of relevant approximations, we provide easy ways to compute them. As a result, it is possible to analyze the influence of important parameters, such as the clutter density, the number of false alarms or the angle between two trajectories. The last result, concerning two crossing targets, is very promising since it can be applied directly to a real situation. For example, assume that a radar has locked onto a moving target and is currently tracking it. Suppose that the target suddenly ejects a decoy. What is the probability that the radar goes on tracking correctly the target, i.e. it keeps getting the right association? How does it depend on the angle of the two trajectories? These are examples of further works that this article allows.

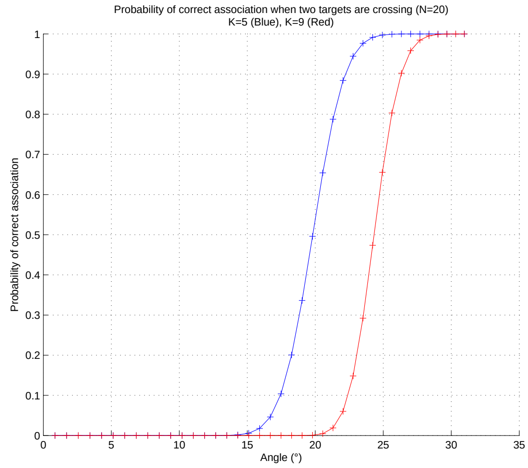

Figure 8. Probability of correct association varying with the angle θ

VIII. Appendix A

The aim of this Appendix is to proove that the density of a **normal** random variable, whose mean is normally distributed (while its variance is constant), is itself a normal random variable and to determine its parameters. Assume that the random variable x has the following (conditional) distribution:

$$X \mid m \sim \mathcal{N}(m, \sigma^2)\ , \tag{19}$$

with $m \sim \mathcal{N}(\theta, s^2)$. Then, integrating over m, we have:

$$\begin{aligned} h(x) &= \int_{\mathbb{R}} f(x \mid m) g(m)\, dm\ , \\ &= \int_{\mathbb{R}} \frac{1}{2\pi\sigma s} e^{-\left(\frac{x-m}{\sqrt{2\sigma^2}}\right)^2 - \left(\frac{m-\theta}{\sqrt{2s^2}}\right)^2} dm \end{aligned} \tag{20}$$

Denting now as $N(x,m,\theta)$ the variable part of the integrand, we have:

$$\begin{aligned}
N(x,m,\theta) &\overset{\delta}{=} \tfrac{(x-m)^2}{\sigma^2} + \tfrac{(m-\theta)^2}{s^2} \ , \\
&= \tfrac{1}{(\sigma s)^2}\Big[s^2x^2 - 2s^2xm + s^2m^2 + \sigma^2 m^2 \\
&\quad -2\sigma^2 m\theta + \sigma^2\theta^2\Big] \ , \\
&= \tfrac{1}{(\sigma s)^2}\Big[m^2(s^2+\sigma^2) - 2m(s^2x+\sigma^2\theta) \\
&\quad +x^2s^2 + \sigma^2\theta^2\Big] \ , \\
&= \tfrac{s^2+\sigma^2}{(\sigma s)^2}\Big[m^2 - 2m\tfrac{s^2x+\sigma^2\theta}{s^2+\sigma^2} + x^2\tfrac{s^2}{s^2+\sigma^2} \\
&\quad +\theta^2\tfrac{\sigma^2}{s^2+\sigma^2}\Big] \ , \\
&= \tfrac{s^2+\sigma^2}{(\sigma s)^2}\Big[\big(m - \tfrac{s^2x+\sigma^2\theta}{s^2+\sigma^2}\big)^2 + x^2\tfrac{s^2}{s^2+\sigma^2} \\
&\quad +\theta^2\tfrac{\sigma^2}{s^2+\sigma^2} - \tfrac{(s^2x+\sigma^2\theta)^2}{(s^2+\sigma^2)^2}\Big] \ .
\end{aligned} \tag{21}$$

It is worth now to isolate the terms involving the m parameter. More precisely, we have:

$$h(x) = \frac{1}{2\pi\sigma s}\int_{\mathbb{R}} \exp\left[-\left(\frac{1}{2\sigma^2}+\frac{1}{2s^2}\right)\left(m - \frac{\frac{x}{\sigma^2}+\frac{\theta}{s^2}}{\frac{1}{\sigma^2}+\frac{1}{\sigma^2}}\right)\right] T_2(x,\theta) \ . \tag{22}$$

