Search Game for a Moving Target with Dynamically Generated Informations

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Abstract – *This paper deals with the tactical planning of the search efforts for a moving target. It refers deeply to the work of Brown-Washburn, related to the multi-step search of a Markovian target. However, this meaningful work is not optimally applicable to tactical situations, where the target may move accordingly to the observations of the possible searcher indiscretions (*eg. *active modes). On the one hand, the probabilistic Markovian model is too restrictive for describing such target motion. In this paper, a more suitable modeling of the target move is presented (*semi-Markovian *model). On the other hand, it is necessary to involve the informational context into the planning. These two paradoxical aspects make the issue uneasy. We introduce a model for handling the context notions into the optimization setting, and apply it to a general search example with* multiple modes management.

Keywords: Tactical planning, Dynamic context, Modes and resource management, Search game, Active/Passive detection.

1 Introduction

The initial framework of Search Theory[1][2][3], introduced by B.O. Koopman and his colleagues, sets the general problem of the detection of a target in a space, in view of optimizing the detection resources. A thorough extension of the prior formalism has been made by Brown and Washburn towards the detection at several periods of search[4][5]. These simple but meaningful formalism were also applied to resource management and data fusion issues[6]. But a probabilistic prior on the target was required. In addition, a Markovian hypothesis is necessary for algorithmic reasons. While this formalism works well for almost "passive" targets, it is inappropriate when the target has a complex (and realistic) move. In a military context especially, the behavior of the "interesting" targets is not neutral and cannot be modeled by a simple probabilistic prior. A conceivable way for enhancing the prior on the target, while involving more properly the complexity or the reactiveness of the target, is to handle the set of the available trajectories, instead of the probabilistic assumption. This yieds a minimax game version of the Koopman or Brown optimization problems. Our interest in this paper is to optimize the minimax detection of a moving target. However, our work is related in many aspects to the Brown's optimization framework, at least to its formalism. For these reasons, the works of Brown (and related) will be quickly presented and assumed as a prerequisite. In section 2 a general game problem is defined involving multi modal strategies for both the target and the searcher. The searcher strategy is determined and then the target strategy. We explain the notion of *context* in section 3 and define therefore an original notion of dynamic search game. This principle is applied in order to solve a general example of search planning in a dynamic context. Finally, an example of application is given in section 4.

The FAB Algorithm (Brown-Washburn): The objective is to detect a target moving in a space E. The detection is done within T periods and the search ends after the first detection. We define $\vec{x} = (x_1, \dots, x_T)$ the position of the target during the periods $1, 2, \dots, T$. The target motion is assumed probabilistic and Markovian, *ie.* $\alpha(\vec{x}) = \prod_{k=1}^{T-1} \alpha_k(x_k, x_{k+1})$. For each period k a given amount of search effort ϕ_k is available. It may be distributed along E. The (local) search effort, applied to the point $x_k \in E$ at time k, is denoted $\varphi_k(x_k)$. The set $\mathcal{R}(\phi)$ of valid choices of φ is thus considered:

$$\mathcal{R}(\phi) = \left\{ \varphi \in \left(\mathbb{R}^{+E}\right)^T \Big/ \forall k,\ \int_E \varphi_k(x_k)\, dx_k \le \phi_k \right\}.$$

Associated with the local effort, $\varphi_k(x_k)$, is defined the conditional probability, $p_{k,x_k}(\varphi_k(x_k))$, not to detect the target within the period k, when its location is indeed x_k. It is assumed, for x_k fixed, that $p'_{k,x_k} < 0$ and that $\log p_{k,x_k}$ is a convex function. This last hypothesis yields the convexity of the problem. It is justified by the law of diminishing return.

The problem is then to find an optimal function $\varphi \in \mathcal{R}(\phi)$ in order to minimize $\mathbf{P}_{nd}(\varphi)$ the global probability of non-detection. We assume the independence of the elementary detections, so that:

$$\mathbf{P}_{nd}(\varphi) = \int_{E^T} \alpha(\vec{x}) \prod_{k=1}^{T} p_{k,x_k}(\varphi_k(x_k))\, d\vec{x}\ . \quad (1)$$

In order to solve this optimisation problem, Brown's algorithm is based on a *Forward And Backward* method[4][5], and uses basically the Markovian assumption relative to α, so as to drastically reduce the computation requirements for the integral (1) and related.

The work of Brown is easily extendable to problems with multiple modes/types of detection resources and multiple running modes for the target. This is accomplished by adding a type index $\rho \in \{1,\dots,r\}$ to the search variables (*eg.* φ_k^ρ, ϕ_k^ρ) and a target state parameter $\vec{\sigma}$ to the target prior (*eg.* $\alpha[\vec{x},\vec{\sigma}]$). The non detection functions are also affected (*eg.* $p_{k,x_k}^{\rho,\sigma_k}$). The definition of the set $\mathcal{R}(\phi)$ is changed this way:

$$\mathcal{R}(\phi) = \left\{ \varphi \in \left(\mathbb{R}^{+E}\right)^{r\times T} \Big/ \forall \rho, k,\ \int_E \varphi_k^\rho \le \phi_k^\rho \right\}.$$

The global non detection probability appears then as follows (S is the set of states):

$$\mathbf{P}_{nd}(\varphi) = \int_{E^T\times S^T} \alpha \begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} \prod_{k,\rho} p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\, d\vec{x} d\vec{\sigma}\ . \quad (2)$$

Based on this multi-type formalism, Dambreville and Le Cadre proposed a linear extension of the search constraints[6], in order to handle the temporal behavior of the various detection resources and their interaction (*data fusion*). Again, the objective functional is given by 2 but the valid choices of φ are now given by the constraint set $\mathcal{R}(A,\psi)$:

$$\mathcal{R}(A,\psi) = \left\{ \varphi \in \left(\mathbb{R}^{+E}\right)^{r\times T} \Big/ A\left(\int_E \varphi_k^\rho\right)_{k,\rho} \le \psi \right\}.$$

An algorithmic solution of this problem have been given by Dambreville and Le Cadre in [6]. This algorithm makes use again of the Markovian assumption.

In the subsequent section, a new type of target prior is considered and we will consider the game aspects of the ploblems introduced here. Thus, the definitions of $\mathcal{R}(\phi)$ and $\mathcal{R}(A,\psi)$ should be kept in mind.

