A N-DIMENSIONAL ASSIGNMENT ALGORITHM TO SOLVE MULTITARGET TRACKING

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ABSTRACT

This paper deals with combinatorial optimization in multitarget multisensor tracking. The cornerstone in any multitarget and/or multisensor tracking problem is the data-association problem. The approach retained in this paper consists to face the combinatorial complexity. It amounts to solve a multi-dimensional assignment problem. Although this problem is known to be $\mathcal{NP}$-Hard, the Lagrangean relaxation provides bounds on the optimal solution by solving successive 2-dimensional assignment problems.
Inherited from commonly used methods in operational research, the N-dimensional assignment problem first applied to multisensor tracking by [1] is revisited. Particularly, issues of dummy measurements to model missed detection and false-alarms are carefully studied. General conditions required to formulate the multitarget multisensor tracking as a multi-dimensional assignment are also discussed.

1. Introduction

Two different problems have to be solved jointly in a multisensor mutitarget tracking problem: data-association and estimation. The approach retained in this paper consists to face the combinatorial complexity of data association.[3], first, and more recently [1] [4] and [5] have formulated multitarget tracking as a N-dimensional assignment problem. Multitarget tracking is converted in finding a feasible partition which maximizes a cost function. However, it is well known from integer programming theory as soon as N$\geq$ 3, such a problem is $\mathcal{NP}$-Hard so that no known algorithm exists which can guarantee a solution in time bounded by a polynomial in N.

A solution consists then in relaxing some constraints in order to produce an upper (lower) bound on the solution for a maximization (minimization) problem. The N-dimensional assignment[1] is thus relaxed to a 2-dimensional assignment problem where efficient algorithms exist (auction algorithm, signature methods...). The dual problem is then solved by using subgradient methods. The main interest of such methods is to give a measure of quality of the solution to the N-dimensional assignment problem. Thus, the algorithm is able to compute a solution to the primal problem at a few percent.

This algorithm is an extension of commonly used methods in operational research [2]. However, unlike classical methods, dummy measurements are defined in order to take into account false-alarms and missed detections in data-association. These measurements are not submitted to constraints in the association process. Consequently, their introduction must be considered carefully in the algorithm derivation.

The algorithm presented in this paper is the same as proposed by [1]. We focus our presentation on some issues that were not discussed there in depth. Particularly, the issues of dummy measurements to express a 2-dimensional and to recover feasible solutions are investigated. More precisely, we present the algorithm in a global way. We consider a N-dimensional assignment problem. We discuss in a first part the conditions required to formulate multisensor multitarget tracking as a combinatorial optimization problem. Next, we solve the N-dimensional problem by relaxing $(N-2)$ set of constraints. This problem is then recast to a 2-dimensional problem and the issues of dummy measurements are discussed in details. The third part deals with the problem of providing a feasible solution to the given problem based upon a solution to the 2-dimensional Lagrangean relaxation. This section is then illustrated by an example of the 3-dimensional assignment problem applied to a 3 sensors problem.

[1] the primal problem

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2. Formulation of the combinatorial optimization problem, notations

This section is devoted to a general formulation of the multisensor multitarget tracking problem in the combinatorial optimization framework. The cost function of the optimization problem is defined and some issues are discussed.

Consider a general data association problem where a set of measurements, $\mathcal{Z}$, has been received. Suppose this set composed of N scans in a multitarget tracking problem or N sensors in a multisensor tracking problem. These two problems are equivalent in terms of data association problem. The first one consists in a temporal association of measurements from different scans whereas the second needs to solve a spacial association of measurements from N sensors at the same time. The aim of the two problems is to estimate the number and the parameters of the targets in the surveillance region. Throughout the paper, we use the notations defined below :

