MODEL REDUCTION AND WIDEBAND ANALYSIS

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ABSTRACT

The aim of this paper is to extend exact model reduction methods to wideband analysis. After a review of the realization methods in the underwater acoustic context, it is shown that the Optimal Realization Method is very well suited to the wideband analysis, thanks to the rational modelling. The basic steps of the proposed method are a multidimensional AR modelling of the sensor outputs, spatial interpolation, and model reduction of a unique model for the frequency band of analysis. It is important to note that this method uses no *a priori* assumption about source locations or source models. Moreover, this one inherits the robustness w.r.t. spatial noise coloration from its narrowband version. Simulation results demonstrate the resolution and detection improvements achieved by the method.

1 - INTRODUCTION

For most of practical situations, in underwater acoustic, sources emit wideband (w.b. for the sequel) signals; therefore, array processing must be expressed in terms of w.b. analysis. The usual and basic method, for that purpose, consists in beamforming and has the great interest to be of very simple implementation. However, its angular resolution is limited by the array aperture. In order to remedy this problem, high resolution (H.R.) methods have been derived; however, they are typically devoted to narrowband (n.b.) analysis. Recently, some extensions of these ones to w.b. analysis have been considered. They often consist in using coherent source-subspace transformations [1] or frequency-dependent models [2]; their main objective being to enhance angular resolution (w.r.t n.b analysis).

However, for numerous practical situations, detection of weak sources is of fundamental importance. For that purpose, we have promoted the use of the theory of exact model reduction for array processing. This kind of method has already been considered for narrow-band array processing [3] by the authors; its performances are quite satisfying, especially for the detection of weak sources (detection gain w.r.t. H.R. methods can be up to 10 dB). Whereas usual array processing methods utilize a rough description of the acoustic field (including HR methods), the special structure (plane waves) of sources is fundamentally taken into account by the (exact) approximation scheme of the model reduction method.

The methods of functional analysis are then the basic tools; especially the methods for function approximation in hardy spaces developed by Adamjan, Arov and Krein (A.A.K for the sequel) which constitute the theoretical framework and yield explicit solutions to the approximation problems (in infinite dimension, however). Fortunately, the tools of linear system analysis (balanced realizations) allow us to transform this infinite dimensional problem into an elementary problem of finite dimensional linear algebra [4].

Extension of this method to w.b. analysis can be achieved by using several ways. The simplest one consists in considering n.b. state space models of the array outputs; it is convenient to deduce from them a unique n.b. model at a given frequency. This is usually achieved by means of spatial interpolation which is straightforward for AR modelling. These interpolated models have rational common factors corresponding to the fact that source bearings are identical whatever the considered frequency, even if the power spectral densities of sources can greatly differ from a frequency to another. These factors can be estimated either by means of model reduction after spatial interpolation or, in a more rigorous way, by means of polynomial algebra, and yield w.b. source bearing estimates.

The advantages of this approach are numerous: there is no *a priori* assumption w.r.t. source modelling (bearings, spectra, etc.), the computation cost is quite reasonable. Furthermore, the proposed method inherits the robustness w.r.t. spatial noise correlation [3] from the n.b. analysis (exact model reduction). These methods have been developed in the array processing context, leading to simple and efficient algorithms.

2 - SIGNAL MODELING

We assume that sources and noise are statistically independent, at a given frequency the spatial density of a sensor output $\{y_i\}$ is the sum of the source and noise spatial density:

$$P_y = P_s + P_n = \frac{1}{A_s(z)\,A_s^*(1/z)} + \frac{B_n(z)\,B_n^*(1/z)}{A_n(z)\,A_n^*(1/z)} \qquad (1)$$

The AR spatial model corresponding to sources has its poles in the vicinity of the unit circle, whereas the noise is modelled by a general ARMA model (this last hypothesis is quite acceptable).

Under this hypothesis, the sensor outputs (at a given frequency) can be modelled by an innovation state-space model [5]:

$$\begin{cases} \mathbf{X}_{i+1} = \mathrm{F}\,\mathbf{X}_i + \mathbf{T}\,w_i \\ y_i = \mathbf{h}^*\,\mathbf{X}_i + w_i \end{cases} \qquad (2)$$

(y_i : i - th sensor output, $\mathbf{X}_i$: state-space vector, w_i : white noise).

