The (MR)MTPF: particle filters to track multiple targets using multiple receivers

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Abstract – *The classical particle filter deals with the estimation of one state process conditionally to a realization of one observation process. We extend it here to the estimation of multiple state processes given realizations of several kinds of observation processes. The new algorithm is used to track with success multiple targets in a bearings-only context. Making use of its abilities to mix different types of observations, we then investigate how to join passive and active measurements to improve tracking results.*

Keywords: Particle filter, Bayesian estimation, Gibbs sampler, multiple targets tracking, multiple receivers, bearings-only tracking .

1 Introduction

Multitarget tracking (MTT) deals with state estimation of an unknown number of moving targets. The available measurements may both arise from the targets if they are detected, and from clutter. To perform multitarget tracking the observer has at his disposal a huge amount of data, possibly collected on multiple receivers. Elementary measurements are receiver outputs, e.g., bearings, ranges, time-delays, Dopplers, etc. But the main difficulty comes from the assignment of a given measurement to a target model. For critical situations, these assignments are unknown, as are the true target models. This is a neat departure from classical estimation problems. Thus, two distinct problems have to be solved *jointly*: data association and estimation. As long as the association is considered in a deterministic way, the hypothesis associations must be exhaustively enumerated, which leads to a NP-hard problem (as in JPDAF and MHT algorithms [3] for instance). As soon as the association variables are considered as stochastic variables and moreover statistically independent like in the Probabilistic MHT (PMHT), the complexity is reduced. However, practically, the above algorithms may suffer from non linear state or measurement models and non Gaussian state or measurement noises. Under such assumptions, particle filters are particulary adapted. They mainly consist in propagating a weighted set of particles which approximates the probability density of the state conditionally to the observations. Particle filtering can be applied under very general hypotheses, is able to cope with heavy clutter, and is very easy to implement. Such filters have been used in very various areas for Bayesian filtering, under different names: the bootstrap filter for target tracking in [4] and the Condensation algorithm in computer vision [6] are two examples among others. Despite a long history of studies, in which the ability of particle filter to track multiple posterior modes is claimed, it is striking that the extension of the particle filter to multiple target tracking has not received much attention. In image analysis, a probabilistic exclusion principle has been developed in [8] to track multiple objects but the algorithm is very dependent on the observation model and seems costly to extend for more than two objects. We propose here a general algorithm for multitarget tracking in the passive sonar context and take advantage of its versatility to extend it to multiple receivers. This work is organized as follows. In section II, we recall the principles of the basic particle filter. In section III, we present the MTPF, our extension of this filter to multiple targets. The new algorithm combines the two major steps, prediction and weighting, of the classical particle filter with a Gibbs sampler-based estimation of the assignment probabilities. Results in bearings-only context with moderate or high clutter validate the algorithm. To end this section, we propose to add two statistical tests to decide if a target has appeared or disappeared of the surveillance area. Carrying on with the approach of the MTPF, the MRMTPF, our extension to multi-receiver data in the context of multiple targets, is presented in section IV. The suitable quantity and distribution of active measurements are then studied in a particular scenario to improve the performances obtained with passive measurements only.

2 The basic particle filter

For the sake of completeness, the basic particle filter is now briefly reviewed. We consider a dynamic system represented by the stochastic process $(X_t) \in \mathbb{R}^{n_x}$ whose temporal evolution is given by the state equation (1). It is observed at discrete times via realizations of the stochastic

- Initialization: $\begin{cases} s_0^n \sim p(X_0) \\ q_0^n = 1/N \end{cases} \quad n = 1, \dots, N.$
- For $t = 1, \dots, T$:
  - Proposal: sample $\tilde{s}_t^n$ from $f(X_t|X_{t-1} = s_{t-1}^n, Y_t = y_t)$ for $n = 1, \dots, N$.
  - Weighting: $\begin{cases} \text{Compute un-normalised weights: } \tilde{q}_t^n = q_{t-1}^n \frac{p(\tilde{s}_t^n|s_{t-1}^n)\ l_t(y_t;\tilde{s}_t^n)}{f(\tilde{s}_t^n|s_{t-1}^n, y_t)} \text{ for } n = 1, \dots, N. \\ \text{Normalise weights: } q_t^n = \frac{\tilde{q}_t^n}{\sum_{n=1}^N \tilde{q}_t^n} \text{ for } n = 1, \dots, N. \end{cases}$
  - Return $\widehat{\mathbb{E}}g(X_t) = \sum_{n=1}^N q_t^n g(\tilde{s}_t^n)$.
  - Calculate $\hat{N}_{eff} = \frac{1}{\sum_{n=1}^N (q_t^n)^2}$.
  - Resampling: if $\hat{N}_{eff} < N_{threshold}$: $\begin{cases} s_t^n \sim \sum_{k=1}^N q_t^k \delta_{\tilde{s}_t^k} \\ q_t^n = 1/N \end{cases}$ else $s_t^n = \tilde{s}_t^n$ for $n = 1, \dots, N$.

