PROPERTIES AND PERFORMANCE OF EXTENDED TARGET MOTION ANALYSIS

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ABSTRACT

Classical array processings are based upon a short-time analysis itself followed by steps of source tracking, TMA and data association. However, for numerous practical applications (e.g. passive sonar), a large amount of spatio-temporal data is available. It seems, then, possible to use globally all these data in order to separate, detect and track the sources by using their respective trajectories. In this spatio-temporal approach, the concept of source trajectory replaces the concept of instantaneous source bearing for the classical analysis. So, this work tries to build a bridge between two major areas of statistical signal processing which are the classical array processing on the first hand and the TMA on the other hand.

1. INTRODUCTION.

Conceptually, the basic problem in target motion analysis (TMA for the sequel) is to estimate the trajectory of an object (i.e. position and velocity) from noise corrupted sensor data. The performance of any classical TMA algorithm is conditioned by the statistical quality (i.e. bias and variance) of the estimated bearings.
Classical array processings correspond to short time analysis and thus, their performance are basically limited by the unstationary nature of the signal. The natural way to overcome this problem consists in considering a true spatio-temporal analysis instead of classical array-processing. A moving source is then parametrized by a (simple) spatio-temporal model. This model corresponds to a long-time analysis.
It is then necessary to incorporate the (unknown) source motion model into the source's one and to define TMA methods using basically this extended source model. The following steps must then be considered:

- derivation of a simplified model of the source motion
- estimation of the source motion parameters
- spatio-temporal analysis (TMA).

The last point is also the more difficult and important one. Classical array processings are based upon a short-time analysis itself followed by steps of source tracking, TMA and data association. However, for numerous practical applications (e.g. passive sonar), a large amount of spatio-temporal data is available. It seems, then, possible to use globally all these data in order to separate, detect and track the sources by using their respective trajectories. In this spatio-temporal approach, the concept of source trajectory replaces the concept of instantaneous source bearing for the classical analysis.
So, this work tries to build a bridge between two major areas of statistical signal processing which are the classical array processing on the first hand and the TMA on the other hand.

2. A SIMPLIFIED MODEL OF THE SOURCE MOTION SEEN BY THE ARRAY.

The notations are those of the reference paper [2] and [1]. Denoting k_t the instantaneous spatial frequency (i.e. $k_t = \sin\theta_t/\lambda$, λ wavelength), the following equality holds (cf figure 1):

$$k_t = \frac{r_0 \sin\theta_0 + tv\delta T \sin\gamma}{\lambda(r_0^2 + (tv\delta T)^2 + 2tr_0 v\delta T\cos(\gamma-\theta_0))^{\frac{1}{2}}}$$

(γ: source heading, r: relative range, v: relative velocity).
Considering the increment $x = \frac{tv\delta T}{r_0}$ then the following first order expansion of k_t is directly obtained:

$$\begin{aligned} k_t &\stackrel{1}{=} k_0 + t\dot{k} \qquad (1)\\ \text{with: } \dot{k} &= [\sin\gamma - \sin\theta_0\cos(\theta_0-\gamma)]v\delta T/r_0 \end{aligned}$$

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$$= \cos\theta_0 \sin(\gamma - \theta_0) v\delta T / r_0$$

$\dot{k}$: is the spatial frequency rate.

Noting that:

$$\cos\gamma = v_y/v\ , \quad \sin\gamma = v_x/v$$

it comes:

$$\dot{k} = \frac{r_y}{\lambda r_0^3}[r_y v_x - r_x v_y]\delta T.$$

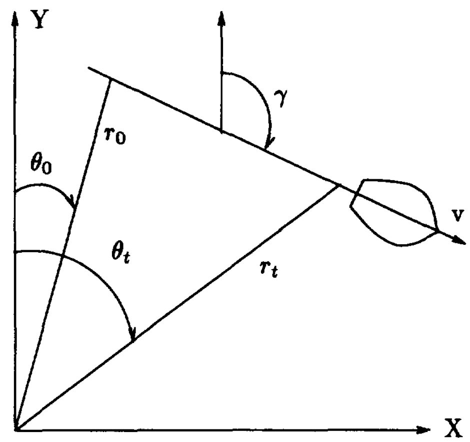

Figure 1: Scenario

Considering the unique source case and the observations made up of N array snapshots $\{X_1, \cdots, X_N\}$ direct calculations yields:

$$\begin{aligned} \mathrm{var}(\hat{k}_0) &\geq \frac{3(1+p\rho)(2N-1)}{p^2\rho^2(p^2-1)\pi^2 d^2 N(N+1)} \\ \mathrm{var}(\hat{\dot{k}}) &\geq \frac{18(1+p\rho)}{p^2\rho^2(p^2-1)\pi^2 d^2 (N^3-N)} \end{aligned}$$

where

ρ: signal to noise ratio.

p: number of sensors (linearly spaced).

N: number of snapshots.

d: intersensor distance.

Actually the two parameters defining the source's trajectory (i.e. k_0 and $\dot{k}$) may be separately estimated. For this purpose, 2D state-space analysis constitute an interesting way.

3. ESTIMATION OF THE $\hat{\dot{k}}$.

Consider the instantaneous spatial correlation of s sources:

$$r(t,m) = \sum_{j=1}^{s} \sigma_j \exp(2i\pi dm(k_j(0) + t\dot{k}_j) + \sigma_\nu^2 \delta(m).$$

Define the source's vector:

$$X(t,m) = \begin{bmatrix} \sigma_1 \exp(2i\pi dm(k_1(0) + t\dot{k}_1) \\ \vdots \\ \sigma_s \exp(2i\pi dm(k_s(0) + t\dot{k}_s) \end{bmatrix},$$

then we have the 2D-state space model [3] for the spatio-temporal sequence (STS) m (the STS m designates here the sequence $\{r(t,m)\}_t$ where the spatial index is constant):

$$\left\{ \begin{aligned} X(t+1,m) &= F_1^m X(t,m)\ , \\ X(t,m+1) &= F_0 F_1^t X(t,m)\ , \\ r(t,m) &= h^T X(t,m)\ , \end{aligned} \right. \tag{2}$$

where :

$$\left| \begin{aligned} F_0 &= \mathrm{diag}\{\exp(2i\pi d k_j(0))\}_{1\leq j\leq N}, \\ F_1 &= \mathrm{diag}\{\exp(2i\pi d \dot{k}_j)\}_{1\leq j\leq N}, \\ h^T &= [1\cdots 1]. \end{aligned} \right. \tag{3}$$

The first equation in (2) stands for the temporal transition, the second one is the space transition.

Let $\mathcal{H}_{t,m}$ be the Hankel matrix build with the STS m, i.e:

$$\mathcal{H}_{t,m} \triangleq \begin{pmatrix} r(t,m) & r(t+1,m) & r(t+2,m) & \cdots \\ r(t+1,m) & r(t+2,m) & r(t+3,m) & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Without consideration of the estimation noise (relative to r) the following factorization holds:

$$\begin{aligned} \mathcal{H}_{t,m} &= \begin{bmatrix} h^t \\ h^t F_1^m \\ h^t F_1^{2m} \\ h^t F_1^{3m} \\ \vdots \end{bmatrix} \begin{bmatrix} X^t(t,m) \\ X^t(t+1,m) \\ X^t(t+2,m) \\ \vdots \end{bmatrix}^t \\ &= \mathcal{O}_m \mathcal{X}_{t,m}. \end{aligned} \tag{4}$$