A link can be made with the classical plane wave model [5]:

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$$y_i = \sum_{k=1}^{m} \alpha_k \exp[-j(i-1)\varphi_k] + v_i \qquad (j^2 = -1) \tag{3}$$

(m : source number, φ_k : phasing factor of the k-th source, α_k : random level of the source, $\{v_i\}$: gaussian mean zero additive noise).

Generally the eigenvalues of the transition matrix include the phasing factors φ_k ($\varphi_k = 2\pi f\, d/c \cos\theta_k$, f : frequency, d : intersensor distance and θ_k bearing of the k-th source).So, we are concerned with the estimation of the matrix F, which can be achieved by using several ways. The more classical ones are based upon approximated covariances; whereas the more promising way considers directly an approximation of the transfer function, associated with the sensor outputs.

3 - APPROXIMATED REALIZATION METHODS (ARM)[5]

Considering the model (2) of the sensor outputs, then the ARM rely upon the vectors $\mathbf{Y}_+$ and $\mathbf{Y}_-$ (future and past at sensor $n_s/2$) defined as follows:

$$\begin{cases} \mathbf{Y}_-^t = (y_{n_2}, y_{n_2-1}, \ldots, y_1) \\ \mathbf{Y}_+^t = (y_{n_2+1}, y_{n_2+2}, \ldots, y_{n_s}) \end{cases} \tag{4}$$

$$n_2 = n_{s/2}$$

The stochastic approach consists in determining a state vector $\mathbf{X}$ (of given dimension p) which sums up the more pertinent part of $\mathbf{Y}_-$ in order to predict $\mathbf{Y}_+$, i.e.:

$$\mathbf{X} = A\,\mathbf{Y}_- \tag{5}$$

with A (p x n_2)

For a markovian system, the orthogonal projection of the future onto the past is given by:

$$\mathbf{Y}_+ \mid \mathbf{Y}_- = \mathcal{O}\mathbf{X} \tag{6}$$

($\mathcal{O}$: observability matrix)

The determination of A yields an estimate of the matrix $\mathcal{O}$ and therefore of the system parameters. The crucial part of the ARM method consists in estimating the matrix A, for that purpose various methods have been considered.

The following functional have been proposed.

1) The predictive efficiency criterion (Arun-Kung method [6]):

$$\underset{\mathbf{X} = A\,\mathbf{Y}_-}{\text{Min}} \left\{ \text{tr}\,[\text{cov}\,(\mathbf{Y}_+ - \mathbf{Y}_+ \mid \mathbf{X})] \right\} \tag{7}$$

2) The information criterion (Desai-Pal method [7]):

$$\underset{\mathbf{X} = A\,\mathbf{Y}_-}{\text{Max}} \; \mathcal{J}(\mathbf{Y}_+, \mathbf{X}) \tag{8}$$

with: $\mathcal{J}(\mathbf{Y}_+, \mathbf{X}) = \mathcal{H}(\mathbf{Y}_+) + \mathcal{H}(\mathbf{X}) - \mathcal{H}\left(\frac{\mathbf{Y}_+}{\mathbf{X}}\right)$

($\mathcal{H}(\mathbf{X})$ entropy of the generic vector $\mathbf{X}$).

Note that these two information criteria differ only from the functional (tr or det). Forgetting the parametric structure of A (i.e.: $A = [\mathbf{T}, (F - \mathbf{T}\,\mathbf{h}^*)\,\mathbf{T}, \ldots, (F - \mathbf{T}\,\mathbf{h}^*)^{n_2-1}\,\mathbf{T}]$) the corresponding optimization problem can be easily solved by means of elementary algebra .

Actually, the information criterion may be expressed in terms of canonical correlations, let be:

$$\mathcal{J}(\mathbf{Y}_+, \mathbf{Y}_-) = -\sum_{k=1}^{p} \log\left(1 - \sigma_k^2\right) \tag{9}$$

The $\{\sigma_k\}$ being the canonical correlations between the normalized future and past [7]. Consider furthermore, a strictly proper power spectrum density P(s), factorize it as : [W(s).W(-s)]; then it has been shown [8] that the Hankel singular values of the function $[W(s) / W(-s)]_+$ are precisely the canonical correlation coefficients of the function P(s).