Figure 1: Basic particle filter with adaptive resampling .

process $(Y_t) \in \mathbb{R}^{n_y}$ governed by the measurement equation (2):

$$X_t = F_t(X_{t-1}, V_t), \tag{1}$$
$$Y_t = H_t(X_t, W_t). \tag{2}$$

The two processes $(V_t) \in \mathbb{R}^{n_v}$ and $(W_t) \in \mathbb{R}^{n_w}$ are only supposed to be independent white noises. Moreover, it is to be noted that no linearity hypothesis on F_t and H_t is done. We will denote by $Y_{0:t}$ the sequence of the random variables $(Y_0, \dots, Y_t)$ and by $y_{0:t}$ one realization of this sequence. Our problem consists in computing at each time t the conditional density L_t of the state X_t given all the observations accumulated up to t, i.e., $L_t = p(X_t|Y_0 = y_0, \dots, Y_t = y_t)$, as well as estimating any functional $g(X_t)$ of the state by the expectation $\mathbb{E}(g(X_t)|Y_{0:t})$. The Recursive Bayesian filter, also named Optimal Filter, resolves exactly this problem in two steps at each time t. Suppose we know L_{t-1}. The *prediction step* is done according to the following equation:

$$\begin{aligned} &p(X_t = x_t|Y_{0:t-1} = y_{0:t-1}) = \\ &\int_{\mathbb{R}^{n_x}} p(X_t = x_t|X_{t-1} = x)L_{t-1}(x)dx. \end{aligned} \tag{3}$$

The observation y_t enables us to correct this prediction using Bayes's rule:

$$L_t(x_t) = \frac{p(Y_t = y_t|x_t)p(X_t = x_t|y_{0:t-1})}{\int_{\mathbb{R}^{n_x}} p(Y_t = y_t|x)p(X_t = x|y_{0:t-1})dx}. \tag{4}$$

These equations provide a closed-form recursion if we assume restrictive hypothesis such as Kalman Filter's ones. The functions F_t and G_t are then supposed to be linear and the noises V_t and W_t to be Gaussian. Unfortunately this modeling is not appropriate in many problems in signal and image processing, which makes the calculation of the integrals in (3) and (4) infeasible (no closed-form). The original particle filter, named bootstrap filter [4], proposes to approximate the densities $(L_t)_t$ by a finite weighted sum of N Dirac densities centred on elements of $\mathbb{R}^{n_x}$, named particles. The application of the particle filter requires that one knows how:

- to sample from initial prior marginal $p(X_0)$;
- to sample from $p(V_t)$ for all t;
- to compute $p(Y_t = y_t|x_t)$ for all t through a known function l_t such that $l_t(y; x) \propto p(Y_t = y|X_t = x)$ where missing normalization must not depend on x.

The algorithm then consists in making evolve the particle set $S_t = (s_t^n, q_t^n)_{n=1,\dots,N}$, where $s_t \in \mathbb{R}^{n_x}$ is the particle and q_t its weight, such that the density L_t can be approximated by the density $L_{S_t} = \sum_{n=1}^N q_t^n \delta_{s_t^n}$. The two steps of prediction and correction are discretized as described in figure 1. The weak convergence of the probability density L_{S_t} towards L_t when $N \to \infty$ with rate $1/\sqrt{N}$ can be proved. To avoid the degeneracy of the particle set, i.e., only few particles with high weights and the others with very small ones, a resampling is done in an adaptive way when the number of effective particles, estimated by $\hat{N}_{eff}$, is under a given threshold [2]. Besides the discretization of the filtering integrals, the use of such particles enables to voice many hypothesis on the position of the object and to keep in the long term only the particles whose position is likely given the sequence of observations. In bootstrap filter, the particles are "moved" by sampling from the dynamics (1), and importance sampling theory shows that the weighting is only based on likelihood evaluations. In the most general setting [2], the displacement of particles is obtained by sampling from an appropriate density f which might depend on the data as well. The complete procedure is summarized in figure 1. The reader will find more details on the different filters in [4], [6] or [2] and on adaptive resampling in [7] and [2]. After these recalls, we propose an extension of this algorithm to multiple-object tracking.

3 Multitarget particle filter

3.1 Notations

Let M be the number of targets to track, assumed to be known and fixed for the moment. The state vector we have to estimate is made by concatenating the state vector of each target. At time t, $X_t = (X_t^1, \dots, X_t^M)$ follows the state equation (1) decomposed in M partial equations:

$$X_t^i = F_t^i(X_{t-1}^i, V_t^i) \qquad \forall\, i = 1, \dots, M. \tag{5}$$

The noises (V_t^i) and $(V_t^{i'})$ are only supposed to be white both temporally and spatially, independent for $i \neq i'$. The observation vector at time t is denoted by $y_t = (y_t^1, \dots, y_t^{m_t})$. Following the seminal ideas of R. Streit and T. Luginbuhl [10], we introduce the stochastic vector $K_t \in \{1, \dots, M\}^{m_t}$ such that $K_t^j = i$ if y_t^j is issued from the i^{th} target. In this case, y_t^j is a realization of the stochastic process:

$$Y_t^j = H_t^i(X_t^i, W_t^j) \text{ if } K_t^j = i. \tag{6}$$

Again, the noises (W_t^j) and $(W_t^{j'})$ are only supposed to be white noises, independent for $j \neq j'$. We assume that the functions H_t^i are such that they can be associated to functional forms l_t^i such that
$l_t^i(y; x) \propto p(Y_t^j = y | K_t^j = i, X_t^i = x)$. We make the assumption that one measurement can originate from one target or from the clutter, and that one target can produce zero or several measurements at one time. For that, we dedicate the model 0 to false alarms. The false alarms are supposed to be uniformly distributed in the observation area. Their number is assumed to arise from a Poisson density of parameter λV where V is the volume of the observation area and λ the number of false alarms per volume unity. Of course, we do not associate any kinematic model to false alarms and then no particles represent their density. Let $\pi_t \in [0, 1]^{M+1}$ be defined by $\pi_t^i = \mathbb{P}(K_t^j = i)$ for all $j = 1, \dots m_t$. This definition implicitly assumes that the probabilities π_t^i are independent of the measurements as their indexation is arbitrary. The assumptions on the observation generation imply that m_t may differ from M and that the association is exclusive and exhaustive. In particular, $\sum_{i=0}^M \pi_t^i = 1$. Furthermore, it is assumed that the assignment vector K_t has independent components (see[10] and [11]). π_t^0 is a constant that can be computed:

$$\pi_t^0 = \mathbb{P}(K_t^j = 0) = \sum_{l=0}^{m_t} \frac{l}{m_t} \exp(-\lambda V) \frac{(\lambda V)^l}{l!} \tag{7}$$

where N_t^0 is the number of measures arising from the clutter at time t. To estimate the density
$L_t = p(X_t = (X_t^1, \dots, X_t^M) | y_{0:t})$, we propose to use particles whose dimension is the sum of the ones of the individual state spaces corresponding to each target. Each of these concatenated vectors then gives jointly a representation of all targets. Let us describe the multiple target particle filter (MTPF). Details on the choices and motivations which have led to the MTPF can be found in [5].

3.2 The algorithm

The initial particle set $S_0 = (s_0^n, 1/N)_{n=1,\dots,N}$ is such that each component $s_0^{n,i}$ for $i = 1, \dots, M$ is sampled from $p(X_0^i)$ independently from the others. Assume we have obtained $S_{t-1} = (s_{t-1}^n, q_{t-1}^n)_{n=1,\dots,N}$ with $\sum_{n=1}^N q_{t-1}^n = 1$. Each particle is a vector of dimension $\sum_{i=1}^M n_x^i$ where we denote by $s_{t-1}^{n,i}$ the i^{th} component of s_{t-1}^n and where n_x^i designates the dimension of target i. The prediction is performed by sampling from some proposal density f. In bootstrap filter case, f coincides with the dynamics (5):

$$\text{For } n = 1, \dots, N, \; \tilde{s}_t^n = \begin{pmatrix} F_t^1(s_{t-1}^{n,1}, v_t^{n,1}) \\ \vdots \\ F_t^M(s_{t-1}^{n,M}, v_t^{n,M}) \end{pmatrix} \tag{8}$$

with $(v_t^{n,i})$ being realizations of (V_t^i). Examine now the computation of the likelihood of the observations conditioned by the n^{th} particle. We can write for all $n = 1, \dots, N$:

$$\begin{aligned} & p(Y_t = (y_t^1, \dots, y_t^{m_t}) | X_t = \tilde{s}_t^n) \\ &= \prod_{j=1}^{m_t} p(y_t^j | \tilde{s}_t^n) \propto \prod_{j=1}^{m_t} [\frac{\pi_t^0}{V} + \sum_{i=1}^M l_t^i(y_t^j; \tilde{s}_t^{n,i}) \pi_t^i]. \end{aligned} \tag{9}$$

It must be noted that the first equality in (9) is true only under the assumption of conditional independence of the measures, which we shall make. Moreover, the normalization factors between $p(Y_t^j = y | K_t^j = i, X_t^i = x)$ and l_t^i must be the same for all i to write the second equality in (9). It remains to estimate the association probabilities $(\pi_t^i)_{i=1,\dots,M}$, which can be seen as the stochastic coefficients of the $M-$component mixture. To estimate them we propose to use a Gibbs sampler whose principles are briefly recalled (see [1] or [9] for more details). For $\theta = (X_t, K_t, \Pi_t)$, it consists in generating a Markov chain that converges to the stationary distribution $p(\theta | Y_{0:t})$ which cannot be sampled directly. Given a partition $\theta^1, \dots, \theta^P$ of θ, one samples alternatively from the conditional posterior distribution of each component of the partition. Assume the τ first elements of the Markov chain $(\theta_1, \dots, \theta_\tau)$ have been drawn. We sample the P components of $\theta_{\tau+1}$ as follows:

- Initialization: $\left\{ \begin{array}{l} s_0^n \sim p(X_0) \\ q_0^n = 1/N \end{array} \right. \quad n = 1, \ldots, N.$

- For $t = 1, \ldots, T$:

  - Proposal: sample $\tilde{s}_t^n$ from $f(X_t | X_{t-1} = s_{t-1}^n, Y_t = y_t)$ for $n = 1, \ldots, N$.
  - Weighting:

    1. Initialization of the Gibbs sampler: $\left\{ \begin{array}{ll} \pi_{t,0}^i = \frac{1-\pi_t^0}{M} & i = 1, \ldots, M; \\ X_{t,0}^i = \sum_{n=1}^N q_{t-1}^n \tilde{s}_t^{n,i} & i = 1, \ldots, M. \end{array} \right.$
    2. For $\tau = 0, \ldots, \tau_{end}$:

       $a.$ $K_{t,\tau+1}^j \sim \; p(K_{t,\tau+1}^j = i) \propto \left\{ \begin{array}{ll} \pi_{t,\tau}^i l_t^i(y_t^j ; x_{t,\tau}^i) & \text{if } i = 1, \ldots, M; \\ \pi_t^0 / V & \text{if } i = 0. \end{array} \right.$

       $b.$ $\pi_{t,\tau+1}^{1:M} \sim \mathcal{D}((1 + n^i(K_{t,\tau+1}))_{i=1,\ldots,M})$, $n^i(K) \triangleq \sharp\{j : K^j = i\}$.

       $c.$ For each i such that $\exists j^1, \ldots, j^i / K_{t,\tau+1}^{j^l} = i$,

       $$\blacktriangleright \left\{ \begin{array}{l} \sigma_{\tau+1}^n = \tilde{s}_t^{n,i} \\ \chi_{\tau+1}^n = \frac{p(y_t^{j^1}, \ldots, y_t^{j^i} | X_t^i = \sigma_{\tau+1}^n) \, q_{t-1}^n}{\sum_{n=1}^N p(y_t^{j^1}, \ldots, y_t^{j^i} | X_t^i = \sigma_{\tau+1}^n) \, q_{t-1}^n} \end{array} \right. \quad n = 1, \ldots, N.$$

       $$\blacktriangleright X_{t,\tau+1}^i \sim \sum_{n=1}^N \chi_{\tau+1}^n \delta_{\sigma_{\tau+1}^n}$$

       $d.$ For each i such that $\nexists j / K_t^j = i$, $X_{t,\tau+1}^i \sim \sum_{n=1}^N q_{t-1}^n \delta_{\tilde{s}_t^{n,i}}$.
    3. $\hat{\pi}_t^i = \frac{1}{\tau_{beg} - \tau_{end}} \sum_{\tau=\tau_{beg}}^{\tau_{end}} \pi_{t,\tau}^i \quad i = 1, \ldots, M.$
    4. $l_t(Y_t = (y_t^1, \ldots, y_t^{m_t}) | \tilde{s}_t^n) = \prod_{j=1}^{m_t} [\frac{\pi_t^0}{V} + \sum_{i=1}^M l_t^i(y_t^j ; \tilde{s}_t^{n,i}) \hat{\pi}_t^i] \quad n = 1, \ldots, N.$
    5. $q_t^n \propto q_{t-1}^n \frac{p(\tilde{s}_t^n) | s_{t-1}^n) \; l_t(Y_t | \tilde{s}_t^n)}{f(\tilde{s}_t | s_{t-1}, y_t)} \quad n = 1, \ldots, N.$
  - Return $\widehat{\mathbb{E}} g(X_t) = \sum_{n=1}^N q_t^n g(\tilde{s}_t^n)$.
  - Calculate $\hat{N}_{eff} = \frac{1}{\sum_{n=1}^N (q_t^n)^2}$.
  - Resampling: if $\hat{N}_{eff} < N_{threshold}$: $\left\{ \begin{array}{l} s_t^n \sim \sum_{k=1}^N q_t^k \delta_{\tilde{s}_t^k} \\ q_t^n = 1/N \end{array} \right. \quad n = 1, \ldots, N$, else $s_t^n = \tilde{s}_t^n$ for $n = 1, \ldots, N$

Figure 2: MTPF: multiple target particle filter with adaptive resampling.

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Draw θ¹_{τ+1} from p(θ¹|Y_{0:t}, θ²_τ, ..., θ^P_τ)
          ⋮                  ⋮
Draw θ^P_{τ+1} from p(θ^P|Y_{0:t}, θ¹_{τ+1}, ..., θ^{P-1}_{τ+1})
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In our case, at a given instant t, the partitionning of θ is:

$$\left\{ \begin{array}{ll} \theta^j = K_t^j & \text{for } j = 1, \ldots, m_t; \\ \theta^{m_t + i} = \pi_t^i & \text{for } i = 1, \ldots, M; \\ \theta^{m_t + M + i} = X_t & \text{for } i = 1, \ldots, M. \end{array} \right. \tag{10}$$

and the algorithm is described in figure 2.

3.3 Application to bearings-only problems with clutter

We first deal with classical bearings-only problem with synthetic data and three targets. In the context of a slowly maneuvering target, we have chosen a nearly-constant-velocity model.