2 Game on moving target

2.1 Problem Setting

A target is moving in a space E, during T periods of search. Each period of search is represented by the suffix $k \in \{1,\dots,T\}$. The position of the target during the period k is denoted x_k. Moreover, at each period k, the target may be in a particular state $\sigma_k \in S$. The set of available target trajectories, denoted $\mathbb{T} \subset E^T \times S^T$, is known. There is no other prior about the target moving behavior. In general, the set $\mathbb{T}$ may be quite big. An extensive definition is not possible. However, for algorithmic reason, it will be necessary to make simplifying hypotheses. A *Markovian*-like definition is thus assumed for the set $\mathbb{T}$:

Definition 1 *Let the sets $m_k \subset E^2 \times S^2$ be given for each $k \in \{1,\dots,T-1\}$. The set of available trajectories $\mathbb{T}$ is defined by:*

$$\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} \in \mathbb{T} \iff \forall k \in \{1,\dots,T-1\},\ \begin{bmatrix}x_k, x_{k+1}\\ \sigma_k, \sigma_{k+1}\end{bmatrix} \in m_k\ .$$

Our interest focuses on a game between the target and the search efforts. A pure target strategy is the choice of a trajectory $[\vec{x},\vec{\sigma}]$. The only constraint put on the target is $[\vec{x},\vec{\sigma}] \in \mathbb{T}$. The target is confronted to several types of search resources. These types are numbered by the suffix $\rho \in \{1,\dots,r\}$. From now on, we define:

$$\mathcal{M} = \{1,\dots,r\} \text{ and } \mathcal{T} = \{1,\dots,T\}\ .$$

A pure search effort strategy is a choice of local resource allocations $\varphi_k^\rho \in \mathbb{R}^{+E}$ for each period $k \in \mathcal{T}$ and each resource type $\rho \in \mathcal{M}$. The set, $\mathcal{R}$, of the valid allocation functions φ is assumed to be *convex.* In practice, we will take $\mathcal{R} = \mathcal{R}(\phi)$ or $\mathcal{R} = \mathcal{R}(A,\psi)$. Associated with the local effort $\varphi_k^\rho(x_k)$, is defined the conditional probability, $p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))$, for the resources ρ not to detect the target within the period k, when its location is indeed x_k and its state is σ_k. It is assumed that $(p_{k,x_k}^{\rho,\sigma_k})' < 0$ and $w_{k,x_k}^{\rho,\sigma_k} = \log p_{k,x_k}^{\rho,\sigma_k}$ is a convex function for x_k, σ_k fixed. The instantaneous visibility parameter $\omega_{k,x_k}^{\rho,\sigma_k} = -(w_{k,x_k}^{\rho,\sigma_k})'$ is a decreasing function, which is an expression of the law of diminishing return. The evaluation function $\mathcal{V}_{nd}$ of the game corresponds to the global probability of non-detection:

$$\forall \begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} \in \mathbb{T},\ \forall\varphi\in\mathcal{R},\ \mathcal{V}_{nd}\left(\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix}, \varphi\right) = \prod_{k,\rho} p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\ .$$

Since $\log p_{k,x_k}^{\rho,\sigma_k}$ is convex, the function $\mathcal{V}_{nd}$ is convex in the variable φ. The aim of the game is to find an optimal couple of strategies, which minimize $\mathcal{V}_{nd}$ for the searcher and which maximize this value for the target. Since the evaluation function $\mathcal{V}_{nd}$ is convex, it has a semi-mixed optimal strategy (α_o, φ_o), where φ_o is a pure strategy and α_o is a mixed strategy. Thus α_o is a probabilistic function on $\mathbb{T}$. Define:

$$\mathcal{P}(\mathbb{T}) = \left\{ \alpha \in \mathbb{R}^{+E^T \times S^T} \Big/ \int_{E^T \times S^T} \alpha = \int_{\mathbb{T}} \alpha = 1 \right\} .$$

The whole problem may be summarized as follows:

$$\begin{cases} \alpha_o \in \arg\max_{\alpha \in \mathcal{P}(\mathbb{T})} \int_{\mathbb{T}} \alpha \begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix} \mathcal{V}_{nd} \left(\begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix}, \varphi_o \right) d\vec{x} d\vec{\sigma} , \\ \varphi_o \in \arg\min_{\varphi \in \mathcal{R}} \int_{\mathbb{T}} \alpha_o \begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix} \mathcal{V}_{nd} \left(\begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix}, \varphi \right) d\vec{x} d\vec{\sigma} . \end{cases} \quad (3)$$

The strategy φ_o may also be defined alone:

$$\varphi_o \in \arg\min_{\varphi \in \mathcal{R}} \mathcal{V}_{nd} ([\vec{x}, \vec{\sigma}], \varphi) . \quad (4)$$

This last equation shows that the determination of φ_o is possible without finding α_o. It is a good thing, because, for complexity reasons, it is uneasy to manipulate α_o explicitly. Actually, α_o may not be Markovian, although $\mathbb{T}$ has such property.

2.2 *Avoiding* the maximization

The very known following approximation of the *max*:

$$\max\{a_1, \ldots, a_M\} = \lim_{\omega \to +\infty} \left(\sum_{m=1}^{M} a_m^\omega \right)^{1/\omega}$$

runs uniformly for all M-uplets $\vec{a} = (a_1, \ldots, a_M)$ of a set $\mathbb{A}$, as soon as this set $\mathbb{A}$ is bounded. A minimization made on $\mathbb{A}$ may thus be inverted with the limit:

$$\min_{\vec{a} \in \mathbb{A}} \max\{a_1, \ldots, a_M\} = \lim_{\omega \to +\infty} \min_{\vec{a} \in \mathbb{A}} \left(\sum_{m=1}^{M} a_m^\omega \right)^{1/\omega} .$$

It is as much easy to derive some minimizers as a limit:

$$\arg\min_{\vec{a} \in \mathbb{A}} \max\{a_1, \ldots, a_M\} \supset \lim_{\omega \to +\infty} \arg\min_{\vec{a} \in \mathbb{A}} \sum_{m=1}^{M} a_m^\omega .$$

Applying the same inversion to the equation (4), it is possible to choose an optimal strategy φ_o as follows:

$$\boxed{\varphi_o \in \lim_{\omega \to +\infty} \arg\min_{\varphi \in \mathcal{R}} \int_{\mathbb{T}} \prod_{k,\rho} \left(p_{k,x_k}^{\rho,\sigma_k} (\varphi_k^\rho (x_k)) \right)^\omega d\vec{x} d\vec{\sigma}} \quad (5)$$