There is a link between canonical correlation coefficients and the singular values of an Hankel matrix. This is also a rationale for the following method.

4 - OPTIMAL REALIZATION METHODS

We are interested in a state space modelling of the sensor outputs; for that purpose consider, in a first time, the largest model. A special model must be chosen for playing the role of initial maximum order model. The maximum entropy (AR) model is convenient for several reasons.

Consider the z-transform of the input-output processes for an innovation model, i.e.: $f_n(z) = y(z) / w(z)$

With the state space notations, this transfer function is given by:

$$f_n(z) = \mathbf{h}^*(zI - F)^{-1}\,\mathbf{T} + 1 \tag{10}$$

The z-function is rational: i.e. $f_n(z) = \frac{n(z)}{d(z)} + 1$.

The function $f_n(z)$ is analytic outside the unit circle and $f_n(z)$ has a Laurent series expansion on that domain:

$$f_n(z) = 1 + \sum_{k=1}^{+\infty} c_k z^{-k} \tag{11}$$

$$\text{with } c_k = \mathbf{h}^* F^{k-1}\,\mathbf{T}$$

The $\{c_k\}$ are named Markov parameters of the system (impulse response) and are also its Fourier coefficients. The theory of approximation leads to consider the infinite Hankel matrix built with the Markov parameters of the system, i.e.:

$$H_{f_n} = \begin{pmatrix} c_1 & c_2 & c_3 & \cdots \\ c_2 & c_3 & \cdots & \cdots \\ c_3 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

Note that the singular values of H_{f_n} are the square roots of the matrix product $G_{\mathcal{O}}.\,G_{\mathcal{C}}$ (respectively observability and reachability gramians).

The initial approximation problem consists in seeking the meromorphic function f_p (with p poles inside $\mathbb{C}$ (0,1)) which is the best approximation of the initial function f_n for the Chebychev norm. Actually, this problem reduces to the following one:

Find a rank p Hankel matrix H_{f_p} s.t.:

$$\underset{M}{\text{Min}} \, \| H_{f_n} - M \|_s \tag{12}$$

(with: M rank p Hankel matrix, $\| \; \|_s$ being the spectral norm).

The existence of such a matrix H_{f_p} has been proven by Adamjan, Arov and Krein [9], furthermore the corresponding value of the minimum is the (p+1)-th singular value of H_{f_n} :

σ_{p+1}. Denoting $H_{\infty,p}$ the set of meromorphic functions $\psi(z)$ s.t.:

$$\psi(z) = \frac{g(z)}{(z-\alpha_1)\ldots(z-\alpha_p)} \tag{13}$$

where g belongs to H_∞ (the subset of L_∞ constituted by the functions which are analytic inside $\mathfrak{D}(0,1)$) and the poles $\{\alpha_i\}_{i=1}^{p}$ are all include inside $\mathfrak{D}(0,1)$.

Then ψ can be partitioned in proper stable and instable parts, i.e.:

$$\psi(z) = [\psi(z)]_+ + [\psi(z)]_-$$

$$([\psi(z)]_+ = \sum_{i=-\infty}^{0} c_i z^{-i})$$

and $H_\psi = H_{[\psi]_-}$

Hence, a rank p Hankel matrix determines a unique rational transfer function $\varphi_p(z) = [\psi_p(z)]_-$ of degree p (= degree of its denominator) which is stable and strictly proper.

Moreover the theorem of Adamjan, Arov and Krein yields an explicit function $\psi_p(z)$. Let $f_n(z)$ be the initial function ($f_n \in L_\infty$), then:

$$\psi_p(z) = f_n(z) - \sigma_{p+1} \frac{u_{p+1}(z)}{v_{p+1}(z)} \tag{14}$$

where the function $u_{p+1}(z)$ and $v_{p+1}(z)$ are the functions corresponding to the Schmidt pair $(\mathbf{u}_{p+1}, \mathbf{v}_{p+1})$ of H_{f_n}, let:

$$H_{f_n} \mathbf{v}_{p+1} = \sigma_{p+1} \mathbf{u}_{p+1}$$

and:

$$\begin{cases} v_{p+1}(z) = \sum_{i=1}^{+\infty} v_{i,p+1} z^{i-1} \\ u_{p+1}(z) = \frac{1}{\sigma_{n+1}} [f_n(z) v_{p+1}(z)]_- \end{cases} \tag{15}$$

However, this theorem is not directly applicable because it uses the infinite Hankel matrix H_{f_n} and its singular vectors. Using linear system properties, a practical algorithm can be derived [4], [3].