To perform the above calculation, it is sufficient to consider the following change of variable:

$$t = \left(m - \frac{\frac{x}{\sigma^2}+\frac{\theta}{s^2}}{\frac{1}{\sigma^2+\frac{1}{\sigma^2}}}\right)\left(\frac{1}{\sigma^2}+\frac{1}{s^2}\right)^{1/2} \ .$$

Then, we have just to factorize conveniently the $T_2(x,\theta)$, so as to render apparent the square:

$$\begin{aligned}
T_2(x,\theta) &= \tfrac{s^2+\sigma^2}{(\sigma s)^2}\Big[x^2\tfrac{s^2}{s^2+\sigma^2} + \theta^2\tfrac{\sigma^2}{s^2+\sigma^2} - \tfrac{(s^2x+\sigma^2\theta)^2}{(s^2+\sigma^2)^2}\Big] \ , \\
&= \tfrac{x^2}{\sigma^2} + \tfrac{\theta^2}{s^2} - \tfrac{1}{\sigma^2+s^2}\Big(\tfrac{s^2}{\sigma^2}x^2 + 2\theta x + \tfrac{\sigma^2}{s^2}\theta^2\Big) \ , \\
&= \tfrac{x^2(\sigma^2+s^2)}{\sigma^2(\sigma^2+s^2)} + \tfrac{\theta^2(\sigma^2+s^2)}{s^2(\sigma^2+s^2)} - \tfrac{s^2x^2}{(\sigma^2+s^2)\sigma^2} \\
&\quad -\tfrac{\sigma^2\theta^2}{s^2(\sigma^2+s^2)} - \tfrac{2\theta x}{(\sigma^2+s^2)}\Big) \ , \\
&= \tfrac{x^2}{\sigma^2+s^2} + \tfrac{\theta^2}{\sigma^2+s^2} - \tfrac{2\theta x}{\sigma^2+s^2} \ , \\
&= \tfrac{(x-\theta)^2}{\sigma^2+s^2} \ .
\end{aligned} \tag{23}$$

The result is that we really have a gaussian density, with mean θ, and variance (σ^2+s^2), i.e. :

$$h(x) = \frac{1}{2\pi(s^2+\sigma^2)}\exp\left[-\frac{1}{2(s^2+\sigma^2)}(x-\theta)^2\right] \ . \tag{24}$$

IX. Appendix B

The aim of this appendix is the calculation of the values of m_0, σ_0^2, v_0 and s_0^2. Calculations are a bit long, we then just express here the main stages to perform the results, doing this toward m_{01}, σ_{01}^2,v_{01} and s_{01}^2, that are parts of the right ones. First, we have:

$$\begin{aligned}
m_{01} &= \mathbb{E}_{XY}\left[\sum_{k=1}^{K}\alpha_N(k,k)(x_k^2+y_k^2-\lambda_k^2)\right] \ , \\
&= \sum_{k=1}^{K}\alpha_N(k,k)\ \mathbb{E}_{XY}\left[(x_k^2+y_k^2-\lambda_k^2)\right] \ , \\
&= \sum_{k=1}^{K}\alpha_N(k,k)\left(\mathbb{E}_X\left[x_k^2\right]+\mathbb{E}_Y\left[y_k^2\right]-\lambda_k^2\right) \ , \\
&= \sum_{k=1}^{K}\alpha_N(k,k)(2-\lambda_k^2) \ .
\end{aligned} \tag{25}$$

Expliciting σ_0^2 is not much more difficult, just notice that X^2 and Y^2 are independant, and then the variance of the sum is equal to the sum of the variance:

$$\begin{aligned}
\sigma_{01}^2 &= \mathbb{V}_{XY}\left[\sum_{k=1}^{K}\alpha_N(k,k)(x_k^2+y_k^2-\lambda_k^2)\right] \ , \\
&= \sum_{k=1}^{K}\alpha_N^2(k,k)\mathbb{V}_{XY}\left[(x_k^2+y_k^2)\right] \ , \\
&= \sum_{k=1}^{K}\alpha_N^2(k,k) \ .
\end{aligned} \tag{26}$$