Algorithm: Let α_u be the uniform distribution on $\mathbb{T}$:

$$\begin{cases} \forall [\vec{x}, \vec{\sigma}] \in \mathbb{T},\, \alpha_u[\vec{x}, \vec{\sigma}] = \dfrac{1}{\mu\mathbb{T}} , \text{ where } \mu\mathbb{T} = \int_{\mathbb{T}} d\vec{x} d\vec{\sigma} , \\ \forall [\vec{x}, \vec{\sigma}] \notin \mathbb{T},\, \alpha_u[\vec{x}, \vec{\sigma}] = 0 . \end{cases}$$

This probability is obviously Markovian:

$$\forall \begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix} \in \mathbb{T},\, \alpha_u \begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix} = \frac{1}{\mu\mathbb{T}} \prod_{k=1}^{T-1} \chi_{m_k}(x_k, x_{k+1}, \sigma_k, \sigma_{k+1}) ,$$

where the functions χ_{m_k} are defined by:

$$\begin{cases} \chi_{m_k}(x_k, x_{k+1}, \sigma_k, \sigma_{k+1}) = 1 \quad \text{if } \begin{bmatrix} x_k, x_{k+1} \\ \sigma_k, \sigma_{k+1} \end{bmatrix} \in m_k , \\ \chi_{m_k}(x_k, x_{k+1}, \sigma_k, \sigma_{k+1}) = 0 \quad \text{else.} \end{cases}$$

Define also the corrected detection functions $p^{(\omega)}$ by scaling their associated visibility factors with ω:

$$\forall \rho \in \mathcal{M},\, \forall k \in \mathcal{T},\, \forall [x_k, \sigma_k] \in E \times S,\, \forall \varphi \in \mathbb{R}^+, \quad p_{k,x_k}^{(\omega)\,\rho,\sigma_k}(\varphi) = \exp\left(-\omega \times w_{k,x_k}^{\rho,\sigma_k}(\varphi) \right) .$$

Then the inner minimization of equation (5) appears clearly as a Brown-like optimization problem, where the target follows the uniform probabilistic prior α_u and the resources run with the scaled visibility factors. The whole problem is rewritten as follows:

$$\varphi_o \in \lim_{\omega \to \infty} \varphi^{(\omega)} ,$$

where:

$$\varphi^{(\omega)} \in \arg\min_{\varphi \in \mathcal{R}} \int_{E^T \times S^T} \alpha_u \begin{bmatrix} \vec{x} \\ \vec{\sigma} \end{bmatrix} \prod_{k,\rho} p_{k,x_k}^{(\omega)\,\rho,\sigma_k} (\varphi_k^\rho(x_k))\, d\vec{x} d\vec{\sigma} . \quad (6)$$

The functions $\varphi^{(\omega)}$ are computable by means of the algorithm of Brown-Washburn when $\mathcal{R} = \mathcal{R}(\phi)$ or by means of the algorithm of Dambreville-Le Cadre when $\mathcal{R} = \mathcal{R}(A, \psi)$. It is easy then to derive from (6) an algorithm for computing a minimax optimum φ_o:

1. Initialization of φ_o and of parameters. In particular ω is set to a positive value;
2. Compute $\varphi^{(\omega)}$ by means of a proper algorithm. Use the current value of φ_o as initialization;
3. Set $\varphi_o = \varphi^{(\omega)}$;
4. Increase ω;
5. Return to 2 until convergence.

This algorithm runs satisfactorilly. Owing to the computer limits due to the real number encoding, the parameter ω was limited in our algorithm to the approximate maximal value 2000. However, it is not a great restriction and the results are sufficiently precise.

2.3 Computing a target strategy

It is of course uneasy to compute $\alpha_o(\vec{x},\vec{\sigma})$ entirely, since the set $\mathbb{T}$ may be huge and complex. And it may be that α_o is not Markovian. However, it is possible to describe the sequence of the marginals of the target strategy, conditionally to the already covered path. More precisely, refering to the actual trajectory:

$$\Diamond_k = \begin{bmatrix} x_1,\dots,x_{k-1} \\ \sigma_1,\dots,\sigma_{k-1} \end{bmatrix} \tag{7}$$

accomplished by the target at the period k, it is sufficient to compute the optimal move of the target for the current period k, that is the conditional marginal $\alpha_o(x_k,\sigma_k|\Diamond_k)$. Such 1-dimensional function is practically easily handable. It will be obtained from a theoretical (but practically unfeasible) construction of the whole strategy α_o. Now, the main difficulty for a theoretical definition of α_o comes from the linearity on α of the global detection probability to be optimized: the variable α disappears from the optimality equation obtained by variational means on α. Now, we will show how to overcome this difficulty by optimizing approximated games, which are convex on φ and concave on α. New evaluation functions $\mathcal{V}_{nd}^{(\omega)}(\alpha,\varphi)$ are proposed:

$$\mathcal{V}_{nd}^{(\omega)}(\alpha,\varphi) = \int_{\mathbb{T}} \alpha^{1-\frac{1}{\omega}}\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} \prod_{k,\rho} p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\, d\vec{x}d\vec{\sigma}\,.$$

The new games considered are minimax on $\mathcal{V}^{(\omega)}$:

$$\begin{cases} \alpha^{(\omega)} = \arg\max\limits_{\alpha\in\mathcal{P}(\mathbb{T})} \mathcal{V}_{nd}^{(\omega)}(\alpha,\varphi^{(\omega)})\,, \\ \varphi^{(\omega)} = \arg\min\limits_{\varphi\in\mathcal{R}} \mathcal{V}_{nd}^{(\omega)}(\alpha^{(\omega)},\varphi)\,. \end{cases}$$

An optimal solution (α_o,φ_o) of the main game is obtained as a limit of the various approximations:

$$(\alpha_o,\varphi_o) \in \lim_{\omega\to+\infty}(\alpha^{(\omega)},\varphi^{(\omega)})$$

