Approximated stochastic realization and model reduction methods applied to array processing by means of state space models

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ABSTRACT

The aim of this paper is to present new methods for passive array processing. The basic idea consists in using a state-space modelling of the sensors output. This paper copes with basic problems as: unknown noise correlations, approximation by a Tœplitz matrix of lower rank, detection of small sources. The presented methods represent considerable improvements with respect to the usual ones and furthermore are quite feasible, some statistical results illustrate these claims.

1. Introduction.

A modelling of the sensors output of an equally spaced linear array by a linear system is shown to be a powerful tool, in particular to take into account fundamental hypotheses about sources and noise. A general frame for approximated stochastic realization methods is presented and connections with existing work and array processing are especially considered.

A state space modelling of the sensors output uses sources and noise properties in a summarizing way and is a powerful means to separate the noise and sources contributions even in the case of an unknown correlated noise. This last property represents a considerable improvement against usual high resolution methods. For that purpose two methods of approximated stochastic realization are considered, the links between these ones and array processing are detailled.

However a fundamental problem remains: find an estimation of the sources subspace that preserves the plane wave structure. We are now facing the basic problem which consists in extracting a low rank matrix (of sources) from an estimated (full rank) one, preserving the special structure (Tœplitz or Hankel) induced by plane wave and spatial stationnarity hypotheses. The Adamjan-Arov-Krein theory is the theoretical frame, the initial problem of functionnal analysis is translated into another simpler and finite dimensional problem by the use of the properties of finite structures and balanced realizations. A further advantage consists in using a L_∞-norm rather than an L_2-norm , much more sensitive to local variations of the spectra.

Furthermore, simulation results present comparisons between state space approaches and usual high resolution methods (as eigenvectors methods) and try to precise their advantages.

2. Additive white noise.

This case is especially simple but allows us to precise our definitions. In the whole paper, the sensors output of a linear equispaced array constituted by n sensors are considered at a given frequency (omitted for the sequel). Under plane wave assumption, the output at the i-th sensor may be written as:

$$y_i = \sum_{k=1}^{m} \alpha_k . \exp(-j.\varphi_k) + v_i \quad (1) \quad (j^2 = -1)$$

where m is the number of sources, α_k and φ_k represent respectively the random level and the deterministic phase of the k-th source, and $\{v_i\}$ is the additive noise.

A rank m linear state space model of (1) is given in the case of an additive white noise by [1]

$$\begin{cases} \mathbf{X}_{i+1} = \mathbf{F} . \mathbf{X}_i & \mathbf{F} : (mxm) \\ y_i = \mathbf{h}^* . \mathbf{X}_i + v_i & (2) \quad \mathbf{h}^* : (1xm) \end{cases}$$

where F, h are deterministic parameters, $\mathbf{X}_i$ is the state vector at sensor i and $\{v_i\}$ is an additive white noise. The matrix F translates the phasing properties of the propagation while h represents its power effects. The phases φ_k are determined from the eigenvalues of the transition matrix F which is similar to diag[exp(-j. φ_k)].

Let Γ_p be the covariances of the sensors output, i.e. : $\Gamma_p = E(y_{i+p} y_i^*)$ then

$$\Gamma_p = \mathbf{h}^* . F^p . P . \mathbf{h} + b^2 . \delta(p) \quad (3)$$

with $P = E(X_i . X_i^*)$, $b^2 = E(|v_i|^2)$.

Furthermore, let Γ_+ be the exact covariance matrix of the sensors output, then

$$\Gamma_+ = \mathbf{O} . P . \mathbf{O}^* + b^2 . I_n \quad (4)$$

where $\mathbf{O}$ is the (nxn) observability matrix

$$\mathbf{O} = \begin{bmatrix} \mathbf{h}^* \\ \mathbf{h}^* . F \\ \vdots \\ \mathbf{h}^* . F^{n-1} \end{bmatrix}$$

The following property can be easily deduced from the structure of the observability matrix:

$$\mathbf{O}^{\uparrow} . F = \mathbf{O}^{\downarrow} \quad (5)$$

($\uparrow$: select the (n-1) last rows, $\downarrow$: select the (n-1) first rows, $\mathbf{O}^{\uparrow}$ and $\mathbf{O}^{\downarrow}$ are ((n-1)xm) matrices), and therefore;

$$F = (\mathbf{O}^{\uparrow})^{\#} . \mathbf{O}^{\downarrow} \quad (6)$$

(#: denoting pseudo-inverse matrix)

Elsewhere, the covariance matrix can also be written in terms of array processing as:

$$\Gamma_+ = \mathbf{D} . \gamma . \mathbf{D}^* + b^2 . I_n \quad (7)$$

with $\mathbf{D}$: source steering matrix

then the plane wave hypothesis yields:

$$\mathbf{D}^{\uparrow} . F = \mathbf{D}^{\downarrow} \text{ or } F = (\mathbf{D}^{\uparrow})^{\#} . \mathbf{D}^{\downarrow} \quad (8)$$

leading to property 1 [2], and a geometric meaning of the observability matrix.