3.3.1 The model

The state vectors X_t^i represent the coordinates and the velocities in the $x - y$ plane: $X_t^i = (x_t^i, y_t^i, vx_t^i, vy_t^i)$ for $i = 1, 2, 3$. For each target, the discretized state equation associated with time period Δt is:

$$X_{t+\Delta t}^i = \left(\begin{array}{ll} Id_2 & \Delta t \, Id_2 \\ 0 & Id_2 \end{array} \right) X_t^i + \left(\begin{array}{l} \frac{\Delta t^2}{2} Id_2 \\ \Delta t \, Id_2 \end{array} \right) V_t, \tag{11}$$

where Id_2 is the identity matrix in dimension 2 and V_t is a Gaussian zero-mean vector of covariance matrix $\Sigma_V = \left[\begin{array}{ll} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{array} \right]$. Let $\hat{X}_t^i$ be the estimation of X_t^i computed by the particle filters with $g(x) = x$, i.e., $\hat{X}_t^i = \sum_{n=1}^N q_t^n \tilde{s}_t^{n,i}$. We use a bootstrap filter, i.e., the importance function f is in fact the prior law $p(x_t | x_{t-1})$. A set of m_t measurements is available at discrete times and can be divided in two subsets:

- A subset of "true" measurements which follow (12) if they are produced by the i^{th} target.

$$Y_t = \arctan\left(\frac{x_t^i - x_t^{obs}}{y_t^i - y_t^{obs}}\right) + W_t, \qquad (12)$$

  where W_t is a zero-mean Gaussian noise of covariance σ_w^2 ($\sigma_w = 0.005$ radians in the sequel) independent of V_t, and x_{obs} and y_{obs} are the Cartesian coordinates of the observer, which are known. We assume that the measurement produced by one target is available with a detection probability P_d ($P_d = 0.9$ in the following simulations).

- A subset of "false" measurements which number follows a Poisson distribution with mean λV.

3.3.2 Results of the MTPF

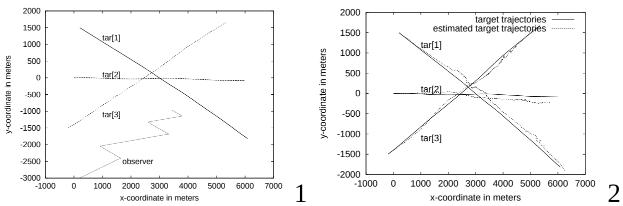

Figure 3: (1) Trajectories of the three targets and of the observer; (2) The true and estimated trajectories of the three targets with 1000 particles, adaptive resampling, $P_d = 0.9$, and $\lambda V = 3$.

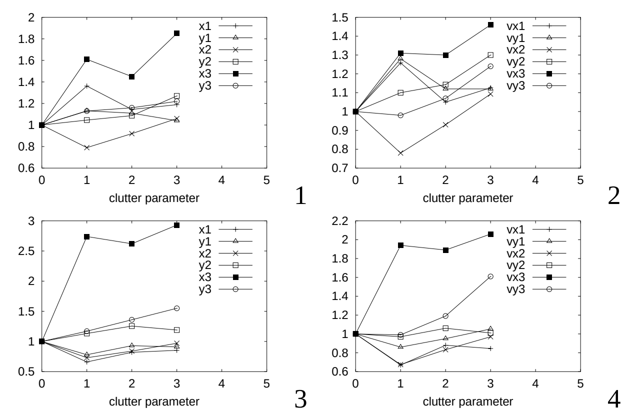

Figure 4: Bias, resp. std, for clutter parameter $= 1, 2, 3$ over bias, resp. std obtained with no clutter obtained with 3000 particles for 30 runs; (1) Bias on x and y position for the three targets; (2) Bias on vx and vy position for the three targets; (3) Std on x and y position for the three targets; (4) Std on vx and vy position for the three targets..

The initial positions of the targets and of the observer are the following, in meters for the positions and ms^{-1} for the velocities: $X_0^{1\,T} = (200,\, 1500,\, 1.0,\, -0.5)$; $X_0^{2\,T} = (0,\, 0,\, 1.0,\, 0)$; $X_0^{3\,T} = (-200,\, -1500,\, 1.0,\, 0.5)$; $X_0^{obs\,T} = (200,\, -3000,\, 1.2,\, 0.5)$. The observer is following a leg by leg trajectory. Its velocity vector is constant on each leg and modified at the following instants, so that:

$$\begin{aligned}(vx^{obs},\, vy^{obs})_{\{200,600,900\}} &= (-0.6,\, 0.3);\\ (vx^{obs},\, vy^{obs})_{\{400,800\}} &= (2.0,\, 0.3).\end{aligned} \qquad (13)$$

The dynamic noise is a normal zero-mean Gaussian vector with $\sigma_x = \sigma_y = 0.001\text{ms}^{-1}$. We use the same dynamic noise to predict the particle. The trajectories of the three targets and of the observer are represented in figure 3.1. The Poisson density mean used is varying between 0 (no clutter) and 3. We consider a total observation volume, i.e., the interval $[-\pi\ ;\pi]$. In this scenario, the data association is particularly difficult: the differences between two bearings issued from two different targets is often lower than the standard deviation of the observation noise. Figure 3.2 shows a particular run of the MTPF with 1000 particles and adaptive resampling. To compare the performances of the algorithm according to the clutter density, we have computed, for $\lambda V = 0, 1, 2, 3$, the bias and the standard deviation for the four components of the vector X for $P = 30$ different runs with 3000 particles defined by:

$$\begin{aligned} bias_l &= \frac{1}{T}\sum_{t=1}^{T}\left|\frac{1}{P}\sum_{p=1}^{P}(\hat{X}_{t,l}^{i,p} - X_{t,l}^{i})\right|,\\ std_l &= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{P}\sum_{p=1}^{P}\hat{X}_{t,l}^{i,p\;2} - \left(\frac{1}{P}\sum_{p=1}^{P}\hat{X}_{t,l}^{i,p}\right)^2\right).\end{aligned} \qquad (14)$$