Solving the approximated game: The equations of saddle points are obtained by variational means:

$$\begin{cases} \alpha\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} > 0 \Rightarrow \alpha^{-\frac{1}{\omega}}\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix}\prod\limits_{k,\rho} p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k)) = \lambda\,, \\ \left.\begin{matrix}\varphi\in\mathcal{R}\\ \varphi+d\varphi\in\mathcal{R}\end{matrix}\right\} \Rightarrow \int_{\mathbb{T}} \alpha^{1-\frac{1}{\omega}}\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix}\prod\limits_{k,\rho} p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k)) \\ \qquad \times \sum\limits_{k,\rho} \dfrac{\left(p_{k,x_k}^{\rho,\sigma_k}\right)'(\varphi_k^\rho(x_k))\,d\varphi_k^\rho(x_k)}{p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))}\,d\vec{x}d\vec{\sigma} \geq 0\,. \end{cases} \tag{8}$$

The first equation reduces to:

$$\alpha\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} > 0 \Rightarrow \alpha\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} = \frac{1}{\lambda^\omega}\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\right)^\omega.$$

It is interesting to characterize the trajectories $[\vec{x},\vec{\sigma}]$ with positive probabilities. The result (9) is proven:

$$[\vec{x},\vec{\sigma}]\in\mathbb{T} \iff \alpha[\vec{x},\vec{\sigma}] > 0 \tag{9}$$

proof: Assume α an optimal strategy. Let $[\vec{c},\vec{\gamma}]$ and $[\vec{d},\vec{\delta}]$ be two trajectories of $\mathbb{T}$ such that $\alpha[\vec{c},\vec{\gamma}]=0$ and $\alpha[\vec{d},\vec{\delta}]>0$. Let dt be a positive variation and let $\tilde{\alpha}$ be the perturbation of α defined by:

$$\begin{cases} \tilde{\alpha}[\vec{c},\vec{\gamma}] = \alpha[\vec{c},\vec{\gamma}]+dt\,,\ \tilde{\alpha}[\vec{d},\vec{\delta}] = \alpha[\vec{d},\vec{\delta}]-dt\,, \\ \tilde{\alpha}[\vec{x},\vec{\sigma}] = 0\,,\ \text{else.} \end{cases}$$

Since $\tilde{\alpha}\in\mathcal{P}(\mathbb{T})$, the inequality $\mathcal{V}_{nd}^{(\omega)}(\alpha,\varphi)\geq\mathcal{V}_{nd}^{(\omega)}(\tilde{\alpha},\varphi)$ holds true. By simplifying the variational decomposition of $\mathcal{V}_{nd}^{(\omega)}(\alpha,\varphi)-\mathcal{V}_{nd}^{(\omega)}(\tilde{\alpha},\varphi)$ then holds:

$$\alpha^{-\frac{1}{\omega}}\begin{bmatrix}\vec{d}\\ \vec{\delta}\end{bmatrix}\prod_{k,\rho} p_{k,d_k}^{\rho,\delta_k}(\varphi_k^\rho(d_k))\,dt \\ -0_+^{-\frac{1}{\omega}}\prod_{k,\rho} p_{k,c_k}^{\rho,\gamma_k}(\varphi_k^\rho(c_k))\,dt \geq 0\,.$$

Since $0_+^{-\frac{1}{\omega}}=+\infty$, the previous inequality is obviously contradictory. The equivalence (9) is then deduced. □□□

The probability α is then entirely defined by the first optimality equation:

$$\begin{cases} \alpha\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} = \dfrac{\prod\limits_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\right)^\omega}{\int_{\mathbb{T}}\prod\limits_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\right)^\omega d\vec{x}d\vec{\sigma}}\,,\ \text{for}\ \begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix}\in\mathbb{T}, \\ \alpha\begin{bmatrix}\vec{x}\\ \vec{\sigma}\end{bmatrix} = 0\,,\ \text{else.} \end{cases}$$

By replacing this optimal value of α in the second equation of (8), the following condition is obtained:

$$\left.\begin{matrix}\varphi\in\mathcal{R}\\ \varphi+d\varphi\in\mathcal{R}\end{matrix}\right\} \Rightarrow \int_{\mathbb{T}}\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\right)^\omega \\ \times\sum_{k,\rho}\frac{\left(p_{k,x_k}^{\rho,\sigma_k}\right)'(\varphi_k^\rho(x_k))\,d\varphi_k^\rho(x_k)}{p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))}\,d\vec{x}d\vec{\sigma}\geq 0\,.$$

This last equation appears also as the optimality condition of the optimization problem:

$$\varphi^{(\omega)}\in\arg\min_{\varphi\in\mathcal{R}}\int_{\mathbb{T}}\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}(\varphi_k^\rho(x_k))\right)^\omega d\vec{x}d\vec{\sigma}\,.$$

These optimization problems were encountered in section 2.2. Applying the algorithm presented previously in this section, we will be able to compute $\varphi^{(\omega)}$. Then, $\alpha^{(\omega)}$ will be deduced.

Convergence to the main game: Now, we intend to check that the sequence of the approximated game minimax converges to minimax solutions of the main game. Assume $(\alpha^{(\infty)},\varphi^{(\infty)})$ to be such limit:

$$\left(\alpha^{(\infty)},\varphi^{(\infty)}\right)\in\lim_{\omega\to\infty}\left(\alpha^{(\omega)},\varphi^{(\omega)}\right)\;.$$

From the optimality of $(\alpha^{(\omega)},\varphi^{(\omega)})$ holds:

$$\forall\omega,\begin{cases}\forall\alpha\in\mathcal{P}(\mathbb{T}),\,\mathcal{V}_{nd}^{(\omega)}\left(\alpha^{(\omega)},\varphi^{(\omega)}\right)\geq\mathcal{V}_{nd}^{(\omega)}\left(\alpha,\varphi^{(\omega)}\right)\;,\\ \forall\varphi\in\mathcal{R},\,\mathcal{V}_{nd}^{(\omega)}\left(\alpha^{(\omega)},\varphi^{(\omega)}\right)\leq\mathcal{V}_{nd}^{(\omega)}\left(\alpha^{(\omega)},\varphi\right)\;.\end{cases}$$

From the continuity of $\mathcal{V}_{nd}$ and $\mathcal{V}_{nd}^{(\omega)}$, and from the convergence $\mathcal{V}_{nd}^{(\omega)}\longrightarrow\mathcal{V}_{nd}$, it follows:

$$\begin{cases}\forall\alpha\in\mathcal{P}(\mathbb{T}),\,\mathcal{V}_{nd}\left(\alpha^{(\infty)},\varphi^{(\infty)}\right)\geq\mathcal{V}_{nd}\left(\alpha,\varphi^{(\infty)}\right)\;,\\ \forall\varphi\in\mathcal{R},\,\mathcal{V}_{nd}\left(\alpha^{(\infty)},\varphi^{(\infty)}\right)\leq\mathcal{V}_{nd}\left(\alpha^{(\infty)},\varphi\right)\;.\end{cases}$$