More precisely denoting $\psi_p(z)$ $(= p(z)/\tilde{m}(z)$, and $\tilde{m}(z) = z^{n-1} m^*(z^{-1}))$ the best order p approximation of $f_n(z)$, then $\psi_p(z)$ is determined by the following polynomial equation:

$$p(z)\, d(z) = n(z)\, \tilde{m}(z) - \lambda\, \tilde{d}(z)\; m(z) \tag{16}$$

The above polynomial equation is the basic equation for the model reduction procedure. It can be easily translated in matricial terms, leading to an eigenvalue-eigenvector problem [4], itself solved by usual packages.

The model reduction procedure is applied to an initial maximum order model itself estimated from the sensor outputs. In our context, the most random extension of the data (i.e. an AR model for an equally spaced line array) seems to be the more convenient. The complete algorithm for narrow-band (n.b) analysis takes the following form:

1) Estimation of maximum order AR model of the sensor outputs.
2) Estimation of $\psi_p(z)$ by means of (16).
3) Compute the roots of the ψ_p denominator, the p roots inside $\mathfrak{D}(0,1)$ provide the source bearing estimates.

The corresponding algorithm is very simple and its computation cost is low. However, its performances are quite satisfying [3]. Especially the detection of weak sources is greatly enhanced w.r.t. MUSIC method [3] ; furthermore the method results are not affected by the noise correlations (which are unknown is the passive array context).

We are now trying to extend these interesting properties of the n.b analysis to w.b. analysis. This task can be achieved by several ways, but it is important to note that, thanks to the simplicity of the initial modelling, the proposed method is well suited to w.b analysis.

5 - WIDEBAND ANALYSIS

We are now concerned with the extension of the previous method to w.b. analysis.

An AR modelling of the sensor outputs is very convenient. This AR model is temporal and multidimensional:

$$\mathbf{X}_t = \sum_{i=1}^{q} A_i \mathbf{X}_{t-i} + \mathbf{N}_t \tag{17}$$

(A_i are the matricial coefficients of the model, $\mathbf{N}_t$: spatially white noise).

The order q of the AR model is usually chosen great; the matricial coefficients A_i are estimated by means of the Levinson-Wiggins algorithm for instance. Then, the basic procedure for w.b analysis takes the following form:

1) Estimate a temporal AR modelling of the sensor outputs.
2) Deduce from it, M cross-spectral matrices associated with sampled temporal frequencies.
3) Interpolate (by any way) the cross-spectral matrices (C.S.M.).
4) Focus the initial estimates at the lowest frequency.
5) Apply the optimal reduction method, eq. (14).

Note that there is no assumption about source locations or source models conversely to usual methods.

Let us now detail the basic steps of the above procedure: The CSM are deduced from the A_i, let:

$$R_{f_i} = \left(\sum_{j=0}^{q} A_j z^j\right)^{-1} \Sigma_q \left(\sum_{j=0}^{q} A_j z^j\right)^{-*} \tag{18}$$

(Σ_q: prediction error matrix, $A_0 \equiv 1$, $z = \exp(-2\, j\, \pi\, f_i)$). The estimated CSM at the f_i frequency.

It can be easily shown that there is a 1:1 mapping between a set $\{R_{f_i}\}_{i=1}^{M}$ and the AR coefficients A_i ($M = (n_s - 1)\, q/2$).

Now a spatial AR model (at the frequency f_i) is deduced from R_{f_i}.

Let f_1 be the reference frequency then the AR spatial model at the frequency f_i is interpolated in order to produce an estimate of the spatial correlation at f_1, i.e.:

$$\hat{r}_{fi}(f_1, q\, d) \equiv \hat{r}\left(f_i, \frac{f_1}{f_i} q\, d\right) \tag{19}$$

(d: intersensor distance, q integer $0 \le q \le n_s - 1$).