A direct consequence of (4) is the following rank property:

$$\mathrm{rank}(\mathcal{H}_{t,m}) = \mathrm{rank}(\mathcal{O}_m) = \dim(X).$$

Practically the estimated observability matrix $\hat{\mathcal{O}}_m$ is obtained from the estimated Hankel matrix $\hat{\mathcal{H}}_{t,m}$ by means of a SVD; an estimated matrix $\hat{F}_1$ is then deduced from $\hat{\mathcal{O}}_m$ by using the classical shift property i.e:

$$\hat{\mathcal{O}}^{\uparrow}\hat{F}_1^{m_0} = \hat{\mathcal{O}}^{\downarrow}.$$

where:

$$\hat{\mathcal{O}}^{\uparrow} = \begin{bmatrix} h^t \\ h^t\hat{F}_1^m \\ \vdots \\ h^t\hat{F}_1^{(L-2)m} \end{bmatrix} \quad \hat{\mathcal{O}}^{\downarrow} = \begin{bmatrix} h^t\hat{F}_1^m \\ h^t\hat{F}_1^{2m} \\ \vdots \\ h^t\hat{F}_1^{(L-1)m} \end{bmatrix}$$

The problem consists now in estimating the matrix $\hat{F}_1$ from the estimated spatial correlation:

$$\hat{r}(t,m) = \frac{1}{N_c - |m|} \sum_{p=0}^{N_c-|m|-1} x_t(p)x_t^*(p+m),$$

where $[x_t(0), \cdots, x_t(N_c-1)]$ is the t-th snapshot. An extension to this method has been developped in [3] which uses multiple STS to estimate $\hat{F}_1$. In that case we have to interpolate the spatial correlation to non-integer time indices in order to have the same reference time for all the process and replace the Hankel matrix $\mathcal{H}_t, m$ by a matrix build with the interpolated spatial correlations:

$$\hat{\mathcal{H}}_{t,m_0,c} \triangleq \begin{pmatrix} r(t,m_0) & r(t,m_0+1) & \cdots & r(t,m_0+c) \\ r(t+1,m_0) & \hat{r}(t+\tau_1,m_0+1) & \cdots & \hat{r}(t+\tau_c,m_0+c) \\ \vdots & \vdots & \vdots & \vdots \\ r(t+r,m_0) & \hat{r}(t+r\tau_1,m_0+1) & \cdots & \hat{r}(t+r\tau_c,m_0+c) \end{pmatrix}$$

with:

$$\tau_l \triangleq \frac{m_0}{m_0+l} \quad 1 \le l \le c$$

The scalars $\tau_1, \tau_2, \cdots, \tau_c$ represent the "compression (of time) factors relative to the STS $m_0+1, \cdots, m_0+c$. In this case the reference STS is m_0 but it can be replaced by any STS.

4. ESTIMATION ALGORITHMS OF THE SOURCE TRAJECTORY PARAMETERS.

As usual, the problem is modelled by the following state-space equation [2]:

$$\mathbf{X}(t_k) = \Phi(t_k, t_{k-1})\mathbf{X}(t_{k-1}) + \mathbf{U}(t_k)$$

with:

$$\Phi(t_k, t_{k-1}) = \begin{pmatrix} \mathrm{Id} & (t_k - t_{k-1})\mathrm{Id} \\ 0 & \mathrm{Id} \end{pmatrix} \qquad \mathbf{X} = \mathbf{X}_s - \mathbf{X}_o.$$

The estimated data depend on the state vector by the following relations:

$$\hat{k}_t = k_t + \nu_t \quad k_t = \cos\theta_t/\lambda \quad \theta_t = \tan^{-1}(r_x(t)/r_y(t))$$
$$\hat{\dot{k}}_j = \dot{k}_j + \omega_j \quad \dot{k}_j = \frac{r_{yj}}{\lambda r_j^3}(r_{yj}v_{xj} - r_{xj}v_{yj}).$$