Thus, $(\alpha^{(\infty)},\varphi^{(\infty)})$ is a minimax of the main game. At last, a method for optimizing both α_o and φ_o is summarized bellow:

$$\boxed{(\alpha_o,\varphi_o)\in\lim_{\omega\to+\infty}\left(\alpha^{(\omega)},\varphi^{(\omega)}\right)}\tag{10}$$

where:

$$\boxed{\begin{cases}\varphi^{(\omega)}=\arg\min_{\varphi\in\mathcal{R}}\int_{\mathbb{T}}\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}\left(\varphi_k^{\rho}(x_k)\right)\right)^{\omega}d\vec{x}d\vec{\sigma}\\ \alpha^{(\omega)}\begin{bmatrix}\vec{x}\\\vec{\sigma}\end{bmatrix}=\dfrac{\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}\left(\varphi_k^{(\omega)\rho}(x_k)\right)\right)^{\omega}}{\int_{\mathbb{T}}\prod_{k,\rho}\left(p_{k,x_k}^{\rho,\sigma_k}\left(\varphi_k^{(\omega)\rho}(x_k)\right)\right)^{\omega}d\vec{x}d\vec{\sigma}}\end{cases}}\tag{11}$$

Deriving a practical target strategy: As yet discussed before, it is not possible to compute entirely α_o, but the conditional marginals are sufficient in practice. Considering $\Diamond_k$, the trajectory already accomplished by the target at period k defined in (7), we define:

$$\mathbb{T}_{\Diamond_k}=\left\{\begin{bmatrix}\vec{y}\\\vec{\varsigma}\end{bmatrix}\in\mathbb{T}\Big/\begin{bmatrix}y_1,\dots,y_{k-1}\\\varsigma_1,\dots,\varsigma_{k-1}\end{bmatrix}=\Diamond_k\right\}\;,$$

its set of possible path completions. The conditional probability $\alpha_o(x_k,\sigma_k|\Diamond_k)$ is defined as follows:

$$\alpha_o(x_k,\sigma_k|\Diamond_k)=\frac{\int_{\mathbb{T}_{\Diamond_{k+1}}}\alpha_o[\vec{x},\vec{\sigma}]\,d\vec{x}d\vec{\sigma}}{dx_kd\sigma_k\int_{\mathbb{T}_{\Diamond_k}}\alpha_o[\vec{x},\vec{\sigma}]\,d\vec{x}d\vec{\sigma}}\;.$$

In particular, by considering $\alpha_o\in\lim_\omega\alpha^{(\omega)}$, we obtain $\alpha_o(|\Diamond_k)$ as a limit of the functions:

$$\begin{bmatrix}x_k\\\sigma_k\end{bmatrix}\mapsto\frac{\int_{\mathbb{T}_{\Diamond_{k+1}}}\prod_{l\geq k,\rho}\left(p_{l,x_l}^{\rho,\sigma_l}\left(\varphi_l^{(\omega)\rho}(x_l)\right)\right)^{\omega}d\vec{x}d\vec{\sigma}}{dx_kd\sigma_k\int_{\mathbb{T}_{\Diamond_k}}\prod_{l\geq k,\rho}\left(p_{l,x_l}^{\rho,\sigma_l}\left(\varphi_l^{(\omega)\rho}(x_l)\right)\right)^{\omega}d\vec{x}d\vec{\sigma}}\;.$$

In fact, the strategy $\alpha_o(x_k,\sigma_k|\Diamond_k)$ will be computed at each period k. In order to do that, it is first necessary to compute the coming optimal search strategies $\varphi_l^{(\omega)\rho}|_{l\geq k,\rho}$ for $\omega\to\infty$. This is done by a FAB-like algorithm. The computations of the integrals is done by a downward method. Defining, for every $\mathbb{S}\subset E^T\times S^T$, the set $\pi_k\mathbb{S}$ by:

$$\pi_k\mathbb{S}=\left\{[x_k,\sigma_k]\Big/\exists[\vec{y},\vec{\varsigma}]\in\mathbb{S},[y_k,\varsigma_k]=[x_k,\sigma_k]\right\}\;,$$

the following computation of $\alpha_o(|\Diamond_k)$ is established:

$$\forall\begin{bmatrix}x_T\\\sigma_T\end{bmatrix}\in\pi_T\mathbb{T},\,D_T^{(\omega)}\begin{bmatrix}x_T\\\sigma_T\end{bmatrix}=\prod_\rho\left(p_{T,x_T}^{\rho,\sigma_T}\left(\varphi_T^{(\omega)\rho}(x_T)\right)\right)^{\omega},$$

$$\forall\begin{bmatrix}x_l\\\sigma_l\end{bmatrix}\in\pi_l\mathbb{T},\,D_l^{(\omega)}\begin{bmatrix}x_l\\\sigma_l\end{bmatrix}=\prod_\rho\left(p_{l,x_l}^{\rho,\sigma_l}\left(\varphi_l^{(\omega)\rho}(x_l)\right)\right)^{\omega}\times\int_{\pi_{l+1}\mathbb{T}_{\Diamond_{l+1}}}D_{l+1}^{(\omega)}\begin{bmatrix}x_{l+1}\\\sigma_{l+1}\end{bmatrix}dx_{l+1}d\sigma_{l+1}\;,$$

$$\alpha_o(|\Diamond_k)\in\lim_{\omega\to+\infty}\left\{\begin{bmatrix}x_k\\\sigma_k\end{bmatrix}\mapsto\frac{D_k^{(\omega)}\begin{bmatrix}x_k\\\sigma_k\end{bmatrix}}{\int_{\pi_k\mathbb{T}_{\Diamond_k}}D_k^{(\omega)}\begin{bmatrix}x_k\\\sigma_k\end{bmatrix}dx_kd\sigma_k}\right\}\tag{12}$$

3 Planning in a dynamic context

During its move, the target will select specific running modes (*eg.* more or less furtive mode) in order to lower the detection capabilities of the search resources. For example, a quick move may speed up the escape of the target but otherwise, it makes the target more visible. Similarly, an active search resource is more efficient against furtive targets but it is easily located by the target and may result in an escape strategy for the target. Otherwise, a good combination of active and passive resources make possible the development of trapping strategies. Since we are optimizing the first detection of the target, it is reasonable to consider that the searcher has no additional information about the target during the search process. Such blindness does not hold for the target, which may obtain some additive informations about the active resources currently used by the searcher (dynamic context). More precisely, it is plausible that the active resources positions are known by the target. In particular, we define:

$$\mathbb{V} \subset \{1, \dots, r\} ,$$

the set of the types *visible* by the target. The aim of the present section is the study of search games, under such dynamic contexts.

