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Spatio-temporal multi-mode information management for moving target detection

Frédéric Dambreville [a], Jean-Pierre Le Cadre [b,*]

[a] DGA/CTA/DT/GIP, Centre Technique d'Arcueil, 16 bis Av. Prieur de la Côte d'Or, 94114 Arcueil, France
[b] IRISA/CNRS, Campus de Beaulieu, 35042 Rennes Cedex, France

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Abstract

This paper deals with the resource management for the detection of a moving target. Based on a generalized linear formalism, an algebraic framework for spatio-temporal optimization of the search efforts is developed, which allows management of multi-modes resources under various rules: *modalization, conditionality, parallelizing*. This formalism is an extension of Koopman/Brown search model and requires a continuous or pseudo-continuous hypothesis about the detection resources. This formalism is sufficiently general to provide a convenient framework for a wide variety of sensor management problems, even if practical applications require additional work for rendering more precise the particular modelling of detection resource.

Keywords: Sensor management; Resource allocation; Optimization; Detection

1. Introduction

This paper deals with the management of modes and resources for detecting a moving target. The searcher has available multiple detection devices (e.g. radar, IR, sonar) which can also work on various modes. These modes can be related to visibility factors (e.g. range, size of search sectors, etc.) and/or to resource constraints (e.g. resource renew, discretion constraints). In this set-up a detection (or search) problem is characterized by three pieces of data: (i) the probabilities of the searched target being in various possible positions, (ii) the local detection probability that a particular amount of local search effort could detect the target, (iii) the total amount of searching effort available. The problem is to find the optimal distribution of the detection (search) effort that maximizes the probability of detection.

The Koopman/Brown general formalism of search theory [1,9,10,12] will be used subsequently and introduced in Section 2. This formalism requires a continuous or pseudo-continuous hypothesis about the detection resources, which restricts the scope of this paper to pseudo-continuous problems. Moreover, this formalism assumes that the maximal level of false alarm is preset for each individual sensor before the search optimization. Various discussion may be done about this choice, and there exists some extension of this model involving other global criteria about false alarm. But whatever, since we are handling numerous sensor, a global criteria is required for tuning false alarms (denoted f.a.). Thus, although the detection/f.a. tuning of the individual sensor is a necessary characterization of this sensor, it is outside the scope of the global management problems we are interested in. The principal contribution of this paper is a versatile and original formalism capable of handling the management of complex and interacting detection systems. In particular, this formalism has to take into account the following main points:

- Search resources of different types (e.g. radar, ESM, IR, sonar) may collaborate in some complex search situations. This modelling concerns both the detec-

* Corresponding author. Tel.: +33-2-9984-7224; fax: +33-2-9984-7171.
E-mail addresses: frederic.dambreville@dga.defense.gouv.fr (F. Dambreville), lecadre@irisa.fr (J.-P. Le Cadre).

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Nomenclature

$k, \rho, x_k \in E$ period of detection, type of resource, a cell of the search space E at period k
$\mathbf{x} = (x_1, \ldots, x_T)$ target trajectory for a move during T periods
$\varphi_k(x_k)$ (resp. $\varphi_k^\rho(x_k)$) local search effort (resp. of type ρ) applied on cell x_k at period k
ϕ_k (resp. ϕ_k^ρ) total amount of search effort (resp. of type ρ) applied on the whole search space E at period k
$\alpha(\mathbf{x})$ probabilistic target distribution
$p_{k,x_k}(\varphi_k(x_k))$ (resp. $p_{k,x_k}^\rho(\varphi_k^\rho(x_k))$) conditional non-detection probability on cell x_k and at period k (resp. for the resource type ρ)
& parallelization operator on detection systems, AND on the constraints
$+$ modalization operator on detection systems, *disjunction* on resources
$\succ$ conditionality operator, dependency between detection systems

tion properties of resources (visibility factors) and their temporal behavior (renewing, redeployment).

- The detection tools themselves may run *simultaneously* in several modes (e.g. Electronically Steered Array (ESA)). Another example is the detection capabilities which, for most systems (radar, IR, sonar) is a compromise between the width of the searched area, the probability of detection versus an acceptable number of false alarms. From an operational point of view, the detection resources are subject to (ambivalent) constraints about their detection power, discretion or moving ability. For these reasons, each operating mode will be characterized by some specific visibility factors and specific temporal behavior.
- In a detection system, various search devices may collaborate, communicate and complex links are then established between these means. In particular, all the resources applied to the search are not necessarily real detection tools. For instance, some will have essentially a logistic function (transport or deployment resources, human means, support, ...), and conditional relations then hold between these types of resources and those specifically devoted to detection.

This formalism will take the form of a *description language* and will apply on some linear constrained problems. The interest of linear constraints for modelling resource evolution is clear when simple actual examples are considered. Imagine that you need two apples (a) and one orange (o) to make a dessert (d). Then, if a, o, d are the respective possible quantity of apple, orange and dessert, you will obtain the following linear constraints $2a \geqslant d$ and $o \geqslant d$, describing the problem. Now, assume that with two oranges, you could also make one juice (j). Then, what are the constraints describing this problem? It is easy to derive the linear constraints $2a \geqslant d$ and $2o \geqslant 2d + j$. Such ideas are guessed in the axiomatic of linear logic, which is a language of resource management. However, we describe here a specific language more appropriate to the resource management problems we are interested in. In this language defined later, our juice and dessert problem may be described by the formula
$(o \succ o' + o'')\&(o'\&2a \succ d)\&(2o'' \succ j)$,
which means a *AND* between several formula:

Formula $o \succ o' + o''$: there are oranges, o', used for dessert and oranges, o'', used for juice (choice of a *using mode* for the oranges). The constraints associated to this formula is simply $o \geqslant o' + o''$,
Formula $o'\&2a \succ d$: one orange is used with two apples to make a dessert (*parallelization* of the orange and apple use). The associated constraints are $2a \geqslant d$ and $o' \geqslant d$,
Formula $2o'' \succ j$: two oranges are needed for a juice. The associated constraint is $2o'' \geqslant j$.

It is easy to check that these four constraints are equivalent to $2a \geqslant d$ and $o \geqslant 2d + j$, by eliminating o' and o''. Of course we will use such principles for modelling management problems.

The definition of this language constitutes the backbone of this paper. Using it, it is possible to manage a class of solvable resource allocation problems, which involve multi-mode, multi-resource scheduling, resource dependency and any combination of these operations [7]. In Section 3, the resource modelling is defined and detailed precisely, as well as the various ways to combine them. The elementary operators (+, &, scalar product, $\succ$) are then presented. An algorithm for solving associated optimization problems is then briefly described and is illustrated on examples involving complex detection systems (see Section 5).