Given the history of measured spatial frequencies $\hat{k}_j$ (i.e. $\hat{\mathbf{K}} \triangleq \left(\hat{k}_1, \hat{k}_2, \cdots, \hat{k}_N\right)^t$) and the estimated values $\hat{\dot{k}}_j$ (i.e. $\hat{\dot{\mathbf{K}}} \triangleq \left(\hat{\dot{k}}_1, \hat{\dot{k}}_2, \cdots, \hat{\dot{k}}_L\right)^t$), the likelihood functional conditional on the state vector $\mathbf{X}$ stands as follows:

$$p(\mathcal{K}|\mathbf{X}) = cst.\exp\left[-\frac{1}{2}(\hat{\mathcal{K}} - \mathcal{K}(\mathbf{X}))^t W^{-1}(\hat{\mathcal{K}} - \mathcal{K}(\mathbf{X}))\right]$$

with:

$$\hat{\mathcal{K}} \triangleq \begin{pmatrix} \hat{\mathbf{K}} \\ - \\ \hat{\dot{\mathbf{K}}} \end{pmatrix}, \quad W = \begin{pmatrix} \sigma_k^2\mathrm{Id} & 0 \\ 0 & \sigma_{\dot{k}}^2\mathrm{Id} \end{pmatrix}.$$

This functional is then maximized by means of any Gauss-Newton algorithm (relatively to $\mathbf{X}$). This algorithm takes the form:

$$\begin{aligned} X_{l+1} &= X_l - s_l\left[\left(\frac{\partial\mathcal{K}}{\partial X}\right)^t W^{-1}\left(\frac{\partial\mathcal{K}}{\partial X}\right)\right]^{-1} \\ &\quad \left(\frac{\partial\mathcal{K}}{\partial X}\right)^t W^{-1}(\hat{\mathcal{K}} - \mathcal{K}) \end{aligned} \tag{5}$$

The utilization of this algorithm needs the computation of the gradient vector at a reference time t_m. The gradient of the i-th spatial frequency is:

$$\frac{\mathrm{d}k_i}{\mathrm{d}K} = \frac{\delta T}{\lambda r_i}\begin{bmatrix} \cos^2\theta_i \\ -\sin\theta_i\cos\theta_i \\ (t_i - t_m)\cos^2\theta_i \\ -(t_i - t_m)\sin\theta_i\cos\theta_i, \end{bmatrix}^t \tag{6}$$

and the derivatives of the j-th spatial frequency rate are:

$$
\begin{aligned}
\frac{\partial \dot{k}_j}{\partial r_x(t_m)} &= -\frac{\cos\theta_j \delta T}{\lambda r_j^2}\left[v_y(t_j)(1-3\sin^2\theta_j)\right. \\
&\qquad \left. + v_x(t_j)(3\sin\theta_j\cos\theta_j)\right], \\
\frac{\partial \dot{k}_j}{\partial r_y(t_m)} &= -\frac{\delta T}{\lambda r_j^2}\left[v_x(t_j)\cos\theta_j(1-3\sin^2\theta_j)\right. \\
&\qquad \left. + v_y(t_j)\sin\theta_j(1-3\cos^2\theta_j)\right], \\
\frac{\partial \dot{k}_j}{\partial v_x(t_m)} &= \frac{\cos^2\theta_j \delta T}{\lambda r_j} \\
&\quad -(t_j-t_m)\frac{\cos\theta_j \delta T}{\lambda r_j^2}\left[v_y(t_j)(1-3\sin^2\theta_j)\right. \\
&\qquad \left. + v_x(t_j)(3\sin\theta_j\cos\theta_j)\right], \\
\frac{\partial \dot{k}_j}{\partial v_y(t_m)} &= \frac{\sin\theta_j\cos\theta_j \delta T}{\lambda r_j} \\
&\quad -(t_j-t_m)\frac{\cos\theta_j v \delta T}{\lambda r_j^2}\left[v_x(t_j)\cos\theta_j(1-3\sin^2\theta_j)\right. \\
&\qquad \left. + v_y(t_j)\sin\theta_j(1-3\cos^2\theta_j)\right], \\
\text{with} &: \cos\theta_j \triangleq \frac{r_y(t_j)}{r_j},\ \sin\theta_j \triangleq \frac{r_x(t_j)}{r_j}
\end{aligned}
\tag{7}
$$