An (almost) general view: Let be given a general search game described by $\mathcal{V}([\vec{x}, \vec{\sigma}], \varphi)$, its evaluation function, and $\mathcal{R}$, $\mathbb{T}$, its constraint sets on the respective variables. We point out that the function $\mathcal{V}$ may contain multiple occurences of the variables $\vec{x}$, $\vec{\sigma}$, φ, and may be rewritten $\mathcal{V}(\{[\vec{x}, \vec{\sigma}], \varphi\}_{\mathrm{i}\in\mathfrak{I}})$. These occurences, i, generally correspond to specific situations in the detection and are also a *built-in* source of context. For example, a one-detection problem[1], $\mathcal{V}_{1d}$, contains many occurences of the variables, distinguished here by (i):

$$\mathcal{V}_{1d}\Big(\big\{\vec{x}^{(\mathrm{i})}, \varphi^{(\mathrm{i})}\big\}_{\mathrm{i}\in\mathcal{T}}\Big) = \sum_{\mathrm{i}\in\mathcal{T}} \Big(1 - p_{\mathrm{i}, x_{\mathrm{i}}^{(\mathrm{i})}}\big(\varphi_{\mathrm{i}}^{(\mathrm{i})}(x_{\mathrm{i}}^{(\mathrm{i})})\big)\Big) \times \prod_{k\neq \mathrm{i}} p_{k, x_k^{(\mathrm{i})}}\Big(\varphi_k^{(\mathrm{i})}(x_k^{(\mathrm{i})})\Big),$$

The occurence i corresponds here to the information *"the target is detected at period* i*"*. Such contextual information is accompanied by a change of the prior on the target (x_{i} is known when the target is actually detected at period i). This example explains how the built-in context evolves during the detection, and how it interacts with the problem formulation. In addition to the build-in context, some context ingredients also result for each period k from the "visible moves" of the players. This is particularly true in multiple-mode problems with active detection modes, $\varphi_l^{\rho}|_{\rho\in\mathbb{V}, l\leq k}$.

The previous example has introduced the notion of context. To handle the context in the search planning, the strategy needs to be a function of the contexts. A target strategy is $(A_k|_k)$, a T-uplet of in-$E \times S$-valued functions depending on target-known (probabilistic) contexts, denoted $*_k$. More precisely, a realisation $[x_k, \sigma_k]$ of $A_k(*_k)$ will represent the future target move for the current search period k. A strategy for the searcher is $(F_k^{\rho}|_{\rho,k})$, a $(T \times r)$-uplet of in-$\mathbb{R}^{+E}$-valued functions depending on searcher-known (probabilistic) contexts, denoted $\star_k^{\rho}$ (here, a vector of contexts is even used for each period). A realisation φ_k^{ρ} of $F_k^{\rho}(\star_k^{\rho})$ will represent the future target move for the current search period k and the type ρ. The contexts $*$ and $\star$ may depend on the previous or current strategies of the target and of the searcher, but also on the occurence position i (buid-in context). In addition the contexts contain (independent) probabilistic components, denoted ξ and ζ. For the (one-mode, furtive) one-detection problem, $*$ and $\star$ are defined as follows:

$$*_k = (x_l|_{l<k}; \xi_k) \text{ and } \star_k = \begin{cases} (\varphi_l|_{l<k}; x_{\mathrm{i}}; \zeta_k) \text{ for } \mathrm{i} < k , \\ (\varphi_l|_{l<k}; \zeta_k) \text{ for } \mathrm{i} \geq k . \end{cases}$$

In particular the definition of $\star_k$ specifies that the prior on the target changes whenever a previous detection of the target happened at the period $\mathrm{i} < k$. For a no-detection problem, there are no built-in context. Nevertheless, taking into account the possible visible search strategy, the context is defined as follows:

$$*_k = (\varphi_l^{\rho}|_{\rho\in\mathbb{V}, l\leq k}; \diamondsuit_k; \xi_k) \text{ and } \star_k^{\varrho} = (\varphi_l^{\rho}|_{\rho, l<k}; \zeta_k^{\varrho}) .$$

The definition of $*_k$ specifies that the target knows the visible searcher moves even for the current period. The probabilistic components, ξ and ζ, are needed to simulate instantaneous mixed strategies.

The minimax optimization on A and F results directly from the game without context. It is necessary, however, to precise which are the constraints to apply on A and F. One (recursive) method is to consider that A (resp. F) maps to $\mathbb{T}$ (resp. $\mathcal{R}$), from any realisation of the context. This will be denoted $A \rightarrowtail \mathbb{T}$ (resp. $F \rightarrowtail \mathcal{R}$). A couple (A_o, F_o) of optimal minimax strategies under context is then defined by:

$$G_{\xi}^{\zeta} : \begin{cases} A_o \in \arg\max_{A\rightarrowtail\mathbb{T}} E_{\xi} E_{\zeta} \mathcal{V}\left(A_k(*_k)\big|_k, F_{o,k}^{\rho}(\star_k^{\rho})\big|_{k,\rho}\right) \\ F_o \in \arg\min_{F\rightarrowtail\mathcal{R}} E_{\xi} E_{\zeta} \mathcal{V}\left(A_{o,k}(*_k)\big|_k, F_k^{\rho}(\star_k^{\rho})\big|_{k,\rho}\right) \end{cases} \tag{13}$$

Otherwise, the strategies may be defined separately:

$$F_o \in \arg\min_{F\rightarrowtail\mathcal{R}} \max_{A\rightarrowtail\mathbb{T}} E_{\xi} E_{\zeta} \mathcal{V}\left(A_k(*_k)\big|_k, F_k^{\rho}(\star_k^{\rho})\big|_{k,\rho}\right) \tag{14}$$