2. A general setting of the detection framework

A target moving in a search space E is to be detected. This space E is considered continuous. [1] In this article, E

[1] We will make a discretization for the practical examples.

is both the space of the target possible positions and of the possible placement areas for the detection tools. The detection is achieved during T periods, each period being brief enough so that the real trajectory of the target may be modeled properly by the vector $\mathbf{x} = (x_1, \dots, x_T) \in E^T$, where x_k represents an *averaged* position of the target during the period k. The probabilistic prior about $\mathbf{x}$ is given by a probabilistic density $\alpha(\mathbf{x})$. Both for algorithmic reasons and model genericity a Markovian assumption is made: α is taken as a product of elementary densities: [2]

$$\alpha(\vec{x}) = \alpha_{1,2}(x_1, x_2) \cdots \alpha_{T-1,T}(x_{T-1}, x_T). \tag{1}$$

A given amount of search effort ϕ_k is available at each period k. In the "classical" setting [3], these effort amounts are fixed and constrain the optimization problem. At each period k, the search effort ϕ_k may be distributed [3] throughout E. The local search effort, applied to the point $x_k \in E$ at time k, is denoted $\varphi_k(x_k)$ and obeys to the following constraints:

$$\forall k \in \{1, \dots, T\}, \quad \varphi_k \geqslant 0 \quad \text{and} \quad \int_E \varphi_k(x_k)\,\mathrm{d}x_k = \phi_k. \tag{2}$$

The local efforts φ condition the local detection probability. We call $p_{k,x_k}(\varphi_k(x_k))$ the conditional probability not to detect within the period k, when the target location is x_k. The problem is to find φ so as to minimize the global probability of non-detection $P_{nd}(\varphi)$ under the constraint (2). An independence hypothesis on elementary detections yields

$$P_{nd}(\varphi) = \int_{E^T} \alpha(\vec{x}) \prod_{k=1}^{T} p_{k,x_k}(\varphi_k(x_k))\,\mathrm{d}\mathbf{x}. \tag{3}$$

The now classic solution of this difficult optimization problem is the Forward And Backward (here denoted FAB) algorithm [3,12]. The main ingredient of this method is the use of the *Markovian assumption* about the density $\alpha(\vec{x})$.

Let us now extend this formalism. From now on, we define $\mathscr{T} = \{1, \dots, T\}$ the set of temporal indices, and $\mathscr{R} = \{1, \dots, r\}$ the set of indices of resource type (or mode). For each index $(\rho, k) \in \mathscr{R} \times \mathscr{T}$ are defined variables of local resources, φ_k^ρ, a variable of global resource, ϕ_k^ρ, and an associated *non-detection* function, p_k^ρ. All these definitions constitute the primary framework of our detection system. In addition, it is necessary to define a set of constraints on the variables of global resources.

A *system of labeled constraints* is defined as a set $\Sigma \subset \mathbb{R}^{\mathscr{T} \times \mathscr{R}} \times \mathbb{R} \times \mathscr{T}$. Further this formal definition, each element $(a, \psi, \tau) \in \Sigma$ is referring to a particular *linear* constraint of the form $\sum_{k,\rho} a_k^\rho \phi_k^\rho - \psi \leqslant 0$ and an associated temporal labeling τ. Associated with this system of labeled constraints is the following optimization problem on the variables φ and ϕ:

Minimize

$$P_{nd}(\varphi) = \int_{E^T} \alpha(\mathbf{x}) \prod_{k=1}^{T} \prod_{\rho=1}^{r} p_{k,x_k}^\rho(\varphi_k^\rho(x_k))\,\mathrm{d}\mathbf{x},$$

under the constraints

$$\begin{aligned} &\varphi \geqslant 0, \quad \phi \geqslant 0, \\ &\forall \rho \in \{1, \dots, r\}, \quad \forall k \in \{1, \dots, T\}, \\ &\quad \int_E \varphi_k^\rho(x_k)\,\mathrm{d}x_k = \phi_k^\rho, \\ &\forall (a, \psi, \tau) \in \Sigma, \quad \sum_{k,\rho} a_k^\rho \phi_k^\rho - \psi \leqslant 0. \end{aligned} \tag{4}$$

Of course, the last constraint $\forall (a, \psi, \tau) \in \Sigma$, $\sum_{k,\rho} a_k^\rho \phi_k^\rho - \psi \leqslant 0$ may be rewritten by means of labeled matrix, and becomes $A\phi \leqslant \psi$, where $A = (a_k^\rho)_{(a,\psi,\tau)\in\Sigma;(k,\rho)\in\mathscr{T}\times\mathscr{R}}$ is the matrix of constraints, with the time labeling τ for each row numbered (a, ψ, τ). It is noticeable that an independence assumption of the detection is made in the definition of P_{nd}. An algorithm for solving (4) will be shortly described in Section 4. In the following section, we define in this new formalism some useful basic detection systems, as well as some operators to mix them.

3. Behavioral resource modelling and resource operators

In the next definitions, we will make a distinction between the resource availability, characterized by the vector of priorly available resources ψ, and the resource properties, characterized by the constraints coefficients a. In the following, we may skip ψ from some constraints definitions, or, inversely, only consider ψ without the coefficients a. The reader should not be surprised by that, since this is purely formal. These apparent contradictions will be solved by the use of operators.

Representation of renewable resources: We consider a type of resources, which are capable of renew after $\Delta T \in \mathbb{N}^* \cup \{\infty\}$ periods (time for replenishment, for moving, etc.). Denote $\varrho \in \mathscr{R}$ the index associated to this resource type. We will define a system of constraints, while taking into account the resource renewing. This definition results in a balance of the resources during the search (sum of the consumed and of the generated efforts) and is obtained recursively. For the detection period 1, the only costs are resultant of the first period of search and are thus equal to ϕ_1^ϱ. For the detection period ℓ, the cost ϕ_ℓ^ϱ of the currently used resources as well as the possibly negative cost $-\phi_{\ell-\Delta T}^\varrho$ (resource renew) are added to the balance of period $\ell - 1$. This yields the

[2] It is noteworthy that the function $\alpha_{1,2}(x_1, x_2)$ contains both the origin location component and the first moving component describing the Markovian movement of the target.

[3] It is assumed that the search amount ϕ_k is indefinitely divisible.

following system of constraints, since the resource balance has to be ever positive:

$$\forall \ell < \Delta T,\ \sum_{k=1}^{\ell} \phi_k^{\varrho} \leqslant 0, \quad \text{and} \quad \forall \ell \geqslant \Delta T,\ \sum_{k=\ell-\Delta T+1}^{\ell} \phi_k^{\varrho} \leqslant 0. \tag{5}$$

For any $\ell \in \mathscr{T}$, define the vectors $\mathbf{a}^{(\ell)}$ by (k: row index):

$\mathbf{a}_k^{(\ell)\varrho} = 1$, when $\max\{0, \ell - \Delta T\} < k \leqslant \ell$;

$\mathbf{a}_k^{(\ell)\rho} = 0$, else.