The calculation of the Fisher-information matrix (FIM) yields some interesting results:

$$
FIM(\hat{\mathcal{K}}) = FIM(\hat{\mathbf{K}}) + FIM(\hat{\dot{\mathbf{K}}})
$$

with:

$$
FIM(\hat{\mathbf{K}}) \simeq \frac{1}{(\sigma_k r\lambda)^2}\sum_{i=1}^{N}\begin{pmatrix} \Omega_i & (i-m)\Omega_i \\ (i-m)\Omega_i & (i-m)^2\Omega_i \end{pmatrix}
$$

$$
\Omega_i = \cos^2\theta_i \begin{pmatrix} \cos^2\theta_i & -\frac{1}{2}\sin 2\theta_i \\ -\frac{1}{2}\sin 2\theta_i & \sin^2\theta_i \end{pmatrix}
$$

$$
FIM(\hat{\dot{\mathbf{K}}}) \simeq \frac{1}{(\sigma_{\dot{k}} r\lambda)^2}\sum_{j=1}^{L}\begin{pmatrix} 0 & 0 \\ 0 & \Omega'_j \end{pmatrix}
$$

$$
\Omega'_j = \cos^2\theta_j \begin{pmatrix} \cos^2\theta_j & -\frac{1}{2}\sin 2\theta_j \\ -\frac{1}{2}\sin 2\theta_j & \sin^2\theta_j \end{pmatrix}
$$

This last expression reveals that it is mainly the estimation of the source velocity which may be improved by using the extended vector of measurements $\mathcal{K}$. This is illustrated by a special case study.

5. PERFORMANCE OF THE EXTENDED TMA, A SPECIAL CASE STUDY.

The utilization of a large linear towed array can restrict seriously the ownship maneuvers. For this special case, the observer moves on a straight line with a constant velocity. It is well-known that the bearings-only TMA problem is unobservable in this case. A way to overcome this problem consists in a parametrization of the solution space. More precisely, the (partial) state space $\mathbf{X}_{m,p}$ is defined as follows:

$$
\mathbf{X}_{m,p} = [r_y(t_m), v_x(t_m), v_y(t_m)]^t
$$

then the FIM (relative to $\mathbf{X}_{m,p}$) may be calculated. For the sake of brevety, the calculation details will be omitted. For a classical scenario (source moving symmetrically w.r.t. to the array broadside) one obtains:

$$
\hat{\mathbf{K}} \left| \begin{array}{lcl} \mathrm{var}(\hat{r}_y) & \simeq & \frac{9}{4}\sigma_\theta^2 N^{-1}(\mathrm{bas}) \\ \mathrm{var}(\hat{v}_x) & \simeq & 57\sigma_\theta^2 N^{-3} r^2 \\ \mathrm{var}(\hat{v}_y) & \simeq & 180\sigma_\theta^2 N^{-3}\left(\frac{r}{\mathrm{bas}}\right)^2 r^2 \end{array}\right. \tag{8}
$$

with: bas $\triangleq Nv\sin\gamma =$ baseline. The same calculation for $\hat{\dot{\mathbf{K}}}$ yields:

$$
\hat{\dot{\mathbf{K}}} \left| \begin{array}{lcl} \mathrm{var}(\hat{v}_x) & = & 3\sigma_{\dot{\theta}}^2 r^2 N^{-3}\left(\frac{L^2(7L^2-4)}{4(L^2-1)}\right) \\ \mathrm{var}(\hat{v}_y) & = & 3\sigma_{\dot{\theta}}^2\left(\frac{r}{\mathrm{bas}}\right)^2 N^{-3}\left(\frac{3L^4}{(L^2-1)}\right) \end{array}\right. \tag{9}
$$