Prop.1: in the asymptotic case, the eigenvalues of the transition matrix F are identical to these of the matrix $[(U_1^{\uparrow})^{\#} . U_1^{\downarrow}]$ where U_1 is the (nxm) matrix

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formed from the eigenvectors corresponding to the m largest eigenvalues of Γ_+.

In practical situation however, the matrix U_1 is replaced by $\widehat{U_1}$ an estimate (from $\widehat{\Gamma_+}$), therefore the presented method "forgets" the special structure of the observability matrix.

3. Approximated realization methods, additive correlated noise.

In the general case, it is possible to take into account the noise correlations by modelling the sensors output as the outputs of a minimum phase innovation model, i.e. :

$$\begin{cases} X_{i+1} = F \cdot X_i + T \cdot v_i \\ y_i = h^* \cdot X_i + v_i \end{cases} \qquad (9)$$

Then the approximated realization methods rely upon consideration of two vectors Y_+ and Y_- (future and past at sensor n/2) defined as follows:

$$\begin{cases} Y_- \equiv (y_{n_2}, y_{n_2-1}, \cdots, y_1) \qquad (n_2 \equiv n/2) \\ Y_+ \equiv (y_{n_2+1}, y_{n_2+2}, \cdots, y_n) \end{cases} \qquad (10)$$

The aim of the methods consists in summarizing the past in order to obtain the more efficient (for a given criterion) prediction of the future. The stochastic approach [3] determines a state vector X (of a given dimension) which sums up the more pertinent part of Y_- in order to predict Y_+, i.e.:

$$X = A \cdot Y_- \quad , \text{ with A (pxn)} \qquad (11)$$

Theoretically one has:

$$A = [\, T, (F - T \cdot h^*) \cdot T, \ldots, (F - T \cdot h^*)^{n2-1} \cdot T] \qquad (12)$$

Furthermore, in the case of a markovian system, the orthogonal projection of the future onto the past is given by:

$$Y_+ \,|\, Y_- = O \cdot X \qquad (O: \text{ observability matrix}) \quad (13)$$

The determination of A allows us to estimate O and therefore the system parameters. The crucial step of these methods consists in the estimation of the matrix A, for that purpose various methods can be considered.

3.1. The predictive efficiency criterion (AK method)

The matrix A is the matrix which minimizes the variance of the predictor error, i.e. [4] :

$$\text{Min}_{X = A \cdot Y_-} \{\text{tr}[\text{cov}(Y_+ - Y_+ \,|\, X)]\}$$

yielding to:

$$\text{Min}_A \{\text{tr}[\Gamma_+ - H \cdot A^* \cdot (A \cdot \Gamma_- \cdot A^*) \cdot A \cdot H^*]\} \qquad (14)$$

$(\Gamma_+ \equiv E(Y_+ \cdot Y_+^*),\ \Gamma_- \equiv E(Y_- \cdot Y_-^*),\ H \equiv E(Y_+ \cdot Y_-^*))$

forgetting special structure of A, the solution of (14) is obtained by elementary algebra, i.e.:

Prop. 2: the solution of the problem (14) has the general form: $A = B \cdot \Sigma_1^{-1} \cdot U_1^* \cdot H \cdot \Gamma_-^{-1}$ (15)

where B is any (pxp) invertible matrix and (Σ_1^2, U_1) are the p principal components of the matrix $H.\Gamma_-^{-1}.H^*$.