- z_{i_1} is the i_1^{th} measurement of the 1^{st} scan in multitarget tracking or the i_1^{th} measurement received by the sensor 1 in multisensor tracking. We denote z_0 the dummy measurement used to model missed detection or false-alarm;
- $Z_{i_1,i_2,\ldots,i_N} = \{z_{i_1}, z_{i_2}, \ldots, z_{i_N}\}$ where $i_1 = 0,\cdots,n_1 \ldots i_N = 0,\cdots,n_N$. n_j is the number of measurements in the j^{th} scan or the number of measurements received by the j^{th} sensor. This set caracterizes an hypothetic track;
- $\gamma = \{Z_{i_1,i_2,\ldots,i_N}\}$ is a feasible partition of the set of measurements $\mathcal{Z}$ with respect to the constraints : for $i_j, i'_j = 1,\ldots,n_j$ and $i_j \neq i'_j$, $j = 1,\ldots,N$ $Z_{i_1,i_2,\ldots,i_N} \cap Z_{i'_1,i'_2,\ldots,i'_N} = \emptyset$; and $\cup Z_{i_1,i_2,\ldots,i_N} = \mathcal{Z}$;
- γ_0 is the partition where all the measurements are false-alarms. A false-alarm is defined as a N-tuple where only one measurement is a non dummy measurement[2];
- $\Gamma = \{\gamma\}$ is the set of all the feasible partition;
- $\mathcal{H}_\gamma = \{\gamma$ is the true partition $\}$ corresponds to the event associated to the partition γ;
- $\mathcal{H} = \{\mathcal{H}_\gamma | \gamma \in \Gamma\}$ is the set of all the feasible events.
- we will denote $\mathcal{X}$ the state vector of the sources in the surveillance region.

[2]this definition may be modified depending on the application

Before defining the cost function of the problem, we introduce a property that it must verify :

Proposition 1 *The cost function of the problem must be broken down in a cumulative sum of the contributions of each N-tuple $Z_{i_1,i_2,\ldots,i_N}$.*

The cost function of the problem is defined based to the probability of the event $\mathcal{H}_\gamma$. The a posteriori probability of $\mathcal{H}_\gamma$ can then be expressed using Bayes'rules as :

$$p(\mathcal{H}_\gamma|\mathcal{Z},\mathcal{X}) = \frac{p(\mathcal{Z}|\mathcal{H}_\gamma,\mathcal{X})\; p(\mathcal{H}_\gamma|\mathcal{X})}{p(\mathcal{Z}|\mathcal{X})} \qquad (1)$$

where $p(\mathcal{Z}|\mathcal{X}) = \sum_{\mathcal{H}_\gamma \in \mathcal{H}} p(\mathcal{Z}|\mathcal{H}_\gamma,\mathcal{X})\; p(\mathcal{H}_\gamma|\mathcal{X})$ is the normalization factor. $p(\mathcal{Z}|\mathcal{H}_\gamma,\mathcal{X})$ is the likelihood of the measurements whereas $p(\mathcal{H}_\gamma|\mathcal{X})$ is the a priori probability of the event. The details of these probabilities depend on the application. However, we stress that the a priori probability of the event contains the information on the probabilities of detection, the density of false-alarms etc. Thus, a first idea is to define the cost function as the likelihood ratio or more precisely as the generalized likelihood ratio :

$$\max_{\mathcal{X}} \frac{p(\mathcal{Z}|\mathcal{H}_\gamma,\mathcal{X})}{p(\mathcal{Z}|\mathcal{H}_{\gamma_0},\mathcal{X})}$$

In [1], the cost function is defined as the generalized likelihood function. However, it includes the probability of detection through the likelihood of 3-tuples defined as :

$$p(Z_{i_1,i_2,i_3}|\mathcal{X}) = \prod_{s=1}^{3} [P_{D_s} p(z_{i_s}|\mathcal{X})]^{1-\delta_{i_s}} [1 - P_{D_s}]^{\delta_{i_s}}$$

This is not a density function corresponding to a likelihood of a 3-tuple since a priori terms (probability of detection) are included in this expression. We show next how to retrieve the definition of this cost function. As a conclusion, the likelihood ratio (or the generalized likelihood ratio) verifies property 1 but don't model correctly the problem. This can lead to incorrect associations in solving the optimization problem.