The interpolated spatial covariances are themselves obtained by Fourier transforms, i.e.:

$$\hat{r}\left(f_i, \frac{f_1}{f_i} q\, d\right) \equiv \int_{-W/2}^{W/2} P\,(f_i, k) \exp\left(-2\, j\, \pi\, k \frac{f_1}{f_i}\right) d\, k \quad (20)$$

(w: spatial bandwidth, P (f_i, k): spatial density, k: wavenumber).

In the above formula the spatial density P (f_i, k) is directly obtained from the spatial AR model.

Obviously the above formula can be replaced by direct interpolation.

Therefore, a set of M spatial covariance lags $\{\hat{r}\,(f_1, q\, d)\}$ can be estimated by the above procedure. Then, a unique order n AR (spatial) model can be derived from all of them in a least square sense. Then, the model reduction procedure provides a multifrequency analysis.

A rationale for this procedure can be derived as follows. After interpolation, the poles associated with sources are identical whatever the considered frequency. Then, the spatial density at the frequency f_i takes the form:

$$P\,(f_i, z) = \sum_{k=1}^{s} \frac{a_{k,\, i}}{z - z_k} + \frac{\bar{a}_{k,\, i}}{z^{-1} - \bar{z}_k} + n_i \quad (21)$$

(z_k: exact source bearings, $a_{k,i}$: source p.s.d, n_i: noise at frequency f_i).

The above formula is assumed to be valid for the interpolated process, then we consider the multifrequency functional:

$$\widetilde{P}\,(z) = \sum_{f_i} P(f_i, z) \quad (22)$$

itself associated to the least square procedure. Then, $\widetilde{P}$ (Z) can also be written as:

$$\widetilde{P}\,(z) = \sum_{k=1}^{s} \left(\frac{\sum_{f_i} a_{k,\, i}}{z - z_k}\right) + \left(\frac{\sum_{f_i} a_{k,\, i}}{z - z_k}\right)^* + \sum n_i \quad (23)$$

The interpolation procedure results generally in a highly (spatially) correlated noise. In order to remedy this problem, it is always possible to whiten it. This constitutes a delicate step for usual multifrequency methods (in particular if the noise model is unknown); but this step is, thanks to the fundamental robustness w.r.t noise correlations, useless for our method.

SIMULATION RESULTS

The output data of a linear array composed with 20 equispaced sensors are simulated to show the performances of the w.b method. The noise is assumed temporally and spatially white, and the sources have flat spectra in the band of analysis [0.22 ; 0.44] in normalized frequency unit. The intersensor distance is $d = c / (2\, f_e)$, where c is the velocity of wave and f_e the sampling frequency; the broadside corresponds to 90 deg. The CSMs R_{f_i}, and an order 19 temporal AR model of the sensor outputs are estimated with the same integration time: $T = 30 / f_e$. The following simulations underline the resolution and detection improvements of the w.b method on n.b ones:

Resolution of two close sources: two sources with same SNR per frequency bin (0 dB) are simulated at bearings 20 and 23 deg. The narrow-band methods (n.b ORM with BT=30 and Root-MUSIC method with BT=30, BT=210) are performed with the CSM at the highest frequency bin of the band: 0.44 ; and the wide-band ORM is performed in the band [0.38 ; 0.44] with 7 frequency bins (and the same integration time), thus BT=30 x 7 = 210. The results are summarized in the following table for 10 independent trials (the assumed source number=2):

n.b ORM BT=30	Root-MUSIC BT=30	Root-MUSIC BT=210	W.B ORM BT=30x7=210
5 / 10	2 / 10	7 / 10	7 / 10

Detection of a weak source: 3 sources are simulated with respective bearing and SNR per frequency bin (70 deg, −10 dB), (78 deg, −3 dB) and (87 deg, −20 dB). With the same processing parameters as in the preceding simulation the different results can be compared in the table below (10 trials, the assumed source number is 3):

n.b ORM BT=30	Root-MUSIC BT=30	Root-MUSIC BT=210	W.B ORM BT=30x7=210
2 / 10	0 / 10	6 / 10	7 / 10

6 - CONCLUSION

A new method for multifrequency analysis has been derived. It requires no assumption about source locations, source models, noise correlations. The complete procedure is very simple and low cost.

The simulation results are quite satisfying and it is important to note the following facts: improvements w.r.t. narrow band analysis can be considerable (angular resolution, weak source detection...), the method is quite robust (no *a priori* hypothesis is used).

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