The system of labeled constraints for this type of resource, renewable after ΔT periods, is given by the set $\Sigma = \{(\mathbf{a}^{(\ell)}, 0, \ell)/\ell \in \mathscr{T}\}$, also denoted by the generic term $\Sigma_{R\Delta T}$. In the sequel, these kind of resources will be generally indexed by the subscript $R\Delta T$. [4] The system $\Sigma_{R\Delta T}$ only describes the temporal behavior. Information about the amount of priorly available resources is obtained by means of the + operator.

Gradually renewable resources: Now let us consider a type of resource, which renew gradually according to a parameter sequence $\gamma = (\gamma_k)_{k\geqslant 1}$. This means that for one amount of resource used at period $k \in \mathscr{T}$, then a $\gamma_{\Delta k}$ resource amount is regenerated at period $k + \Delta k$ ($\sum_k \gamma_k \leqslant 1$). Denote $\varrho \in \mathscr{R}$ the index associated to this resource type. For the detection period 1 (constraint C_1^{ϱ}), the only costs are resultant of the first period of search and are thus equal to ϕ_1^{ϱ}. For the detection period ℓ (constraint C_ℓ^{ϱ}), the cost ϕ_ℓ^{ϱ} of the currently used resources as well as the possibly negative costs $-\gamma_k \phi_{\ell-k}^{\varrho}$ (resource renew) are added to the balance of period $\ell - 1$, i.e.,

$$\begin{cases} \phi_1^{\varrho} \leqslant 0, \\ \vdots \\ \sum_{k=1}^{\ell} \phi_k^{\varrho} - \sum_{k=2}^{\ell} \sum_{j=1}^{k-1} \gamma_j \phi_{k-j}^{\varrho} \leqslant 0, \\ \vdots \\ \sum_{k=1}^{T} \phi_k^{\varrho} - \sum_{k=2}^{T} \sum_{j=1}^{k-1} \gamma_j \phi_{k-j}^{\varrho} \leqslant 0. \end{cases} \tag{6}$$

For any $\ell \in \mathscr{T}$, define the vectors $\mathbf{a}^{(\ell)}$ by

$$\mathbf{a}_k^{(\ell)\varrho} = 1 - \sum_{k=1}^{\ell-k} \gamma_k, \text{ when } 1 \leqslant k \leqslant \ell; \quad \mathbf{a}_k^{(\ell)\rho} = 0, \text{ else.}$$

The system of labeled constraints for this type of γ-gradually renewable resource is given by the set $\Sigma = \{(\mathbf{a}^{(\ell)}, 0, \ell) | \ell \in \mathscr{T}\}$, also denoted by the generic term $\Sigma_{R\gamma}$ or $\Sigma_R(\gamma)$. The following example [5] refers to resources of type $\Sigma_R(\frac{1}{3}, \frac{1}{2})$, that is $(\frac{1}{3}, \frac{1}{2})$-gradually renewable, where the values of the sequence are taken as zero after $\frac{1}{2}$:

$$\Sigma_R\left(\frac{1}{3}, \frac{1}{2}\right) : \begin{matrix} 1 \to \\ 2 \to \\ 3 \to \\ 4 \to \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2/3 & 1 & 0 & 0 \\ 1/6 & 2/3 & 1 & 0 \\ 1/6 & 1/6 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} \phi_1^{\varrho} \\ \phi_2^{\varrho} \\ \phi_3^{\varrho} \\ \phi_4^{\varrho} \end{pmatrix} \leqslant 0.$$

Representation of an amount of priorly available resources: Priorly available resources are seen as negative priorly consumed resources. Let $\lambda \in \mathbb{R}$ be an amount of priorly consumed resources. Associated with λ are defined *constant* constraints for each period of search:

$$\forall \ell \in \mathscr{T}, \quad \lambda \leqslant 0. \tag{7}$$

The system of labeled constraints representing an amount λ of priorly consumed resources is given by the set $\Sigma = \{(0, \lambda, \ell)/\ell \in \mathscr{T}\}$, also denoted by the term λ.

Resource operators: We now define the operators acting on labeled constraints systems. These operators will be used with the basic systems just defined previously. From now on, and until the end of the section, we will often refer to the following objects:

- The numbers $\lambda, \mu \in \mathbb{R}$.
- A system $\{\Sigma^{\alpha}, \Sigma^{\beta}, \Sigma^{\gamma}\}$ made of three labeled constraints systems.

Operator +: The purpose of this *modalization* operator is to split a given type of resource into two running modes. This operator applies on two systems of labeled constraints and works by summing each constraints of the first system to each constraints of the second system, in so far as they have the *same temporal label*:

$$\Sigma^{\alpha} + \Sigma^{\beta} = \{(a^{\alpha} + a^{\beta}, \psi^{\alpha} + \psi^{\beta}, k) | (a^{\alpha}, \psi^{\alpha}, k) \in \Sigma^{\alpha} \text{ and } (a^{\beta}, \psi^{\beta}, k) \in \Sigma^{\beta}\}. \tag{8}$$

Example: Consider some resources, which priorly amount is equal to 100, and which may run either as non-renewable resources ($R\infty$) or as $(\frac{1}{3}, \frac{1}{2})$-gradually renewable resources ($R(\frac{1}{3}, \frac{1}{2})$). These resources are simply described by the system:

$$\Sigma_{R\infty} + \Sigma_{R\left(\frac{1}{3},\frac{1}{2}\right)} + (-100)$$

$$: \begin{matrix} 1 \to \\ 2 \to \\ 3 \to \\ 4 \to \end{matrix} \begin{pmatrix} 1000 & 1 & 0 & 0 & 0 \\ 1100 & 2/3 & 1 & 0 & 0 \\ 1110 & 1/6 & 2/3 & 1 & 0 \\ 1111 & 1/6 & 1/6 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} \phi^{R\infty} \\ \phi^{R\gamma} \end{pmatrix} \leqslant \begin{pmatrix} 100 \\ 100 \\ 100 \\ 100 \end{pmatrix}.$$

Operator &: The AND or *parallelizing* operator applies on two systems of labeled constraints and just puts together the constraints of both systems:

$$\Sigma^{\alpha} \& \Sigma^{\beta} = \Sigma^{\alpha} \cup \Sigma^{\beta}. \tag{9}$$

When the constraints of each basic system operate on distinct variables, this operation & may be described by means of block-diagonalizing.

[4] The suffix R stands for Renewable, while ΔT means that the resource renews after ΔT periods.

[5] We are taking $\mathscr{T} = \{1, 2, 3, 4\}$ for many examples of this section.