These different quantities, normalized by their values obtained with no clutter, are plotted against the clutter parameter in figure 4. For the y positions of the three targets, the ratio of the bias with clutter over the bias with no clutter is contained between 1 and 1.2, that is very low. For the x positions, the changes are more important but the bias is not necessarily rising with the clutter parameter. Except for the x and vx component of the third target, the standard deviation is neither very sensitive to clutter. Morever the absolute values of the standard deviation are very low: about 5m for the x and y positions and about 0.003 ms^{-1} for the vx and vy positions. The clutter will not be taken into account in the next sections where we focus on the issue of varying the number of targets, and in the next part where the problem of multiple receivers is adressed.

3.4 Varying number of targets

Until then, the number of targets to track was considered constant and known, and the MTPF estimates the state processes through the indissociable steps of data association and of estimation.

6. Disappearing test:

   $\triangle$ Calculate $\begin{cases} \hat{D}_t^i = 1 \text{ if } \hat{\pi}_t^i > D_{threshold} \\ \hat{D}_t^i = 0 \text{ otherwise} \end{cases} \quad i = 1, \ldots, M$

   $\triangle$ For $i = 1, \ldots, M$, test the hypothesis (H_0^D) against (H_1^D) with a χ^2.
   If (H_1^D) is decided, replace M by $M - 1$ and remove the concerned particle components.

7. Appearing test:

   $\triangle$ Calculate $\hat{N}_t^0 = \frac{1}{\tau_{beg} - \tau_{end}} \sum_{\tau=\tau_{beg}}^{\tau_{end}} n_t^0(K_{t,\tau})$.

   $\triangle$ Test the hypothesis (H_0^A) against (H_1^A) with a χ^2.
   If (H_1^A) is decided, replace M by $M + 1$ and initialize the new particle components.

Figure 5: Disappearing and appearing tests for the MTPF with varying target number.

The vector π_t can then in turn help the estimation: the disparition of one target from the surveillance area (noted SA in the following) can be detected by a drop of corresponding π_t component. We will use the estimation of π_t to decide between the two following hypotheses:
• (H_0^D) The target is present in the SA.
• (H_1^D) The target is not present in the SA.
If the target is still present in the surveillance area, the fall of π_t can only be due to its non-detection, which occurs with a probability $1 - P_d$. Let D_t^i be the binary variable equal to 1 if the i^{th} target has been detected at time t and 0 otherwise. Over a test interval $[t_1, \ldots, t_d]$ and for a given target i, the variables $D_{t_1}^i, \ldots, D_{t_d}^i$ are distributed according to a multinomial law of parameters $(P_d, 1 - P_d)$. These variables are unknown but we can use the estimates $\hat{\pi}_t^i$ to estimate them. Let us define:

$$\begin{cases} \hat{D}_t^i = 1 \text{ if } \hat{\pi}_t^i > D_{threshold}, \\ \hat{D}_t^i = 0 \text{ otherwise.} \end{cases} \tag{15}$$

where $D_{threshold}$ is a probability threshold. The χ^2 test with the variables $\hat{D}_{t_1}^i, \ldots, \hat{D}_{t_d}^i$ decides on the true hypothesis. In practice, the length of the interval $t_d - t_1$ must be chosen such that $(t_d - t_1)(1 - P_d) > 4$. As far as the algorithm is concerned, this reduction only leads to update M (the number of targets) and to remove the components of the particles related to the disapearred target. On the other hand, the arrival of a new target might be related to an observation whose likelihood is low whatever target it is associated with. As a result, assignment variables simulated by the Gibbs sampler might be more often equal to 0. We propose to use the values of the assignment variables to decide between the two following hypotheses:
• (H_0^A) A new target is arriving inside the SA.
• (H_1^A) Any new target is arriving inside the SA.
Let $\hat{N}_t^0$ be the estimation of N_t^0, the number of measures arising from the clutter at time t, supplied by the Gibbs sampler $\hat{N}_t^0 = \frac{1}{\tau_{beg} - \tau_{end}} \sum_{\tau=\tau_{beg}}^{\tau_{end}} n_t^0(K_{t,\tau})$ where $n_t^0(K) \triangleq \sharp\{j : K^j = 0\}$. Over an interval $[t_1, \ldots, t_d]$, a χ^2 test enables to test the adequation between the Poisson law of parameter λV followed by $(N_t^0)_{t=t_1,\ldots,t_d}$ and the empirical law of the variables $(\hat{N}_t^0)_{t=t_1,\ldots,t_d}$. These two tests, summarized in figure 5 can be integrated to the MTPF after the step 5. Nevertheless, the initialization of the new target based on the observation sets is a tricky problem which we have not solved yet. We will now extend the MTPF to deal with observation processes of different kinds.