A second idea is thus to consider a bayesian cost :

$$\max_{\mathcal{X}} \frac{p(\mathcal{H}_\gamma|\mathcal{Z},\mathcal{X})}{p(\mathcal{H}_{\gamma_0}|\mathcal{Z},\mathcal{X})}$$

It has the advantage to take into account the a priori probability of the different partitions (through eq.(1))

but does not verify now property 1 since $p(\mathcal{H}_\gamma|\mathcal{X})$ can not split up in each N-tuple. Consequently, a compromise solution must be retained between a good modelisation of the physical problem and the required property of the cost function. This solution is obtained by making the assumption that terms in $p(\mathcal{H}_\gamma|\mathcal{X})$ which can not be broken down are equally probable. In this way, we obtain finally the cost function as defined in [1]. We do not go further into details of the calculation. Based to the appropriate cost function, the problem may be stated as choosing the partition which maximizes the cost function. Since all the N-tuples must be considered, binary variables $\rho_{i_1 i_2 \cdots i_N}$ are introduced to indicate whether the corresponding N-tuple $Z_{i_1 i_2 \cdots i_N}$ belongs to the partition. The contribution of the N-tuple to the cost of a partition is denoted $c_{i_1 i_2 \cdots i_N}$. As soon as property 1 is verified, this leads to the following mathematical problem :

$$\Phi_N = \max_{\rho_{i_1 i_2 \cdots i_N}} \sum_{i_1=0}^{n_1} \cdots \sum_{i_N=0}^{n_N} c_{i_1 i_2 \cdots i_N}\, \rho_{i_1 i_2 \cdots i_N} \tag{2}$$

subject to

$$\begin{cases} \sum_{i_2=0}^{n_2} \cdots \sum_{i_N=0}^{n_N} \rho_{i_1 i_2 \cdots i_N} & = 1,\ i_1 = 1 \cdots n_1 \\ \vdots & \vdots \\ \sum_{i_1=0}^{n_1} \cdots \sum_{i_{N-1}=0}^{n_{N-1}} \rho_{i_1 i_2 \cdots i_N} & = 1,\ i_N = 1 \cdots n_N \end{cases} \tag{3}$$

The N sets of constraints (3) are the mathematical formulation of the constraints in the building of the feasible partitions. Thus, the multisensor multitarget tracking problem has been expressed as a combinatorial optimization problem more precisely as a N-dimensional assignment problem. We stress that this problem is feasible since the N-tuples in γ_0 verify the constraints (3). A Lagrangean relaxation algorithm is proposed as in [1] to solve this difficult $\mathcal{NP}$-Hard problem. This method is the most efficient since it provides the tightest bounds on the solution of the given problem.

3. The Lagrangean relaxation of the N-dimensional assignment problem

This problem is $\mathcal{NP}$-Hard. So, it does not exist an algorithm which provides a solution in time bounded by a polynomial. Lagrangean relaxation is commonly used in integer programming since it consists in including the difficult constraints into the objective function. Although, in general the solution to the Lagrangean relaxation is not the solution to the given problem, it provides a good estimation of it. In our problem, it is an easy task to show that the two solutions are equalled since the constraint matrix is totally unimodular [6].

The aim is thus to relax the sets of constraints which make the problem $\mathcal{NP}$-Hard in a new problem which can be solved efficiently. Since polynomial algorithms exist for the 2-dimensional assignment problem, $(N-2)$ sets of constraints are relaxed in (2). The relaxed problem is then recast in a 2-dimensional assignment problem. However, this new formulation needs some comments since the dummy measurements are not submitted to constraints.

So, Suppose the last $(N-2)$ sets of constraints be relaxed. The Lagrangian relaxation of the N-dimensional assignment problem is of the form :

$$\begin{aligned} \Phi_u^2(u_3, \ldots, u_N) = & \\ & \max_{\rho_{i_1 \cdots i_N}} \sum_{i_1=0}^{n_1} \cdots \sum_{i_N=0}^{n_N} (c_{i_1 \cdots i_N} - u_{3,i_3} - \cdots \\ & - u_{N,i_N})\, \rho_{i_1 \cdots i_N} + \sum_{i_3=0}^{n_3} u_{3,i_3} + \cdots + \sum_{i_N=0}^{n_N} u_{N,i_N} \end{aligned} \tag{4}$$

subject to

$$\begin{cases} \sum_{i_2=0}^{n_2} \cdots \sum_{i_N=0}^{n_N} \rho_{i_1 \cdots i_N} & = 1,\ i_1 = 1 \cdots n_1 \\ \sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \cdots \sum_{i_N=0}^{n_N} \rho_{i_1 \cdots i_N} & = 1,\ i_2 = 1 \cdots n_2 \end{cases} \tag{5}$$