Property 1 *There are two probabilistic parameters ξ and ζ, which yield a (pure) solution to the game G_{ξ}^{ζ}.*

proof: Let $(\mathbf{A}_o, \mathbf{F}_o)$ a couple of mixed minimax strategies of the "deterministic" game $G_{\emptyset}^{\emptyset}$. The contexts $*$ and $\star$ will refer to $G_{\emptyset}^{\emptyset}$. Set $\xi_k = \mathbf{A}_o$ and $\zeta_k^{\rho} = \mathbf{F}_o$. Define the strategies $\widehat{A}$ and $\widehat{F}$ of G_{ξ}^{ζ} by:

$$\widehat{A}_k(*_k, \xi_k) = (\xi_k)_k(*_k) \text{ and } \widehat{F}_k^{\rho}(\star_k^{\rho}, \zeta_k^{\rho}) = (\zeta_k^{\rho})_k^{\rho}(\star_k^{\rho}) .$$

It is easy to check the following equalities:

$$\max_{A\rightarrowtail\mathbb{T}} E_{\xi} E_{\zeta} \mathcal{V}\left(A_k(*_k, \xi_k)\big|_k, \widehat{F}_k^{\rho}(\star_k^{\rho}, \zeta_k^{\rho})\big|_{k,\rho}\right) = \max_{A\rightarrowtail\mathbb{T}} E_{\zeta} \mathcal{V}\left(A_k(*_k)\big|_k, \widehat{F}_k^{\rho}(\star_k^{\rho}, \zeta_k^{\rho})\big|_{k,\rho}\right) = \max_{A\rightarrowtail\mathbb{T}} E_{\mathbf{F}_o} \mathcal{V}\left(A_k(*_k)\big|_k, \mathbf{F}_{o,k}^{\rho}(\star_k^{\rho})\big|_{k,\rho}\right) .$$

[1] For a simple presentation, only one mode for the target and the searcher is considered.

Similarly holds:

$$\min_{F\rightarrowtail\mathcal{R}} E_\xi E_\zeta \mathcal{V}\left(\widehat{A}_k(*_k,\xi_k)\big|_k, F_k^\rho(\star_k^\rho,\zeta_k^\rho)\big|_{k,\rho}\right) = \min_{F\rightarrowtail\mathcal{R}} E_{\mathbf{A}_o}\mathcal{V}\left(\mathbf{A}_{o,k}(*_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) .$$

Since $(\mathbf{A}_o,\mathbf{F}_o)$ is a (mixed) saddle point for $G_\emptyset^\emptyset$, it follows then that $(\widehat{A},\widehat{F})$ is a pure saddle point for G_ξ^ζ. □□□

This proof shows that a mixed strategy may always be simulated by a probabilistic *switch* of pure strategies. We now denote $\succ$, whenever there is an onto mapping s so that $Y = s(X)$.

Property 2 *Let $\xi_1,\xi_2,\zeta_1,\zeta_2$ be probabilistic parameters such that $\zeta_{2,k}^\rho \succ \zeta_{1,k}^\rho$ and $\xi_{2,k}\succ\xi_{1,k}$. Assume there is a pure solution to the game $G_{\xi_1}^{\zeta_1}$. Then, there is a pure solution to the game $G_{\xi_2}^{\zeta_2}$.*

proof (partial): Define the onto mappings r and s such that $\xi_{2,k} = r_k(\xi_{1,k})$ and $\zeta_{2,k}^\rho = s_k^\rho(\zeta_{1,k}^\rho)$. Let (A_o,F_o) a solution of G_ξ^ζ. Simply define $(\widehat{A}_o,\widehat{F}_o)$ by:

$$\begin{cases}\widehat{A}_{o,k}(*_k,\xi_{2,k}) = A_{o,k}(*_k,r_k(\xi_{1,k}))\ ,\\ \widehat{F}_{o,k}^\rho(\star_k^\rho,\zeta_{2,k}^\rho) = F_o(\star_k^\rho, s_k^\rho(\zeta_{1,k}^\rho))\ .\end{cases}$$

It is easy to check that $(\widehat{A}_o,\widehat{F}_o)$ is a solution to $G_{\xi_2}^{\zeta_2}$. □□□

Thus, the use of sufficiently *general* and wide probabilistic parameters ensure the existence of a pure solution. This hypothesis is made in the sequel.

The no-detection case: Since $\mathcal{V}_{nd}$ is convex on F, the following inequality then hold:

$$E_\xi\mathcal{V}_{nd}\left(A_k(*'_k)\big|_k, E_\zeta F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) \le E_\xi E_\zeta \mathcal{V}_{nd}\left(A_k(*'_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) ,$$

where $*'$ denote a $*$-context defined outside E_ζ (it does not depend on ζ). Since $*'$ contains less information than a $*$-context defined inside E_ζ, it follows:

$$\max_{A\rightarrowtail\mathbb{T}} E_\xi E_\zeta\mathcal{V}_{nd}\left(A_k(*'_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) \le \max_{A\rightarrowtail\mathbb{T}} E_\xi E_\zeta\mathcal{V}_{nd}\left(A_k(*_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) ,$$

$$\min_{F\rightarrowtail\mathcal{R}}\max_{A\rightarrowtail\mathbb{T}} E_\xi\mathcal{V}_{nd}\left(A_k(*'_k)\big|_k, E_\zeta F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) \le \min_{F\rightarrowtail\mathcal{R}}\max_{A\rightarrowtail\mathbb{T}} E_\xi E_\zeta\mathcal{V}_{nd}\left(A_k(*_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) .$$

This signifies that an optimal strategy F_o may be chosen independently of any probabilistic parameter ζ. The definition of F_o reduces to:

$$F_o \in \arg\min_{F\rightarrowtail\mathcal{R}}\max_{A\rightarrowtail\mathbb{T}} E_\xi\mathcal{V}_{nd}\left(A_k(*_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) .$$

Now, for a given F and $[\vec{x},\vec{\sigma}]\in\mathbb{T}$, it is always possible to choose $A\rightarrowtail\mathbb{T}$ such that $A_k(*_k)\big|_k = [\vec{x},\vec{\sigma}]$ whatever the value of ξ, and it follows then:

$$\max_{A\rightarrowtail\mathbb{T}} E_\xi\mathcal{V}_{nd}\left(A_k(*_k)\big|_k, F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) \ge \max_{[\vec{x},\vec{\sigma}]\in\mathbb{T}} \mathcal{V}_{nd}\left([\vec{x},\vec{\sigma}], F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) .$$