4 Multireceiver multitarget PF

4.1 The MRMTPF

A natural extension is to consider that observations can be issued from multiple receivers. Let R be their number. We will see that we can easily adapt the particle filter to this situation. We always consider that the M targets (their number is fixed again) obey (5). Some useful notations must be added to modify the measurement equations. The observation vector at time t will be denoted by $y_t = (y_{t,r^1}^1, \ldots, y_{t,r^{m_t}}^{m_t})$ where r^j refers to the receiver which received the j^{th} measure. This measure is then a realisation of the stochastic process:

$$Y_{t,r^j}^j = H_{t,r^j}^i(X_t^i, W_t^j) \text{ if } K_t^j = i. \tag{16}$$

We assume the independence of the observations issued from the different receivers. We denote by $l_{t,r^j}^i(y; x)$ the functions which are proportionnal to $p(Y_{t,r^j}^j = y | K_t^j = i, X_t^i = x)$. The likelihood of the observations conditionned by the n^{th} particle is readily obtained:

$$p(Y_t = (y_{t,r^1}^1, \ldots, y_{t,r^{m_t}}^{m_t}) | X_t = \tilde{s}_t^n) = \prod_{j=1}^{m_t} p(y_{t,r^j}^j | \tilde{s}_t^n)$$
$$\propto \prod_{j=1}^{m_t} [\frac{\pi_t^0}{V} + \sum_{i=1}^{M} l_{t,r^j}^i(y_{t,r^j}^j; \tilde{s}_t^{n,i}) \pi_t^i]. \tag{17}$$

There is no strong limitation on the use of the particle filter for multireceiver and multitarget tracking: the MRMTPF is obtained from the MTPF by replacing the likelihood functions $l_t^i(y;x)$ by the functions $l_{t,r^j}^i(y;x)$. Moreover it can deal with measurements of various periodicities. We present in the next section a scenario where bearings are available at all times whereas range measurements are only available at some times.

4.2 Application to problems with active and passive measurements

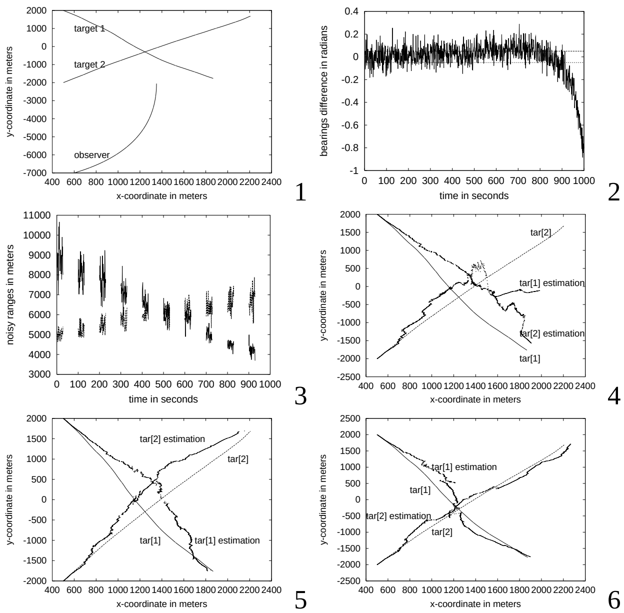

Figure 6: (1) Trajectories of the targets and of the observer; (2) Difference between the noisy bearings issued from the targets compared to the standard deviation of the measurement noise = 0.05, i.e., 2.8 degrees; (3) Noisy ranges simulated for $T = 30$ and $P = 100$; (4) Estimation obtained with only bearings-measurements; (5) Estimation obtained with bearings-measurements and 20% of range measurements; (6) Estimation obtained with bearings-measurements and 50% of range measurements.

We consider a scenario with two targets whose bearings made with an observer are always very closed (see figure 6.2. The trajectories of the targets and of the observer are plotted in figure 6.1 and the initial positions are: $X_0^1 = (500,\, 2000,\, 0.3,\, -0.7)$;
$X_0^2 = (500,\, -2000,\, 0.3,\, 0.7)$;
$X_0^{obs} = (600,\, -7000,\, 0.5,\, 0.3)$. It is to be noted that the observer does not follow a leg by leg trajectory, that renders the estimation of the trajectories quite difficult, and a lot of runs of the MTPF lost the track as illustrated by figure 6.4. To solve this problem, we study the impact of adding active measurements (here ranges). We assume that noisy ranges are available during intervals of length T every P times, i.e., if the current time t is such that $t \; mod \, P \in [0\;; T]$. A noisy range issued from the i^{th} target is supposed to follow the equation:

$$R_t^j = \sqrt{x_t^i + y_t^i} + Z_t \tag{18}$$

where Z_t is a Gaussian noise of standard deviation equal to $\sigma_z(x_t^i + y_t^i)$ with $\sigma_z = 10^{-5}$. This noise modeling seems more realistic than the constant standard deviation modeling generally in use for such contexts. For instance, for $T = 30$ and $P = 100$, the simulated ranges of the two targets are shown in figure 6.3. The evolution of the bias and the standard deviation of the estimation errors has been studied according to the quantity of active measures on the one hand and to their distribution on the other hand.