We can see from (9) that L must be as small as possible. This remark is tempered by the validity of the approximation (1). Similar computations have been conducted for $\theta_c = \pi/4$:

$$
\hat{\mathbf{K}} \left| \begin{array}{lcl} \mathrm{var}(\hat{r}_y) & \simeq & \frac{27}{7}\sigma_\theta^2 N^{-1} r^2 \\ \mathrm{var}(\hat{v}_x) & \simeq & \mathrm{var}(\hat{v}_y) \simeq \frac{6}{7}90\sigma_\theta^2 N^{-3}\left(\frac{r}{bas}\right)^2 r^2 \end{array}\right. \tag{10}
$$

$$
\hat{\dot{\mathbf{K}}} \left| \begin{array}{lcl} \mathrm{var}(\hat{v}_x) & \simeq & \mathrm{var}(\hat{v}_y) \simeq \lambda\sigma_{\dot{k}}^2\left(\frac{r}{bas}\right)^2 r^2\left(\frac{3L}{L^2-1}\right) \end{array}\right. \tag{11}
$$

with: $bas \triangleq Nv\sin(\gamma-\theta_c)$ (= baseline).
These formula show that the use of the spatial frequency rate allows a reduction of the estimation variance of the source's parameter. This is confirmed by simulation results.

6. SIMULATION.

Many simulations have been done, all of them show that the standart deviation is divided by at least 2 when we use the spatial frequency rate in the algorithm. We present here one of those.

- Number of sensors: 32.
- Duration of the simulation: 1600s.
- At $t = 0$, the observer is at the origin and the source is at (10km, 40km).
- Observer's velocity: $v_{ox} = 5\text{m/s}$, $v_{oy} = 0\text{m/s}$.
- Target's velocity: $v_{tx} = 1\text{m/s}$, $v_{ty} = 7\text{m/s}$.
- There is 120 $\hat{k}$ estimations and 4 $\hat{\dot{k}}$ estimations.

The table below shows you the amelioration you can have using the spatial frequency with its derivative. The gain on the standard deviation of the estimation of the source's parameter is essentially visible on the velocity parameters. It is intellectually satisfying because the $\dot{k}$ is a dynamic measure. The factor 5 or 6 obtained on the standard deviation is representative of the amelioration generally achieved with the inclusion of the $\hat{\dot{k}}$ in the estimator.

$\rho = -20dB$			
	$r_{ty}(t_m)$	v_x	v_y
True values			
	51200	1	7
Estimator ($\hat{k}$)			
Mean	51199.70	0.91	6.6
Standard deviation	23	1.12	4.41
CRB	25	1.26	4.96
Estimator($\hat{k}$ and $\hat{\dot{k}}$)			
Mean	51200.82	0.96	6.84
Standard deviation	10.51	0.26	0.91
CRB	11.37	0.25	0.90

7. MULTIPLE TARGET MOTION ANALYSIS.

Standard TMA permormances may be degraded by the presence of multiple sources in a reduced area. The spatio-temporal analysis presented in section 3 is a good answer to these difficulties:

- The parameters $\{k_{0,i}\}_{i=1}^{s}$ and $\{\dot{k}_i\}_{i=1}^{s}$ may be separatly and directly estimated by a non-iterative and simple procedure.
- Better temporal association between the $\hat{k}$ because of the estimation of the $\hat{\dot{k}}$.
- Temporal association between the $\hat{\dot{k}}$ is simplified due to their relative closeness.

8. CONCLUSION.

The spatial frequency rate has been introduced in the TMA. Besides the addition of a new measurement for improving TMA performance, it represents a fundamental change for TMA since the notion of source trajectory replaces the classical one based on instantaneous bearings. The extension of TMA to these new measurements has been presented, but overall the statistical performance study has been stressed, especially in the case of a non-maneuvering observer for which analitical calculation have been conducted. In all the cases, the improvements of the TMA performance due to these new measurements may be important.

9. REFERENCES

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