3.2. Information criterion method (DP method)

This approach is based upon the fact that the whole information contained in Y_+ which is explained by Y_- can be expressed in terms of few parameters $\{\sigma_k\}_{k=1,p}$, called the canonical correlation coefficients:

$$\mathfrak{I}(Y_+, Y_-) = -\sum_{k=1}^{p} \log(1 - \sigma_k^2)$$

Denoting $\mathfrak{I}(Y_+, X)$ the mutual information between Y_+ and X defined as follows:

$$\mathfrak{I}(Y_+, X) = \mathfrak{H}(Y_+) + \mathfrak{H}(X) - \mathfrak{H}\left[\frac{Y_+}{X}\right] \qquad (16)$$

where $\mathfrak{H}(X)$ is the entropy of the gaussian vector X, then the estimation of $X = A \cdot Y_-$ amouts to minimize the following criterion [5]:

$$\text{Min}_A \{\det[\Gamma_+ - H \cdot A^* \cdot (A \cdot \Gamma_- \cdot A^*) \cdot A \cdot H^*]\} \qquad (17)$$

Note that both criteria differ only by the functional (tr or det) , the minimization of this criterion is achieved by means of elementary algebra leading to:

Prop. 3: the solution of the problem (17) is given by: $A = B \cdot V_1^* \cdot \Gamma_-^{-1/2}$ (18)

where V_1 is the matrix formed from the p right principal singular vectors of the matrix $\{\Gamma_+^{-1/2} \cdot H \cdot (\Gamma_-^{-1/2})^*\}$.

Therefore the solution of (17) is nothing but the solution obtained by maximization of canonical correlations as advocated by Desai and Pal [5].

The matrix A being estimated by means of Prop.2 or 3, F is staightforwardly deduced by two ways since:

- $O \cdot A = H \cdot \Gamma_-^{-1}$, and $O^{\uparrow} \cdot F = O^{\downarrow}$.
- $X_{i+1} \,|\, X_i = F \cdot X_i$.

The links with classical methods of array processing are much more subtle than in §2. In the white noise case, one has [6]: $H \cdot \Gamma_-^{-1} \cdot H^* = \mathfrak{D} \cdot \Delta \cdot \mathfrak{D}^*$ therefore the matrix $H \cdot \Gamma_-^{-1} \cdot H^*$ plays the role of Γ_+ in Prop. 1; but this is not very relevant of the general case. For that purpose, it is more appropriate to consider the predictive efficiency criterion in terms of the intersection of two linear subspaces, i.e. [6] :

Prop. 4: the rows of the matrix A approximate a basis of $\text{Range}(H^* \cdot H) \cap \text{Range}(\Gamma_-)$.

Prop. 4 means that the rows of A minimize the principal angles between $\text{Range}(\Gamma_-)$ and $\text{Range}(H^*.H)$.

Under the assumption of a shortly correlated additive noise, the matrix H is perturbed only by a triangular matrix, the vectors of $\text{Range}(H^*.H) \cap \text{Range}(\Gamma_-)$ are mainly related to source parameters.

To sum up, the presented methods are efficient to estimate the source bearings in presence of an additive noise with unknown correlations, however they do not use fundamentally the plane wave hypothesis for the estimation of A. We shall now cope with this problem.

4. Optimal realization methods.

4.1. A general framework

The approximation of a linear system by a lower order one represents an important part of control systems litterature. However this field seems to be not very relevant for array processing, but the following general framework permits us to use the results of control systems theory.

The array outputs are modelled by a maximal order parametric model (e.g.: an AR model of order n), note that there is no information loss since it is possible to construct the corresponding covariances (Γ) from them. It is simply another representation of the covariances. A state space model can be easily obtained from the model and it is then possible to use the optimal model reduction methods [7].

We want an exact approximation, preserving the structure and avoiding the classical least squares approximations (leading generally to principal component analysis). This is basically a problem of approximation. In fact, the theorem of Adamjan, Arov and Krein (AAK) is the foundation of such an

approximation, although it is mainly theoretical and in particular involves infinite matrices. Fortunately it is possible to translate this problem into another finite dimensional problem (by use of finite structure) and to present a practical algorithm.

4.2. Problem formulation

Given a scalar transfert function (e.g. : $f(z) = h^* . (z . I - F)^{-1} . T + 1$) we seek an approximation (for a certain norm) of this function by a lower order system. The use of L_2-norm is classical but is not as powerful as L_∞-norm for our problem where sources may have very different powers. The inclusion of special structure leads to the following problem:

Pb 1: given $f \in L_\infty$ (defined on $\mathbb{C}(0,1)$) and a positive integer k, find

$$\inf\{ \| f - \varphi \|_\infty : \varphi \in H_{\infty,k} \} \qquad (19)$$

and a function φ_k (of $H_{\infty,k}$) for which the infinimum is attained; $H_{\infty,k}$ being the set of meromorphic functions in L_∞ which can be written as:

$$\varphi(z) = \frac{g(z)}{(z-\alpha_1) \dots (z-\alpha_k)} \qquad (20)$$

where : $(\alpha_1, \dots, \alpha_k) \in \mathbb{D}(0,1)$ and $g \in H_\infty$; H_∞ being the subspace of L_∞ for which $c(n) = 0$ for $n < 0$ ($c(n)$ being the n-th Fourier coefficient of f w.r. to the complete basis (z^n), $z = \exp(-j . \theta)$).