The calcultation of v_0 is quite similar to the m_0's one, just notice that the first order moment of Y is null:

$$\begin{aligned}
v_{01} &= \mathbb{E}_{XY}\left[4\sum_{k=1}^{K}\theta(k,k)(x_k^2+(y_k-\lambda_k)^2)\right] \ , \\
&= 4\sum_{k=1}^{K}\theta(k,k)\mathbb{E}_{XY}\left[(x_k^2+(y_k-\lambda_k)^2)\right] \ , \\
&= 4\sum_{k=1}^{K}\theta(k,k)\mathbb{E}_X\left[x_k^2\right]+\mathbb{E}_Y\left[(y_k-\lambda_k)^2\right] \ , \\
&= 4\sum_{k=1}^{K}\theta(k,k)\left(\mathbb{E}_X\left[x_k^2\right]+\mathbb{E}_Y\left[y_k^2-2\lambda_k y_k+\lambda_k^2\right]\right) \ , \\
&= \sum_{k=1}^{K}4\theta(k,k)(2+\lambda_k^2) \ .
\end{aligned} \tag{27}$$

The last calculation is a bit more intricated:

$$\begin{aligned} s_{01}^2 &= \mathbb{V}_{XY}\left[4\sum_{k=1}^{K}\theta(k,k)(x_k^2+(y_k-\lambda_k)^2)\right] , \\ &= 16\sum_{k=1}^{K}\theta^2(k,k)\mathbb{V}_{XY}\left[(x_k^2+(y_k-\lambda_k)^2)\right] , \\ &= 16\sum_{k=1}^{K}\theta^2(k,k)\Big(\mathbb{V}_X\left[x_k^2\right]+\mathbb{V}_Y\left[(y_k-\lambda_k)^2\right]\Big) \end{aligned} \quad (28)$$

The (small) problem we have to solve is the calculation of the second term. This is achieved via classical reults about moments of a normal random variable:

$$\begin{aligned} \mathbb{V}_Y\left[(y_k-\lambda_k)^2\right] &= \mathbb{E}\Big[(y_k-\lambda_k)^4\Big]-\mathbb{E}^2\Big[(x_k-\lambda_k)^2\Big] \\ &= \mathbb{E}\Big[x_k^4-4x_k^3\lambda_k+6x_k^2\lambda_k^2-4x_k\lambda_k^3+\lambda_k^4\Big] \\ &\quad -(1+\lambda_k^2)^2 \\ &= 3+6\lambda_k^2+\lambda_k^4-1-2\lambda_k^2-\lambda_k^4 \\ &= 2+4\lambda_k^2 \end{aligned} \quad (29)$$

And then, we have:

$$s_{01}^2 = 64\sum_{k=1}^{K}\theta^2(k,k)(1+\lambda_k^2) \quad (30)$$

REFERENCES

[1] A. ICKOWICZ, J.-P. LE CADRE and P. MINVIELLE, On the Effect of Data Contamination for Multitarget Tracking, Part I. *International Conference on Information Fusion ISIF 2007*, Quebec, July 2007.

[2] K.-C. CHANG, S. MORI and C.Y. CHONG, Performance Evaluation of Track Initiation in Dense Target Environment. IEEE *Trans. on Aut. Control*, vol. ac 29, no. 7, July 1984, pp. 585–594.

[3] A. BERMAN and A. HAMMER, False Alarm Effects on Estimation in Multitarget Trackers. IEEE *Trans. on* AES, vol. 27, no. 4, July 1991, pp. 675–681.

[4] Y. BAR-SHALOM and X. RONG LI, Effectiveness of the Likelihhod Function in Logic-Based Track Formation. IEEE *Trans. on* AES, vol. 27, no. 1, Jan. 1991, pp. 184–187.

[5] Y. BAR-SHALOM and E. TSE, Tracking in a Cluttered Environment With Probabilistic Dat Association. *Automatica* , vol. 11, no. 1, Jan. 1991, pp. 451–460.

[6] R.J. FITZGERALD, Simple Tracking Filters: Closed-Form Solutions. IEEE *Trans. on* AES, vol. 17, no. 6, Nov. 1981, pp. 781–785.

[7] S.N. GUPTA and S.M. AHN, Closed-Form Solutions of Target-Tracking Filters with Discrete Measurements. IEEE *Trans. on* AES, vol. 19, no. 4, July 1983, pp. 532–538.

[8] S.R. ROGERS, Diffusion Analysis of Tracl Loss in Clutter. IEEE *Trans. on* AES, vol. 27, no. 2, March 1991, pp. 380–387.

[9] H. SOLOMON and M.A. STEPHENS, Distribution of a Sum of Weighted Chi-Square Variables. *Journal of the American Statistical Association*, vol. ac 72, no. 360, December 1977, pp. 881–885.