Example:

$$(\Sigma_{R2}+(-50))\&(\Sigma_{R\infty}+(-100)):\begin{pmatrix}\mathbf{A}^{R\infty} & 0\\ 0 & \mathbf{A}^{R2}\end{pmatrix}\begin{pmatrix}\phi^{R\infty}\\ \phi^{R2}\end{pmatrix}$$
$$\leqslant (100,100,100,100,50,50,50,50)^t$$

Scalar product operator:

$$\lambda\Sigma^\alpha=\{(\lambda a^\alpha,\lambda\psi^\alpha,k)/(a^\alpha,\psi^\alpha,k)\in\Sigma^\alpha\}. \tag{10}$$

When $\lambda=-1$, the system $\lambda\Sigma^\alpha$ is just denoted $-\Sigma^\alpha$.

Example: Consider a detection involving two running modes, say $R\infty$ (non-renewable) and $R2$ (renewable after 2 periods), with priorly amount of available resource equal to 50. Assume moreover that the mode $R2$ needs a double use of resources. This is described by the system

$$\Sigma_{R\infty}+(2\Sigma_{R2})+(-50)$$
$$:\begin{matrix}1\rightarrow\\2\rightarrow\\3\rightarrow\\4\rightarrow\end{matrix}\begin{pmatrix}1\,0\,0\,0 & 2\,0\,0\,0\\ 1\,1\,0\,0 & 2\,2\,0\,0\\ 1\,1\,1\,0 & 0\,2\,2\,0\\ 1\,1\,1\,1 & 0\,0\,2\,2\end{pmatrix}\begin{pmatrix}\phi^{R\infty}\\ \phi^{R2}\end{pmatrix}\leqslant\begin{pmatrix}50\\50\\50\\50\end{pmatrix}.$$

Operator $\succ$: This *conditionality* operator conditions the use of one type of resources to the use of another type of resources. This operator applies on two systems of labeled constraints and just substracts the *diagonal* elements of these systems accordingly to the temporal labeling:

$$\Sigma^\alpha\succ\Sigma^\beta=\{(\delta_{jk}a_j^\rho|_{j,\rho\in\mathcal{T},\mathcal{R}},0,k)/\exists\psi\in\mathbb{R},$$
$$(a,\psi,k)\in\Sigma^\beta+(-\Sigma^\alpha)\}, \tag{11}$$

where $\delta_{jk}=1$ for $j=k$ and $\delta_{jk}=0$ for $j\neq k$. More precisely, $\Sigma^\alpha\succ\Sigma^\beta$ means that the use of one resource of type Σ^β needs the use of one resource of type Σ^α. This operator is useful in combination with other constraints of the problem.

Example: Just consider the previous example, but, in addition, assume that it is necessary to use 3 resources in mode $R2$ so as to be able to use 2 resources in mode $R\infty$. Since the scalar product is a multiplier of the *resource need*, this conditioning yields the weighs $\frac{1}{3}$ and $\frac{1}{2}$ on the respective types $R2$ and $R\infty$ in order to render a proper comparison of the resources. The whole problem is described by the system

$$(\Sigma_{R\infty}+(2\Sigma_{R2})+(-50))\&\left(\left(\frac{1}{3}\Sigma_{R2}\right)\succ\left(\frac{1}{2}\Sigma_{R\infty}\right)\right),$$

where the system $(\frac{1}{3}\Sigma_{R2})\succ(\frac{1}{2}\Sigma_{R\infty})$ is written:

$$\left(\frac{1}{3}\Sigma_{R2}\right)\succ\left(\frac{1}{2}\Sigma_{R\infty}\right)$$
$$:\begin{matrix}1\rightarrow\\2\rightarrow\\3\rightarrow\\4\rightarrow\end{matrix}\begin{pmatrix}\frac{1}{2}\,0\,0\,0 & -\frac{1}{3} & 0 & 0 & 0\\ 0\,\frac{1}{2}\,0\,0 & 0 & -\frac{1}{3} & 0 & 0\\ 0\,0\,\frac{1}{2}\,0 & 0 & 0 & -\frac{1}{3} & 0\\ 0\,0\,0\,\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{3}\end{pmatrix}\begin{pmatrix}\phi^{R\infty}\\ \phi^{R2}\end{pmatrix}\leqslant\begin{pmatrix}0\\0\\0\\0\end{pmatrix}.$$

Using the previous definitions, it is possible to construct algebraic rules for combining resources. These rules are summarized below.

Proposition 1. *General properties: Let Σ^α, Σ^β, Σ^γ and Σ^δ be four systems of labeled constraints and let $\lambda,\mu\in\mathbb{R}$ be two scalar. The following relations hold*:

$$\begin{aligned}
&\Sigma^\alpha+\Sigma^\beta=\Sigma^\beta+\Sigma^\alpha\\
&\Sigma^\alpha\&\Sigma^\beta=\Sigma^\beta\&\Sigma^\alpha\\
&(\Sigma^\alpha\&\Sigma^\beta)\&\Sigma^\gamma=\Sigma^\alpha\&(\Sigma^\beta\&\Sigma^\gamma)\\
&(\Sigma^\alpha+\Sigma^\beta)+\Sigma^\gamma=\Sigma^\alpha+(\Sigma^\beta+\Sigma^\gamma)\\
&\lambda(\Sigma^\alpha+\Sigma^\beta)=(\lambda\Sigma^\alpha)+(\lambda\Sigma^\beta)\\
&\lambda(\Sigma^\alpha\&\Sigma^\beta)=(\lambda\Sigma^\alpha)\&(\lambda\Sigma^\beta)\\
&1\Sigma^\alpha=\Sigma^\alpha\\
&(\Sigma^\alpha\&\Sigma^\beta)+\Sigma^\gamma=(\Sigma^\alpha+\Sigma^\gamma)\&(\Sigma^\beta+\Sigma^\gamma)\\
&(\Sigma^\alpha\succ\Sigma^\gamma)\&(\Sigma^\beta\succ\Sigma^\gamma)=(\Sigma^\alpha\&\Sigma^\beta)\succ\Sigma^\gamma\\
&(\Sigma^\alpha\succ\Sigma^\beta)\&(\Sigma^\alpha\succ\Sigma^\gamma)=\Sigma^\alpha\succ(\Sigma^\beta\&\Sigma^\gamma)\\
&(\Sigma^\alpha\succ\Sigma^\beta)+(\Sigma^\gamma\succ\Sigma^\delta)=(\Sigma^\alpha+\Sigma^\gamma)\succ(\Sigma^\beta+\Sigma^\delta)\\
&\Sigma^\alpha\succ(\Sigma^\beta\succ\Sigma^\gamma)=(\Sigma^\alpha+\Sigma^\beta)\succ\Sigma^\gamma\\
&(\Sigma^\alpha\succ\Sigma^\beta)\succ\Sigma^\gamma=\Sigma^\beta\succ(\Sigma^\alpha+\Sigma^\gamma)\\
&\forall\lambda,\mu\neq0,\quad\exists\Sigma^\alpha,\quad(\lambda+\mu)\Sigma^\alpha\neq(\lambda\Sigma^\alpha)+(\mu\Sigma^\alpha)
\end{aligned}\tag{12}$$

Proof is left to the reader since it is a straightforward application of definitions. These properties show similarities with *Linear Logic*, an other resource language.