where $u_3 = \{u_{3,i_3}\}_{i_3=0 \cdots n_3}, \ldots, u_N = \{u_{N,i_N}\}_{i_N=0 \cdots n_N}$ are the Lagrange multiplier vectors for the relaxed constraints. For convenience of notation, we introduce a Lagrange multiplier $u_{j,0}$ $(j = 3 \cdots N)$. Since it does not correspond to any relaxed contraint, $u_{j,0} = 0,\ \forall j = 3 \cdots N$. This problem can then be recast in a 2-dimensional assignment problem as follows [2] [1] :

$$\begin{aligned} \overline{\Phi}_u^2(u_3, \ldots, u_N) = & \\ & \max_{w_{i_1 i_2}^2} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} d_{i_1 i_2}^2 w_{i_1 i_2}^2 + \sum_{i_3=0}^{n_3} u_{3,i_3} \cdots + \sum_{i_N=0}^{n_N} u_{N,i_N} \end{aligned} \tag{6}$$

subject to

$$\begin{cases} \sum_{i_2=0}^{n_2} w_{i_1 i_2}^2 & = 1,\ i_1 = 1 \cdots n_1 \\ \sum_{i_1=0}^{n_1} w_{i_1 i_2}^2 & = 1,\ i_2 = 1 \cdots n_2 \end{cases} \tag{7}$$

where $d^2_{i_1 i_2}$ and $w^2_{i_1 i_2}$ are given below :

$$d^2_{i_1 i_2} = \begin{cases} \max_{i_3} \cdots \max_{i_N} (c_{i_1 \cdots i_N} - u_{3,i_3} - \cdots \\ \quad -u_{N,i_N}) & , \forall (i_1, i_2) \neq (0,0) \\ 0 & , (i_1, i_2) = (0,0) \end{cases}$$

$$w^2_{i_1 i_2} = 0, 1$$

Proof :

Let i_1 and i_2 be fixed such that $(i_1, i_2) \neq (0,0)$. In order to verify one[3] or the two sets of constraints in (5)[4], it must exist a unique tuple $(i_3, \cdots, i_N)$ for a i_1 and a i_2. Thus, we choose the best one that is the one which maximizes $(c_{i_1 \cdots i_N} - u_{3,i_3} - \cdots - u_{N,i_N})$. Next consider $(i_1, i_2) = (0,0)$, none of the two constraints needs to be satisfied. Thus, $(0,0)$ does not take part directly in the maximizing problem, it can be considered separately as we will show. Finally, the problem can be recast in a maximizing problem with respect to i_1 and i_2 where the constraints have been modified in (7). Since the case $(0,0)$ has not been considered in (6), the two problems (4) and (6) are not equivalent. Nevertheless, it is straightforward to build the solution of (4) based upon the solution of (6). Just add in the solution the N-tuples of the form $(0, 0, i_3, \cdots, i_N)$ for which the reduced cost $(c_{i_1 \cdots i_N} - u_{3,i_3} - \cdots - u_{N,i_N}) > 0$.

Problem (6) is solved using 2-dimensional assignment algorithm (auction, signature method ...). Notice that the use of the auction requires some modifications to take into account dummy measurements [1]. The main interest in relaxation methods and more precisely in Lagrangean relaxation relies upon the following property :

Proposition 2

$$\Phi_N \leq \Phi^{(2)*} = \min_{u_3 \cdots u_N} \Phi_u^{(2)} \leq \Phi_u^{(2)}.$$

This property is deduced from the fact that the set of feasible solutions of (2) is included in the set of feasible solutions of (4). The difference $(\Phi^{2*} - \Phi_N)$ is called the duality gap in the litterature. Thus based upon a feasible solution of (2), the method provides an overestimate[5] of the duality gap. As we mentioned above, in the multi-dimensional assignment problem, there's no duality gap. Thus, the solution will be at a few percent of the optimal one depending on the rate of convergence of the algorithm used to optimize the dual function. The interest is the opportunity to define a stopping criteria from which data-association does not need to be pursued due to the noise level.

[3] If i_1 or i_2 equals zero

[4] The set of feasible solutions of (4) is nonempty since the N-tuples in γ_0 verify the constraints.