This problem has been solved by AAK, its solution consists in using the Hankel matrix of a function f w.r. to the basis $z^{-1}, z^{-2}, \dots$ in $L_2 \ominus H_2$ defined as:

$$H_f = \begin{bmatrix} c(-1) & c(-2) & c(-3) & \dots \\ c(-2) & c(-3) & c(-4) & \dots \\ c(-3) & c(-4) & c(-5) & \dots \\ \dots & \dots & \dots & \end{bmatrix} \qquad (21)$$

then a first result is available:

Res. 1: $\| H_f \|_\infty = \mathrm{dist}_{L_\infty}(f, H_\infty)$

the function of H_∞ which minimizes that distance being given by Nehari's theorem.

Furthermore, the Kronecker's theorem allows us to precise the problem:

Kronecker's theorem : let $f \in L_\infty$, then H_f has finite rank $\leq k$ if and only if $f \in H_{\infty,k}$.

The theorem of AAK gives an explicit solution to Pb. 1.

Theorem (AAK): let $f \in L_\infty$ and $k \geq 0$. Then $\mathrm{dist}_{L_\infty}(f, H_{\infty,k}) = \sigma_{k+1}(H_f)$, where the σ_i are the singular values of H_f ordered in decreasing value; furthermore this distance is attained at a unique function $\varphi_k \in H_{\infty,k}$ and if v_{k+1} is the singular vector of H_f corresponding to $\sigma_{k+1}(H_f)$ then:

$$\varphi_k(z) = f(z) - [H_f . v_{k+1}](z) / v_{k+1}(z)$$

where $H_f . v_{k+1} \equiv \Pi_-(v_{k+1} . f)$ (22)

Defining the projection operators Π_+ and Π_- by

$$\Pi_-\Big(\sum_{i=-\infty}^{+\infty} \alpha_i . z^{-i}\Big) = \sum_{i=1}^{+\infty} \alpha_i . z^{-i} \quad , \quad \Pi_+ = I - \Pi_- \quad , \text{one obtains}$$

$$\varphi_k(z) = f(z) - \sigma_{k+1} . \frac{\mu_{k+1}(z)}{v_{k+1}(z)}$$

with:

$$\mu_{k+1}(z) = \sum_{j=1}^{+\infty} u_j^{k+1} . z^j$$

$$v_{k+1}(z) = \sum_{j=1}^{+\infty} v_j^{k+1} . z^{1-j} \qquad (23)$$

these results are interesting but not directly useful because they involve mainly the infinite Hankel matrix H_f and its singular vectors. The implication of finite structure allows us to solve explicitely the initial Pb. 1.

4.3. Implication of finite structure

Firstly, recall the classical equality: $H = O . C$, therefore the square of the i-th singular value σ_i of the (infinite) matrix H is

$$\sigma_i(H^* . H) = \sigma_i(C^* . O^* . O . C) = \sigma_i(C . C^* . O^* . O)$$
$$= \sigma_i(P . Q) \qquad (24)$$

where P and Q are respectively the controllability and observability gramians. The matrices P and Q depend strongly on state-space coordinates but not the eigenvalues of the product P . Q , furthermore it has the great advantage to be finite-dimensional.

Let the strictly proper part of the transfert function be: $f(z) = h^* . (z . I - F)^{-1} . T = \frac{n(z)}{d(z)}$ (25)

where $d(z) = \det(z . I - F)$. By use of balanced system transformation, it is possible to obtain simpler expressions for thez functions $\mu_i(z)$ and $v_i(z)$.

More precisely, let $H = H(f)$ and $\mathfrak{B}$ a balancing transform: $(F, h^*, T) \xrightarrow{\mathfrak{B}} (F_b, h_b^*, T_b)$ (balanced triple).Hence,

if $H = U . \Sigma . V$, then $O_b = U . \Sigma^{-1/2}$ and $C_b = \Sigma^{1/2} . V$, therefore :

$$\mu_i(z) = \sigma_i^{-1/2} . h_b^* . (I_n - z . F_b)^{-1} . e_i \qquad (26)$$

where e_i denotes the i-th column of the (nxn) identity matrix.