An example of multiple use of the operators: Consider some detection resources running with the two following modes:

- a complex mode, parallelizing $R\infty$ and $R2$ (such resource will run in this mode like two parallelized resources, some non-renewable and some renewable after 2 periods.),
- a pure mode, where resources renew after 3 periods, $R3$.

This situation is represented by the system of constraints $(\Sigma_{R\infty}\&\Sigma_{R2})+\Sigma_{R3}+(-75)$, itself equivalent to $(\Sigma_{R\infty}+\Sigma_{R3})\&(\Sigma_{R2}+\Sigma_{R3})+(-75)$:

$$(\Sigma_{R\infty}\&\Sigma_{R2})+\Sigma_{R3}+(-75)$$
$$:\begin{matrix}1\rightarrow\\2\rightarrow\\3\rightarrow\\4\rightarrow\\ \\1\rightarrow\\2\rightarrow\\3\rightarrow\\4\rightarrow\end{matrix}\begin{pmatrix}1\,0\,0\,0 & 0\,0\,0\,0 & 1\,0\,0\,0\\ 1\,1\,0\,0 & 0\,0\,0\,0 & 1\,1\,0\,0\\ 1\,1\,1\,0 & 0\,0\,0\,0 & 1\,1\,1\,0\\ 1\,1\,1\,1 & 0\,0\,0\,0 & 0\,1\,1\,1\\ & & \\ 0\,0\,0\,0 & 1\,0\,0\,0 & 1\,0\,0\,0\\ 0\,0\,0\,0 & 1\,1\,0\,0 & 1\,1\,0\,0\\ 0\,0\,0\,0 & 0\,1\,1\,0 & 1\,1\,1\,0\\ 0\,0\,0\,0 & 0\,0\,1\,1 & 0\,1\,1\,1\end{pmatrix}\begin{pmatrix}\phi^{R\infty}\\ \phi^{R2}\\ \phi^{R3}\end{pmatrix}\leqslant\begin{pmatrix}75\\75\\75\\75\\75\\75\\75\\75\end{pmatrix}.$$

We point out that these constraints, by definition of +, result from a "merging" of Σ^{R3} and −75 with both $\Sigma_{R\infty}$ and Σ_{R2}.

4. Numerical resolution

In the FAB algorithm [3], convergence toward the optimal solution is achieved by successively optimizing each period alone with the other fixed. More precisely, for a particular period κ, $P_{nd}(\varphi)$ can also be written:

$$P_{nd}(\varphi) = \int_E \beta_\kappa^\varphi(x_\kappa) p_{\kappa,x_\kappa}(\varphi_\kappa(x_\kappa))\,\mathrm{d}x_\kappa, \quad \text{where}$$

$$\beta_\kappa^\varphi(x_\kappa) = \int_{E^{T-1}} \alpha(\mathbf{x}) \prod_{1\leqslant k\leqslant T}^{k\neq\kappa} (p_{k,x_k}(\varphi_k(x_k))\,\mathrm{d}x_k). \tag{13}$$

This shows that, when the search efforts are fixed for all periods, except for a period κ, the optimization problem becomes the following 1-period problem:

$$\text{Minimize} \quad P_{nd}(\varphi_\kappa) = \int_E \beta_\kappa^\varphi(x_\kappa) p_{\kappa,x_\kappa}(\varphi_\kappa(x_\kappa))\,\mathrm{d}x_\kappa,$$

$$\text{subject to} \quad \int_E \varphi_\kappa(x_\kappa)\,\mathrm{d}x_\kappa = \phi_\kappa \quad \text{and} \quad \varphi_\kappa \geqslant 0. \tag{14}$$

The following optimality conditions scaled by the parameter η (de Guenin's equations [5]) are obtained:

$$\begin{cases} \beta_\kappa^\varphi(x_\kappa) p'_{\kappa,x_\kappa}(\varphi_\kappa(x_\kappa)) = \eta_\kappa & \text{if } \beta_\kappa^\varphi(x_\kappa) > \eta_\kappa / p'_{\kappa,x_\kappa}(0), \\ \varphi_\kappa(x_\kappa) = 0 & \text{else.} \end{cases} \tag{15}$$

The correct value of η_κ [6] is obtained by a dychotomic process on φ_κ^η. The whole process requires only a few iterations. It uses basically the Markovian assumption relative to α, so as to drastically reduce the computation requirements for the integral [3,14]. As we shall see now, it is again this algorithm (i.e. the FAB) which will be the workhorse for solving problems involving complex resource interaction.

4.1. Algorithm

Theoretical aspects of this algorithm may be found in [4]. Let us present its main steps now. The first step is to transform (4) into a problem with only equality linear constraints. Adding the positive slack variables ϕ_c^s ($c \in \Sigma$), we obtain a problem equivalent to (4):

[6] i.e. such that $\int_E \varphi_\kappa(x_\kappa)\,\mathrm{d}x_\kappa \leqslant \phi_\kappa$.

Minimize

$$P_{nd}(\varphi) = \int_{E^T} \alpha(\mathbf{x}) \prod_{k=1}^{T} \prod_{\rho=1}^{r} p_{k,x_k}^\rho(\varphi_k^\rho(x_k))\,\mathrm{d}\mathbf{x},$$

under the constraints

$$\varphi \geqslant 0, \quad \phi \geqslant 0, \quad \phi^s \geqslant 0,$$

$$\forall \rho \in \{1,\ldots,r\},\ \forall k \in \{1,\ldots,T\}, \quad \int_E \varphi_k^\rho(x_k)\,\mathrm{d}x_k = \phi_k^\rho,$$

$$\forall(\mathbf{a},\psi,k) \in \Sigma, \quad \sum_{k,\rho} \mathbf{a}_k^\rho \phi_k^\rho - \psi + \phi_{(\mathbf{a},\psi,k)}^s = 0. \tag{16}$$

The last constraints will be considered in this section in its matrix form $\mathbf{B}\phi = \psi$. The optimal value of ϕ will be obtained by a gradient method (in this case, the *gradient projection method of Rosen* [13]). The main ingredients of this method are the evaluation of the objective functional $P_{nd}(\phi) = \min_{\int\varphi=\phi,\varphi\geqslant 0} P_{nd}(\varphi)$ for the current choice of global resources ϕ, and the calculation of its differential $dP_{nd}(\phi)$. The FAB algorithm is the basic tool for determining the optimal *spatial* distribution φ associated with ϕ. Constraint $\mathbf{B}\phi = \psi$ is taken into account by constructing a matrix $\mathbf{B}^{\sim}$ and a vector ϕ_0, such that $\mathbf{B}\phi = \psi \iff [\exists v, \phi = \phi_0 + \mathbf{B}^{\sim} v]$; i.e. $\mathbf{B}^{\sim}$ columns form a basis of ker($\mathbf{B}$). Thus, the use of the new variable v instead of ϕ is instrumental for performing optimization in the constraint space. The descent will then be done on the new variable v. The optimization algorithm itself is based on the following property, proven in [4]:

Property 1. *The variation of the probability of non-detection around φ is given by*

$$\mathrm{d}P_{nd}(\phi) = \mathbf{v}^t \mathbf{B}^{\sim} \mathrm{d}v, \tag{17}$$

where $\mathbf{v}$ *is the vector defined by*

$$\begin{cases} \mathbf{v}_k^\rho = \eta_k^\rho, \quad \text{when } \phi_k^\rho > 0, \\ \mathbf{v}_k^\rho = \min_{x_k\in E}(\beta_{k,\rho}^\varphi(x_k)(p_{k,x_k}^\rho)'(0)), \quad \text{when } \phi_k^\rho = 0, \\ \mathbf{v}_c^s = 0, \quad \text{when } c \in \Sigma. \end{cases} \tag{18}$$

The whole algorithm may be sum up as follows:

1. Compute $B^{\sim}$, ϕ_0 and initialize $v = 0$, $\phi = \phi_0$.
2. Run Brown's algorithm for ϕ.
3. Compute $\mathrm{d}P_{nd}(\phi)$ with the solution φ obtained in 2.
4. Find the descent Δv according to the method of Rosen.
5. Update v and ϕ by $v := v + \Delta v$ and $\phi := \phi_0 + \mathbf{B}^{\sim} v$.
6. Return to 2 until convergence.

Our algorithm is rather fast. Actually, its computation load is of the same order than the FAB one.

5. Examples

The use of this general resource management framework is illustrated by the three following examples. They will be presented within the general framework developed in Sections 2–4. Real word applications will be briefly presented in the last subsection.

5.1. Three examples of general resource management

The space search E is a square of 30×30 cells. The target trajectories are simulated through a start position and a motion model. The target starting position is represented by s, a uniform density in the 10×10 square with top-left vertex on the point $(1,1)$, i.e.,

$$s(x_1) = \frac{1}{100}, \text{ when } (1,1) \leqslant x_1 \leqslant (10,10);$$
$$s(x_1) = 0, \text{ else.}$$

At each period the (Markovian) target motion is an uniform diffusion (toward down and right) represented by the function m on the 2D motion vector:

$$\begin{cases} m(0,0) = m(3,3) = \frac{3}{14} \quad \text{and} \quad m(2,3) = m(3,2) = \frac{2}{14}, \\ m(0,3) = m(3,0) = m(1,3) = m(3,1) = \frac{1}{14}, \\ m(x_{k+1} - x_k) = 0, \quad \text{else.} \end{cases}$$

3	0	0	1
0	0	0	1
0	0	0	2
1	1	2	3

Of course, this motion is not limitative and more sophisticated models can be used without major changes. All non-detection functions we consider here will be exponential, i.e. $p^{\rho}_{k,x}(\varphi) = \exp(-\omega^{\rho}_x \varphi)$. The visibility parameter ω^{ρ}_x is independent of the detection period.

First example:

$$(2\Sigma^a) + \Sigma^b + (-200), \quad \text{where } \Sigma^a = \Sigma^{R3} \text{ and } \Sigma^b = \Sigma^{R\infty}.$$

Meaning: The global resource prior amount is 200 and may run in a renewable mode (renewable after 3 periods) or in a non-renewable mode.

Results: The visibility parameters ω^a_x and ω^b_x are represented in Fig. 1. Thus, p^a is more efficient down-left, efficiency decreasing with the radius, while the maximum visibility for p^b is stronger near the up-right side.

The optimal spatial distributions are obtained after roughly 40 iterations of the main algorithm (see Section 4) and are represented in Fig. 1 (φ^a up, φ^b down, from the periods 1 to 7). For the type a, the global amounts of resources (at the optimum), i.e. ϕ^a_k, are $100, 0, 0, 48, 0, 0, 0$, when k varies from the periods 1 to 7. For the type b, the global amounts, ϕ^b_k, are $0, 0, 0, 104, 0, 0, 96$.

The evolution of the detection probability is presented in Table 1.

Some comments: These results agree with the renewing constraints. Since the target is dispersive, it is not surprising that the maximum of resources is used in the first periods. However, it is better to use resources in the renewable mode a at the beginning, and in the non-renewable mode b (b is more powerful and profitable but is resource exhausting) at the end. Spatially, the resources a are rather placed down-left, whereas the resources b are rather up-right, accordingly to the visibility parameters. Some slight surrounding occurs at the first periods.

Real word interpretation: Various visibility factors (like ω^a_x and ω^b_x) may be related to the positions of the two detectors (e.g. the signal-to-noise ratio varies as a factor of $1/r^n$, r: target-receiver range). The renewable resource ($\Sigma^a = \Sigma^{R3}$) may corresponds to a rotation of the radar antenna, while the non-renewable resource ($\Sigma^b = \Sigma^{R\infty}$) is associated with short-time living sensors.

Second example:

$$(\Sigma^c \succ (\Sigma^d + \Sigma^e))\&\Sigma^c\&\Sigma^d\&\Sigma^e,$$

where

$$\Sigma^c = \Sigma^{R2} + (-150),$$
$$\Sigma^d = \Sigma^{R\infty} + (-200) \quad \text{and}$$
$$\Sigma^e = \Sigma^R\left(\frac{1}{8}, \frac{1}{4}, \frac{1}{8}\right) + (-100).$$

Meaning: Two-mode resources are running simultaneously. The first mode, d, describes non-renewable

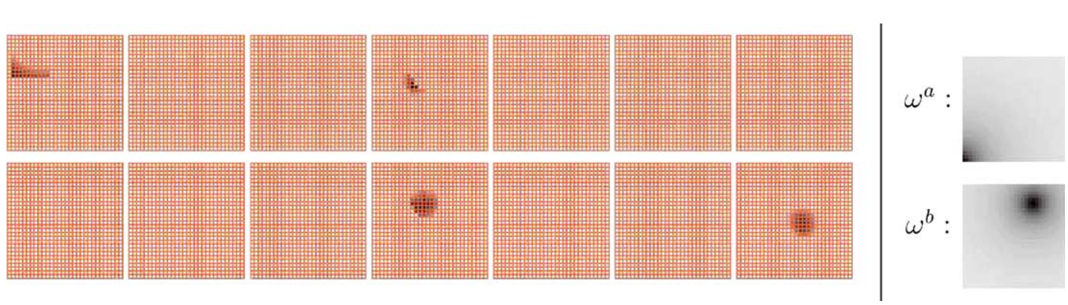

Fig. 1. First example: $(2\Sigma^a) + \Sigma^b + (-200)$.