[5] we refer to it as the approximated gap

The minimization of the dual function Φ_u^2 is obtained by using subgradient methods since this function have the nice property of being a continuous piecewise linear convex function[6]. So having a starting point, e.g., $u^{(0)} = \{u_j^{(0)}\}_{j=1 \cdots N}$ we may solve the minimization problem by calculating a sequence of points :

$$u_{j,i_j}^{(k+1)} = u_{j,i_j}^{(k)} + \lambda^{(k)} g_{j,i_j}^{(k)} \; \forall j = 3 \cdots N \; i_j = 1 \cdots n_j \; .$$

Different strategies may be employed to choose the step $\lambda^{(k)}$ in the direction of a subgradient $g_j^{(k)}$. It exists a subgradient which is very easy to compute since it corresponds to the violated constraints :

$\forall j = 3 \cdots N \; i_j = 1 \cdots n_j$

$$g_{j,i_j}^{(k)} = \tag{8}$$

$$1 - \sum_{i_1=0}^{n_1} \cdots \sum_{i_{j-1}=0}^{n_{j-1}} \sum_{i_{j+1}=0}^{n_{j+1}} \cdots \sum_{i_N=0}^{n_N} \rho^*_{i_1 \cdots i_j \cdots i_N} \, . \tag{9}$$

In (9), $\rho^*_{i_1 \cdots i_N}$ is the optimal solution of the Lagrangean relaxation (4). We stress that the global performance of the N-dimensional assignment problem depends mainly upon the rapidity of convergence of the algorithm used to optimize the dual problem.

For clarity of presentation, the primal problem was relaxed by including the $(N-2)$ last sets of constraints into the objective function. This relaxation may be also archieved by relaxing one set of constraints at a time[1]. We thus obtain a recursive formulation of the 2-dimensional assignment problem (4) which is the form retained for the algorithm derivation. Starting from $r = N$ to $r = 3$, the r^{th} Lagrangean relaxation $\Phi_u^{(r-1)}$ is expressed based upon $\Phi_u^{(r)}$ by relaxing the last set of constraints[6]. Then, $\Phi_u^{(r-1)}$ is recast to $\overline{\Phi}_u^{(r-1)}$ defining a recursive form based to $\overline{\Phi}_u^{(r)}$. Property 2 is then recast to :

Proposition 3

$$\Phi_N \leq \Phi^{(N-1)*} = \min_{u_N} \Phi_u^{(N-1)} \leq \Phi_u^{(N-1)} \leq \cdots$$
$$\leq \Phi^{(2)*} = \min_{u_3 \cdots u_N} \Phi_u^{(2)} \leq \Phi_u^{(2)}.$$

Note that the discussion on the issues of the dummy measurements still remain valid. Finally, we propose in the next section to detail the method for obtaining a feasible solution to the given problem. We recall that this feasible solution serves to estimate the approximated gap.

[6] the set of constraints on i_r

4. Obtaining feasible solutions

From the solution of the 2-dimensional assignment algorithm, we show that a feasible solution to the primal problem is obtained by successively solving 2-dimensional algorithms. Constraints are enforced one at a time. The issues of dummy measurements for recovering feasible solutions are investigated. We denote W^{2*} the set of solutions of the 2-dimensional assignment problem (6). First, based upon W^{2*}, constraints on i_3 are enforced by solving a 2-dimensional problem, in order to provide a feasible solution to $\overline{\Phi}_u^{(3)}$. This procedure is then applied to the solution of this new 2-dimensional problem, W^{3*}, to enforce constraints on i_4 and is repeated till enforcing constraints on i_N.

Let us describe in detail this procedure. W^{2*} is defined as an ordered set :

$$W^{2*} = \left\{ w^{2*}_{1\alpha_1}, \cdots, w^{2*}_{n_1\alpha_{n_1}}, w^{2*}_{0\alpha_{n_1+1}}, \cdots, w^{2*}_{0\alpha_L} \right\} .$$

Note that the length L of this finite set is greater or equal to n_1 due to the fact that dummy measurements may have been associated to multiple measurements in order to satisfy constraints (7) in problem (6). Moreover, since the optimal solution W^{2*} satisfies constraints (7), $\{\alpha_j\}_{j=1\cdots L}$ verify :

$$\begin{aligned} \forall(\alpha_j, \alpha_{j'}) \neq (0,0)\,, \qquad & \alpha_j \neq \alpha_{j'} \\ \forall \alpha_j \neq 0\,, \qquad & \cup \alpha_j = \{1, \cdots, n_2\} \,. \end{aligned}$$