4.2.1 Quantity of active measurements

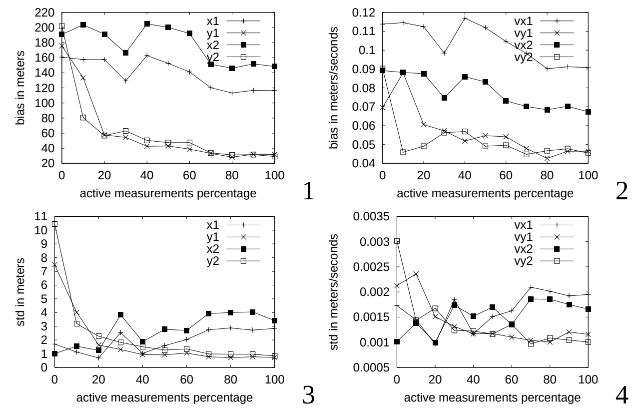

Figure 7: Bias and std on the estimation of the hidden states (x, y, v_x, v_y) with 1000 particles over 20 runs: (1) Bias on x and y position of target 1 and 2; (2) Bias on vx and vy position of target 1 and 2; (3) Std on v_x and v_y position of target 1 and 2; (4) Std on v_x and v_y position of target 1 and 2.

For these experimentations, we have fixed $P = 100$ and taken $T = 10, 20, \ldots, 100$. The figures 6.(5,6) show two particular runs with resp. 20% and 50% of active measurements. Figure 7 summarizes the evolution of the bias and the standard deviation of the estimation errors as a function of the active measurement percentage. First, the addition of active measurements particularly improves the estimation of the components y and vy judging from the bias and std on the total trajectories. The x and vx-positions of the two targets are actually very close. The bearings measurements do not help to dissociate them because of the difficulty of data association. The range measurements enable to discriminate the targets because of the differences betwen their y-position. However, for this scenario, x and vx

measurements are considerably less informative than y and vy ones. The improvement is larger for the standard deviation than for the bias. The percentage of 20% (of active measurements) appears to be a good compromise between a significant improvement of the estimation and a reasonable quantity of active measurements.

4.2.2 Distribution of active measurements

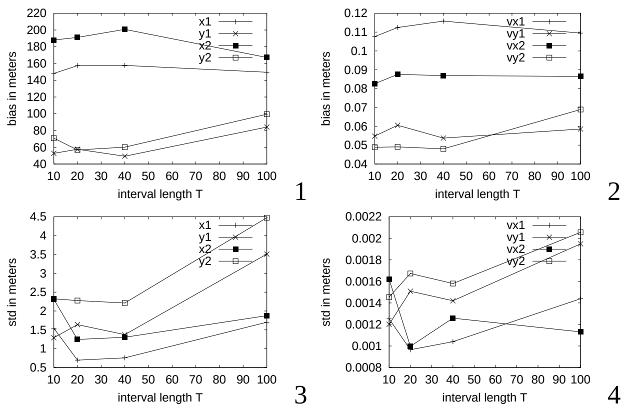

Figure 8: Bias and std on the estimation of the hidden states (x, y, v_x, v_y) with 1000 particles over 20 runs: (1) Bias on x and y position of target 1 and 2; (2) Bias on vx and vy position of target 1 and 2; (3) Std on x and y position of target 1 and 2; (4) Std on v_x and v_y position of target 1 and 2.

We now look at the impact of the distribution of the active measurements: the ratio of passive over active measurements is fixed to 5 (i.e., to 20% of active measurements). The interval length T and the period P during which active measures are available take respectively the values $10, 20, 40, 100$ and $50, 100, 200, 500$. Figure 8 presents the evolution of the bias and of the standard deviation of the estimation errors.

First of all, if the state evolution was deterministic, the better distribution would be to obtain active measurements at the beginning and at the end of the scenario. In our case, the state evolution is stochastic. We observe that the bias and the standard deviation increase with the interval length T, except the bias on the x-position and the std on the vx-position for the second target. As explained before, the scenario implies we have few information about these two components from both the bearings and the ranges. The active measurements should then be available as frequently as possible, by little sets.

5 Conclusion

Two major extensions of the classical particle filter have been presented in order to deal first with multiple targets (MTPF) and then with multiple receivers (MRMTPF). Considering the data association from a stochastic point of view, Gibbs sampling is the workhorse for estimating association vectors, thus avoiding combinatorial drawbacks. Moreover, the particle filtering performs satisfactorily even in the presence of dense clutter. A next step would be to deal with more realistic clutter models. Two statistical tests have also been proposed for detecting changes of the target states (emitting or not). Even if the MTPF is quite versatile, it can suffer from initialization problems. This drawback cannot be completely avoided in the multitarget context and it will be the context for future studies. Finally, MTPF has been extended to multiple receivers and multiple measurements (here passive and active). In this area, the effects of active measurement distribution have been investigated. Preliminary results on this aspect show all the importance of measurement scheduling.

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