Table 1
First example: evolution of the detection probability versus the iteration

Iter.	1	2	3	4	5	6	7	8	9	10
Proba.	12.49	12.53	12.55	12.57	12.61	12.67	12.76	12.88	13.05	13.30
Iter.	11	12	13	14	15	16	17	19	22	
Proba.	13.66	14.18	14.90	15.90	17.24	19.03	19.76	19.78	19.79	

resource with the global prior amount 200. The second type, e, corresponds to $(\frac{1}{8},\frac{1}{4},\frac{1}{8})$-gradually renewable resource with the global prior amount 100. These two types are both *controlled* by a third type of resources, c: the use of either 1 resource d or 1 resource e requires the use of 1 resource c. These control resources are renewable after 2 periods with the prior amount 150. The visibility parameters of types d and e are represented in Fig. 2. Thus, p^d is stronger down-left, while p^e is stronger down-right, while the control resource does not have own detection capabilities.

Results: The optimal spatial repartitions are represented in Fig. 2 (φ^d upper row, φ^e lower row). The values of ϕ_k^c, are $147, 3, 135, 15, 54, 16, 8$, the values of ϕ_k^d, are $84, 3, 74, 0, 37, 2, 0$ and the values of ϕ_k^e, are $63, 0, 61, 15, 17, 14, 8$. Some surrounding occurs at the first periods.

Real word interpretation: The control resource plays here a fundamental role. Practically, it may correspond to communication or commanding facilities. Another interpretation is the need for support e.g. carrier or replenishment facilities. These control resources are generally renewable as considered here.

Multiple constraints and multiple interactions: Let us consider the following (intricate) set of "inequations" specifying spatio-temporal constraints as well as resource interactions. The aim of this example is to emphasize the generality of our description language.

$$\Sigma^1\&(\Sigma^1 \succ (\Sigma^{1\beta} + \Sigma^{1\gamma}))\&((\Sigma^{2\alpha}\&\Sigma^{3\alpha}) \succ \Sigma^\alpha)$$
$$\&\Sigma^2\&(\Sigma^2 \succ (\Sigma^{2\gamma} + \Sigma^{2\alpha}))\&((\Sigma^{3\beta}\&\Sigma^{1\beta}) \succ \Sigma^\beta)$$
$$\&\Sigma^3\&(\Sigma^3 \succ (\Sigma^{3\alpha} + \Sigma^{3\beta}))\&((\Sigma^{1\gamma}\&\Sigma^{2\gamma}) \succ \Sigma^\gamma)$$

Meaning: Three types of elementary resources, Σ^1, Σ^2 and Σ^3 are available. These resources do not work alone and need to be combined: to be efficient, *one* resource has to be combined with *one* resource of a *different nature*. These combined types are denoted Σ^α, Σ^β, Σ^γ. To explain in our language this (1;1) to 1 combination, the intermediate types $\Sigma^{1\beta}$, $\Sigma^{1\gamma}$, $\Sigma^{2\gamma}$, $\Sigma^{2\alpha}$, $\Sigma^{3\alpha}$, $\Sigma^{3\beta}$ (*splitted primary resources*) are involved. As a first step, the constraint $\Sigma^1 \succ (\Sigma^{1\beta} + \Sigma^{1\gamma})$ is just switching from each primary resource 1 to a splitting β or γ. Similar constraints hold for the types 2 and 3. The actual combination of the switched resources $\Sigma^{3\alpha}$ and $\Sigma^{2\alpha}$ is controlled by the constraint $(\Sigma^{3\alpha}\&\Sigma^{2\alpha}) \succ \Sigma^\alpha$. Similar constraints hold for the types β and γ. The whole process, involving the switching/combination for all types, is summarized in Fig. 3 and results in the above formula.

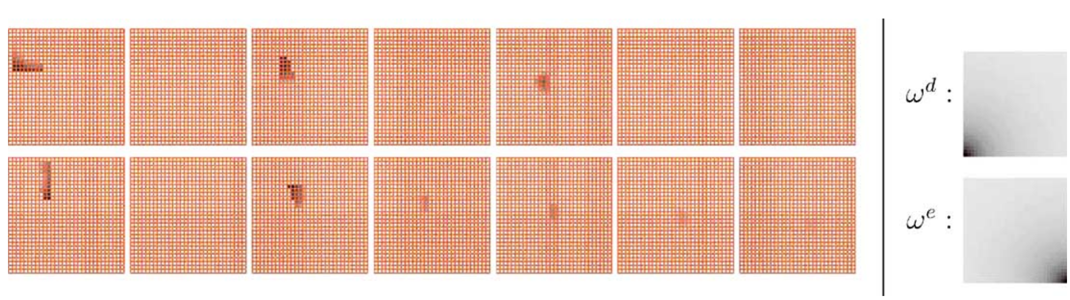

Fig. 2. Second example: $(\Sigma^c \succ (\Sigma^d + \Sigma^e))\&\Sigma^c\&\Sigma^d\&\Sigma^e$.

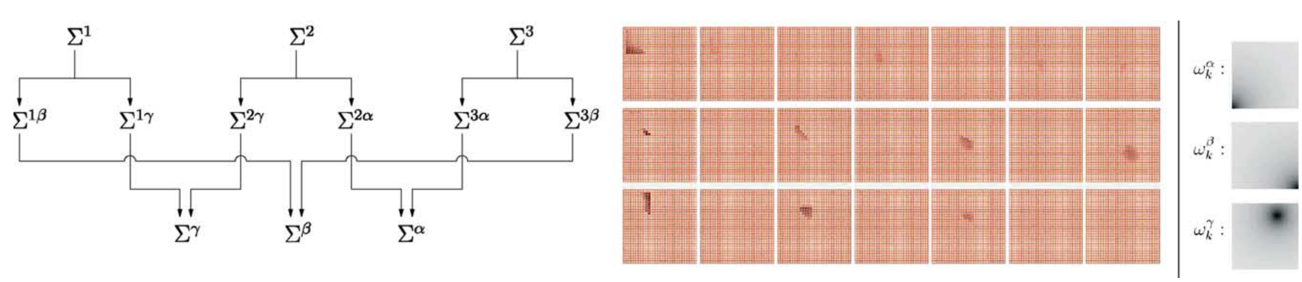

Fig. 3. Diagram of the resource combination/resulting optimal choice for $\varphi^{\{\alpha,\beta,\gamma\}}$.

Results: The detection parameters ω^α, ω^β, ω^γ for the *combined modes* α, β, γ are represented in Fig. 3. The primary resources $\Sigma^1, \Sigma^2, \Sigma^3$ are defined as $\Sigma^{R2} + (-100)$, $\Sigma^R(\frac{1}{8}, \frac{1}{4}, \frac{1}{8}) + (-200)$ and $\Sigma^{R4} + (-300)$ respectively. For the modes α (resp. β, resp. γ), the resulting global amounts of resources from the periods 1 to 7 are $141, 25, 4, 38, 0, 19, 12$ (resp. $41, 0, 51, 0, 79, 0, 100$, resp. $59, 0, 49, 0, 21, 0, 0$). The local distributions are represented in Fig. 3 (φ^α up, φ^β mid, φ^γ down).