Since we know how to associate an element of $\{i_1\}_{0\cdots n_1}$ to a unique element of $\{i_2\}_{0\cdots n_2}$, we would like to associate each couple (i_1, i_2) to a unique element $\{i_3\}_{0\cdots n_3}$ so that each element $\{i_3\}_{0\cdots n_3}$ would be associated to a unique couple (i_1, i_2). Thus, we only need to define a cost function on two variables: one describing the first index set and another describing the third index set. Since, multiple measurements from the second index set may be assigned to the dummy measurement of the first index set, we introduce a new index $l = 0 \cdots L$ and define new costs $\beta_{l\,i_3}$ as :

$$\beta_{l\,i_3} = \begin{cases} \max\limits_{i_4} \cdots \max\limits_{i_N} (c_{00 i_3 \cdots i_N} - u_{4,i_4} \\ \qquad - \cdots - u_{N,i_N}) = d^3_{00 i_3} & l = 0 \\ \max\limits_{i_4} \cdots \max\limits_{i_N} (c_{i_1 \cdots i_N} - u_{4,i_4} \\ \qquad - \cdots - u_{N,i_N}) = d^3_{l\alpha_l i_3} & 1 \le l \le L \\ \max\limits_{i_4} \cdots \max\limits_{i_N} (c_{0\alpha_l \cdots i_N} - u_{4,i_4} \\ \qquad - \cdots - u_{N,i_N}) = d^3_{0\alpha_l i_3} & l \ge n_1 + 1 \end{cases}$$

The best[7] feasible solution to the 3-dimensional assignment problem, based upon a solution to the 2-dimensional Lagrangean relaxation, is thus obtained by solving the following 2-dimensional assignment problem :

$$\max_{x_{li_3}} \sum_{l=0}^{L} \sum_{l=0}^{L} \beta_{li_3} x_{li_3} \tag{10}$$

subject to

$$\begin{cases} \sum\limits_{l=0}^{L} x_{li_3} & = 1,\ i_3 = 1 \cdots n_3 \\ \sum\limits_{i_3=0}^{n_3} x_{li_3} & = 1,\ l = 1 \cdots L \\ x_{li_3} = 0, 1 \end{cases} \tag{11}$$

Proof:

We give some elements to show that the solution of this 2-dimensional assignment problem is a feasible solution to the 3-dimensional Lagrangean relaxation. First, we recall the expression of $\Phi_u^{(3)}$:

$$\begin{aligned} &\Phi_u^{(3)}(u_4, \ldots, u_N) = \\ &\quad \max_{\rho_{i_1 \cdots i_N}} \sum_{i_1=0}^{n_1} \cdots \sum_{i_N=0}^{n_N} (c_{i_1 \cdots i_N} - u_{4,i_4} - \cdots \\ &\quad - u_{N,i_N})\, \rho_{i_1 \cdots i_N} + \sum_{i_4=0}^{n_4} u_{4,i_4} + \cdots + \sum_{i_N=0}^{n_N} u_{N,i_N} \end{aligned} \tag{12}$$

subject to

$$\begin{cases} \sum\limits_{i_2=0}^{n_2} \cdots \sum\limits_{i_N=0}^{n_N} \rho_{i_1 \cdots i_N} & = 1,\ i_1 = 1 \cdots n_1 \\ \sum\limits_{i_1=0}^{n_1} \sum\limits_{i_3=0}^{n_3} \cdots \sum\limits_{i_N=0}^{n_N} \rho_{i_1 \cdots i_N} & = 1,\ i_2 = 1 \cdots n_2 \\ \sum\limits_{i_1=0}^{n_1} \sum\limits_{i_2=0}^{n_2} \sum\limits_{i_4=0}^{n_4} \cdots \sum\limits_{i_N=0}^{n_N} \rho_{i_1 \cdots i_N} & = 1,\ i_3 = 1 \cdots n_3 \,. \end{cases} \tag{13}$$