Since $\star_k^\rho$ does not depend on A or $[\vec{x},\vec{\sigma}]$, the reversed inequality trivially holds true. Hence:

$$F_o \in \arg\min_{F\rightarrowtail\mathcal{R}}\max_{[\vec{x},\vec{\sigma}]\in\mathbb{T}} \mathcal{V}_{nd}\left([\vec{x},\vec{\sigma}], F_k^\rho(\star_k^\rho)\big|_{k,\rho}\right) .$$

The optimal contextual strategies are equivalent to an optimal non-contextual strategy :

$$\boxed{F_o = \varphi_o \in \arg\min_{\varphi\in\mathcal{R}}\max_{[\vec{x},\vec{\sigma}]\in\mathbb{T}} \mathcal{V}_{nd}([\vec{x},\vec{\sigma}],\varphi)} \tag{15}$$

Then, the contextual strategy A_o is defined as a semi-contextual saddle point:

$$\begin{cases}A_o \in \arg\max_{A\rightarrowtail\mathbb{T}} E_\xi\mathcal{V}_{nd}\left(A_k(*_k)\big|_k,\varphi_o\right)\\ \varphi_o\in\arg\min_{\varphi\in\mathcal{R}} E_\xi\mathcal{V}_{nd}\left(A_{o,k}(*_k)\big|_k,\varphi\right)\end{cases}$$

In these equations, $A_k(*_k)\big|_k$ may be replaced by anything else. It is thus obvious that **the contextual strategy A_o is equivalent to the non-contextual strategy described by α_o.** Intuitively, the visible resources φ_o^ρ for $\rho\in\mathbb{V}$ are the only external information gained by the target. When the searcher plays optimally, this information is yet deductible from the planned searcher strategy, which is a pure strategy. Then, no external information or context is needed, and it is sufficient to use a planned strategy for the target. But the situation is different, if we admit that the searcher use suboptimal strategies.

4 Results

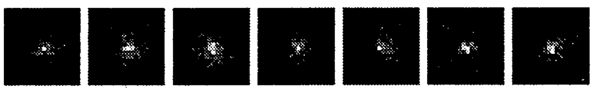
We present here an example for a two-type non-contextual game. The search space E is a set of 20×20 cells. The number of search periods is $T = 7$. The target moves from the 5×5 top-left sub-square down to the 10×10 down-right sub-square of E. The set of allowed movements is especially oriented down-right and is uniform with the start position and the start period. More precisely, the set $\mathbb{T}$ is characterized by:

$$\forall\vec{x}\in\mathbb{T},\begin{cases}(1,1)\le x_1\le(5,5)\ ,\\ (11,11)\le x_7\le(20,20)\ ,\\ \forall k\in\{1,\dots,6\},\ (x_{k+1}-x_k)\in\mathbb{m}\ ,\end{cases}$$

where $\mathbb{m}$ is a set of 2D motion vector. In our example, the set $\mathbb{m}$ is composed by the pairs (i,j) of integers labeled by a star $\star$ in the following table:

$i \backslash j$	−3	−2	−1	+0	+1	+2	+3	+4
−3								
−2				⋆				
−1			⋆	⋆	⋆			
+0		⋆	⋆	⋆	⋆	⋆		
+1			⋆	⋆	⋆	⋆		
+2				⋆	⋆	⋆	⋆	
+3						⋆	⋆	
+4								

The number of available target trajectories is, in this case, about 90000000. The two resource types are named a and b. The visibility functions for these resources are taken linear and are thus of the form $w^{\rho}_{k,x_k}(\varphi) = \omega^{\rho}_{k,x_k} \times \varphi$. The visibility coefficients ω^a_k and ω^b_k are respectively represented in figure 1 and 2. Low values for ω^{ρ}_{k,x_k} correspond to dark cells whereas bright cells represent high values.



ω^a_1 ω^a_2 ω^a_3 ω^a_4 ω^a_5 ω^a_6 ω^a_7

Figure 1: Visibility Parameter for the type a

ω^b_1 ω^b_2 ω^b_3 ω^b_4 ω^b_5 ω^b_6 ω^b_7

Figure 2: Visibility Parameter for the type b

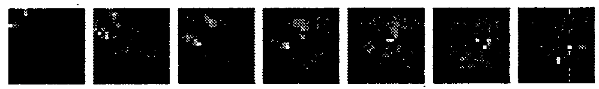
The minimax optima are given in figure 3 for type a and in figure 4 for type b. Interpretation of such results is uneasy. However, there is a splitting of the detection between the two resource types, according to their respective visibility parameters. Particularly, the resources a tend to be used in the center of the space, while the resources b are more concentrated on the borders of the target move. At last, a comparison of the results can be done with a Brown's optimization solution. In the case of a Brown's optimization of the resources, and for a target with a diffusive Markovian probabilistic prior, the obtained optimal sharing functions φ present some surrounding behavior. In the present case, the functions φ_o are almost not surrounding. In other word, it seems that the target strategy avoids the surrounding of the searcher.

5 Conclusion

Our aim was to solve a spatial resource allocation problem for a moving target, including the management of several modes and types. In this framework,

$\varphi^a_{o,1}$ $\varphi^a_{o,2}$ $\varphi^a_{o,3}$ $\varphi^a_{o,4}$ $\varphi^a_{o,5}$ $\varphi^a_{o,6}$ $\varphi^a_{o,7}$

Figure 3: Minimax optimum for the type a



$\varphi^b_{o,1}$ $\varphi^b_{o,2}$ $\varphi^b_{o,3}$ $\varphi^b_{o,4}$ $\varphi^b_{o,5}$ $\varphi^b_{o,6}$ $\varphi^b_{o,7}$

Figure 4: Minimax optimum for the type b

we intended to enhance the target behavior representation. A set modeling of the target trajectories appeared much more realistic than a simple probabilistic model. This model resulted in games between the target and the searcher. We solved these game by an approximating method. The principle of this quite general method allows to translate the minimax optimization into one-sided optimizations. At last, we defined a general formalism to handle the dynamic context evolution into the search planning. Using this model, we proved the equivalence of the contextual and non-contextual game, when the evaluation function is convex and no information about the target is obtained until detection. This work should be enhanced soon for solving more complex cases.

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