Similarly, one obtains:

$$v_i(z) = \sigma_i^{-1/2} . e_i^* . (z . I_n - F_b^t)^{-1} . T_b \qquad (27)$$

This last equality can be usefully transformed by means of the following property:

Prop. 5: for a balanced SISO system, the following equalities are satisfied: $F_b^t = Q . F_b . Q^*$; $T = Q^* . h$, Q being a unitary diagonal matrix.

From (26) we see that $\mu_i(z)$ has a rational form, i.e.: $\mu_i(z) = \frac{m(z)}{d(z)}$

then (27) and Prop. 5 yield the following fundamental property:

$$v_i(z) = q_i . z^{-1} . \mu_i(z^{-1}) = q_i . \frac{\tilde{m}(z)}{\tilde{d}(z)} \qquad (\text{with } |q_i|^2 = 1)$$

where q_i is the i-th element of the diagonal Q and $\tilde{m}(z)$, $(\tilde{d}(z))$ the reversed polynomials of m(z) (resp. d(z)) , i.e. $\tilde{m}(z) = z^{n-1} . m(z^{-1})$

Using AAK theorem the best L_∞ approximation of order k is given by:

$$\varphi_k(z) = \frac{n(z)}{d(z)} - q_{k+1} . \sigma_{k+1} . \frac{m(z) . \tilde{d}(z)}{\tilde{m}(z) . d(z)}$$

$$= \frac{q(z)}{d(z) . \tilde{m}(z)} \qquad (28)$$

Furthermore it can be easily proved that the rational fraction $q(z)/d(z)$ is actually a polynomial $p(z)$ of degree $< n$, as a consequence the following equality is valid:

$$\varphi_k(z) = \frac{p(z)}{\tilde{m}(z)}$$

with : $$p(z) = \frac{n(z) . \tilde{m}(z) - \lambda . \tilde{d}(z) . m(z)}{d(z)}$$

$$(\lambda = q_{k+1} . \sigma_{k+1})$$

This last form of $\varphi_k(z)$ leads to determine the coefficients of m(z) by solving the following

polynomial equation:

$$p(z) \cdot d(z) = n(z) \cdot \tilde{m}(z) - \lambda \cdot \tilde{d}(z) \cdot m(z) \qquad (29)$$

equating the coefficients of powers of z [7], one obtains the two following matricial equations (30) and (31):

$$J \cdot S_2 \cdot J \cdot p = R_2 \cdot \tilde{m} - \lambda \cdot J \cdot \tilde{S}_1 \cdot J \cdot m \quad (z^i: i = 0, \dots, n-1)$$

$$S_1 \cdot p = R_1 \cdot \tilde{m} - \lambda \cdot \tilde{S}_2 \cdot m \quad (z^i: i = n, \dots, 2n-1)$$

where :

$$n(z) = c_n + c_{n-1} \cdot z + \dots + c_1 \cdot z^{n-1}$$

$$d(z) = a_n + a_{n-1} \cdot z + \dots + a_1 \cdot z^{n-1} + z^n$$

$$m(z) = m_{n-1} + m_{n-2} \cdot z + \dots + m_0 \cdot z^{n-1}$$

$$p(z) = p_{n-1} + p_{n-2} \cdot z + \dots + p_0 \cdot z^{n-1}$$

and:

$$S_1 \equiv \begin{bmatrix} a_{n-1} & a_{n-2} & \dots & 1 \\ a_{n-2} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & 0 \end{bmatrix} \qquad S_2 \equiv \begin{bmatrix} a_1 & a_2 & \dots & a_n \\ a_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_n & \cdot & \cdot & 0 \end{bmatrix}$$

$$R_1 \equiv \begin{bmatrix} 0 & c_1 & \dots & c_{n-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & c_1 & \cdot \\ 0 & \cdot & \cdot & 0 \end{bmatrix} \qquad R_2 \equiv \begin{bmatrix} c_n & \dots & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ c_2 & \cdot & \cdot \\ c_1 & c_2 & \dots & c_n \end{bmatrix}$$

and J the antidiagonal matrix (32)

from (30) and (31) one obtains (33):

$$(J.S_2.J.S_1^{-1}.R_1 - R_2).\tilde{m} = \lambda .(J.S_2.J.S_1^{-1}.\tilde{S}_2 - J.\tilde{S}_1.J).m$$

$$m^t = (m_0, m_1, \dots, m_{n-1})$$

which appears as a generalized eigenvalue-eigenvector equation. In order to solve (33), numerous subroutines are available. Once the coefficients of m(z) have been calculated, those of p(z) are straightforwardly deduced.