It is recast in :

$$\begin{aligned} &\overline{\Phi}_u^{(3)}(u_4, \ldots, u_N) = \\ &\quad \max_{w^3_{i_1 i_2 i_3}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} d^3_{i_1 i_2 i_3} w^3_{i_1 i_2 i_3} + \sum_{i_4=0}^{n_4} u_{4,i_4} \\ &\quad + \cdots + \sum_{i_N=0}^{n_N} u_{N,i_N} \end{aligned} \tag{14}$$

subject to

$$\begin{cases} \sum\limits_{i_2=0}^{n_2} \sum\limits_{i_3=0}^{n_3} w^3_{i_1 i_2 i_3} & = 1,\ i_1 = 1 \cdots n_1 \\ \sum\limits_{i_1=0}^{n_1} \sum\limits_{i_3=0}^{n_3} w^3_{i_1 i_2 i_3} & = 1,\ i_2 = 1 \cdots n_2 \\ \sum\limits_{i_1=0}^{n_1} \sum\limits_{i_2=0}^{n_2} w^3_{i_1 i_2 i_3} & = 1,\ i_3 = 1 \cdots n_3 \end{cases} \tag{15}$$

[7] in terms of maximization

In (15), $d^3_{i_1 i_2 i_3}$ and $w^3_{i_1 i_2 i_3}$ are given below :

$$d^3_{i_1 i_2 i_3} = \begin{cases} \max\limits_{i_4} d^4_{i_1 i_2 i_3 i_4} & , (i_1, i_2, i_3) \neq (0,0,0) \\ 0 & , (i_1, i_2, i_3) = (0,0,0) \end{cases}$$

$$w^3_{i_1 i_2 i_3} = 0, 1$$

Now, suppose a solution to (10), x^{3*}, is obtained. Note that the feasible set of (10) may be empty due to variables x_{li_3} preassigned to 0. This may arrive when a preprocessing is used to limit the number of candidate N-tuples. Nevertheless, it is possible to avoid this drawback. So we consider for the sequel that the feasible set is not empty. $W^{3*} = \{w^{(3)*}_{i_1 i_2 i_3}\}$ is then deduced from x^{3*} as follows :

$$w^{(3)*}_{l\alpha_l i_3} = \begin{cases} 1, \forall x_{li_3} \in x^{3*} \\ 0, \text{ otherwise} \end{cases}$$

It is straightforward to verify that W^{3*} verify constraints (15).

We present on fig.1 an example of the application of the 3-dimensional assignment problem to the 3 passive sensors. The cost function was defined as in [1]. The measurements were azimuths and the measurement error variance was 1 deg. This scenario was generated for the ideal case $Pd = 1$ and $Pfa = 0$. The relative approximated gap was $\simeq 2\%$ at the end of the algorihtm.

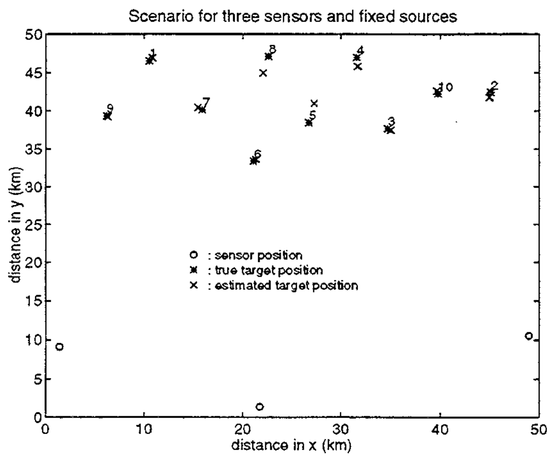

Figure 1: Example of a 3-dimensional assignment problem for the 3 passive sensors

5. Conclusion

The multisensor multitarget tracking has been formulated as a N-dimensional assignment problem. The approach retained consists to face the combinatorial complexity of data association. This approach was first proposed by [1]. In this paper, we have focussed the discussion on the conditions required to formulate the problem as a multi-dimensional assignment problem and on the definition of the cost function to optimize. Moreover, the different steps in the algorithm have been studied carefully, particularly the issues of dummy measurements in the algorithm derivation. Proofs have been given.

However, some more work need to be pursued for applying the algorithm to real applications, more precisely for the passive sonar. We stress that this method is particularly appealing since Lagrangean relaxation provides information of how close the solution is from the optimal solution of the given problem.

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