Real word interpretation: Resource combination plays here the fundamental role. Various devices can be used altogether. For example, active (radar) and passive measurements (IR, ESM) can be fused for improving detection or location. Thus, a radar measurement can be combined either with an IR or an ESM measurement. Combined modes represent the system behavior at the fused system level.

General ideas we used will now be illustrated on two practical examples.

Practical example 1. It is well known that for a radar system the probability of detection (P_d) rapidly decreases with the target-receiver distance. Let T be the time devoted to detection in a given beam. A first way for enhancing P_d is to increase T (the integration time). Unfortunately, the two following problems strongly limit the interest of this way:

- For P_d relatively great the slope $\frac{\partial}{\partial T} P_d$ is small.
- For large value of T, a moving target cannot be reasonably considered as stationary (i.e. remaining in the beam).

Another approach is to divide the total time T in N (equal) parts and to perform elementary detection on time periods, each of T/N length. If we assume that elementary detections (named P_{de}) are independent, [7] then we have

$$P_d = 1 - (1 - P_{de})^N.$$

Now, let us assume that we have a Swerling 1 for the target detection, i.e. $P_d = P_{fa}^{\frac{1}{1+\rho}}$. Where P_{fa} denotes the probability of false alarm and ρ the signal to noise ratio (e.g. $\rho = \alpha T \frac{\cos^2(\theta)}{r^4}$). Then [6] there exists an optimal value of the parameter N, for which the following relation holds:

$$P_d = 1 - \exp\left(-\frac{T}{\tau}\right), \quad \text{where } \tau = \frac{r^4 \ln(P_{fa}^{-1})}{\alpha(\cos(\theta)\ln(2))^2}. \tag{19}$$

Note that this detection model is of the general type we have considered here. Moreover, it has also the interest to explicit the relationship between P_d, P_{fa} and T. For instance, the parameter T is the search effort allocated to a given beam. We thus see that our general framework is quite relevant for this type of application. More detailed system models can be found in [2,11] (especially Chapter 15).

In this model, the local effectiveness parameter [8] is $1/\tau$. There are situations where this model may appear restrictive and we refer to [8] for an extension.

Practical example 2. Consider a passive detection system working in two modes:

- WIDE FIELD of view (WF mode),
- NARROW FIELD of view (NF mode).

That means that in the (WF mode, it can detects with a ap_{WF} degree aperture and up to a r_{WF} range; idem for the NF mode (r_{NF} range and a_{NF} aperture). This system is fixed but can turn around the horizon with an increment of a_{NF} degrees. So, $360/a_{\mathrm{NF}}$ positions are available. At each place the sensor stops, it can take an image which is ultimately processed.

In fact processing an image has a certain cost. From a scheduling point of view that means that we want to optimize the sequence of both the directions and modes (field of view). From a control perspective, we shall also consider that an image (or a look) corresponds to a *period*. The total number of images (or periods) is fixed (and denoted T) and corresponds to the value of the search effort. *This is the constraint*. The search area is divided into elementary zones corresponding to the possible lines of sight of the detection system. For a given zone, a reasonable assumption is that the probability of detection is given by the following formulas:

$$\begin{cases} P_{d,\mathrm{WF}}(c_i) = \int_{\theta=0}^{ap_{\mathrm{WF}}} \int_{r=0}^{r_{\mathrm{WF}}} \alpha(r,\theta) h_{\mathrm{WF}}(r)\,\mathrm{d}r\,\mathrm{d}\theta, \\ P_{d,\mathrm{NF}}(c_i) = \int_{\theta=0}^{ap_{\mathrm{NF}}} \int_{r=0}^{r_{\mathrm{NF}}} \alpha(r,\theta) h_{\mathrm{NF}}(r)\,\mathrm{d}r\,\mathrm{d}\theta, \\ \quad i = 1, \ldots, N. \end{cases} \tag{20}$$

In (20), c_i denotes the cell indexed by i; while $h_{\mathrm{WF}}(r,\theta)$ and $h_{\mathrm{NF}}(r,\theta)$ correspond to *known* visibility functions (e.g. $h(r) = \frac{c}{r^2}$). With the above definitions, this prior updating (reallocation) is described by

$$\alpha_{(t+1)}(r,\theta) = \frac{\alpha_t(r,\theta) P_{nd}(r,\theta,d_t(\theta))}{\int_{\mathscr{E}} \alpha_t(r,\theta) P_{nd}(r,\theta,d_t(\theta))\,\mathrm{d}r\,\mathrm{d}\theta}, \quad t = 2, \ldots, T-1, \tag{21}$$

where $P_{nd}(r,\theta,d_t(\theta))$ is the probability that the target remains undetected if it is searched at period t, with the mode $d_t(\theta)$ and $\mathscr{E}$ is the search space. The decision $d_t(\theta)$ is the observer decision (WF or NF mode) associated with θ, at the time period t.

Definition 1. A target is said undetected if it has not been detected at *any* time period t, $t \in \{1, \ldots, T\}$.

[7] e.g. this is the case if frequency is changed on each T/N period.

[8] For the search.

The objective function we want to optimize [9] is

$$\mathscr{P}_{nd}(\{d_2,\ldots,d_{T-1}\},T)=\int_{\mathscr{E}} P_{nd}(r,\theta,\{d_2,\ldots,d_{T-1}\},T)\,\mathrm{d}r\,\mathrm{d}\theta, \tag{22}$$

where $P_{nd}(r,\theta,\{d_2,\ldots,d_{T-1}\},T)$ is deduced from recursions 20 and 21. So that the problem we have to deal with is defined as

Definition 2. Determine the sequence of decisions [10] $\{d_2,\ldots,d_{T-1}\}$ which minimizes $\mathscr{P}_{nd}(\{d_2,\ldots,d_{T-1}\},T)$.

This is the basic formulation, yet involving multimode management. Of course, this elementary problem can be complicated if we consider a moving target or multiple receivers but our general formalism will remain relevant.

6. Conclusion

This paper is centered around the spatio-temporal management of complex detection systems for detection/tracking of moving targets. For that purpose, an original formalism has been developed. Its major aim is to provide an algebraic framework for resource combination under spatial and temporal constraints. Its underlying semantic even permits to include conditional aspects for resource allocation problems and to introduce controls. It has been shown that the corresponding optimization problems are quite efficiently solved by means of an extension of the FAB algorithm. Improvement of the probability of detection on real situations may be quite real, while the computation load remain limited even for complex systems.

[9] Here minimize.
[10] Mode and direction.

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