$$J \cdot S_2 \cdot J \cdot p = R_2 \cdot \tilde{m} - \lambda \cdot J \cdot \tilde{S}_1 \cdot J \cdot m \quad (z^i: i = 0, \dots, n-1)$$

$$S_1 \cdot p = R_1 \cdot \tilde{m} - \lambda \cdot \tilde{S}_2 \cdot m \quad (z^i: i = n, \dots, 2n-1)$$

4.4. Practical implementation:

Practical utilization of the previous method needs a prior state-space model which will be reduced. The more "random" model (or maximum entropy) seems to be the most convenient, it summarizes all the available informations (i.e. : the estimated covariance of the outputs) and gives the more "random" extension of them. For a linear equispaced array of sensors it is similar with an AR modelling. The practical algorithm takes the following form after DFT of sensors:

1) estimation of the covariances of the sensors output (n+1 sensors); i.e. : $\hat{r}(0), \hat{r}(1), \dots, \hat{r}(n)$.

2) estimation of the corresponding AR model coefficients: $(1, \hat{a}_1, \dots, \hat{a}_n ; b^2)$.

3) solve the generalized eigenvalue-eigenvector problem (33) with:

$a_1 = \hat{a}_1, \dots, a_{n-1} = \hat{a}_{n-1}, a_n = \hat{a}_n$

$c_1 = 0, \dots, c_{n-1} = 0, c_n = 1$

select the (k+1)-th generalized eigenvector (to obtain a rank k approximation) and calculate $m_{k+1}(z)$.

4) compute the roots of $m_{k+1}(z)$ and their arguments (note that by definition of $H_{\infty,k}$ there are k roots in $\mathcal{D}(0,1)$).

5. Simulation results.

Due to the lack of space, we present briefly simulation results, details are available in [6].

5.1. Approximated realization methods (ARM):

The case of an additive white noise is firstly considered. Three uncorrelated sources impinge a linear array constituted by 16 equispaced sensors (at $\lambda/2$). The covariance matrix of the outputs is estimated by averaged periodogram (BT = 100). The source bearings are 56 , 60 , 65 degrees with respective levels -1, -11, -1 dB. The assumed sources number is 4, the results are summarized below:

Detection of a weak source 100 trials	MUSIC	ARM
	15/100	40/100

5.3. Optimal realization method (ORM):

Simulation 1: the additive noise is spatially white, 3 sources are simulated with respective powers and bearings (30 deg , 0 dB), (36 deg , -10 dB), (45 deg , 7 dB); the number of averaged snapshots (BT) is equal to 300. Afterwhat the poles of Music method and ORM are computed, the results are presented below (for ten trials)

Detection of the weak source	MUSIC	ORM
	0/10	8/10

Note that for MUSIC method the pole associated to the weak source (-10 dB) can not be seen generally or at best is very small ($\cong 0.5$), conversely the pole associated to that source is very close to the strongest ones for ORM (typically 0.98 !).

Simulation 2: in order to judge ability of ORM to separate 2 close sources, two sources of same powers (0 dB) have been simulated at bearings 10 and 13 deg (BT = 300), with spatially white noise; results are summarized below:

Separation of the two sources	MUSIC	ORM
	4/10	9/10

Simulation 3: we consider now the simulation of §5.2, the additive noise is strongly correlated. For ten trials, the sources are correctly seen at their exact bearings for ORM.

6. Conclusion.

The simulation results have demonstrate the effectiveness of a state-space modelling of the sensors output. Robustness versus noise correlations is greatly enhanced by use of ARM (w.r. "usual " high resolution method), it is their main advantage. Detection of weak source can be slightly enhanced also but estimation of sources subspace remains based upon eigenvector decomposition.

Conversely, ORM takes advantage of the estimation of sources subspace by use of their special structure and present considerable improvements for angular separation of close sources, detection of weak sources. Furthermore, the method is very robust w.r. noise correlations, its computation cost is similar to MUSIC method.

Obviously the presented tools are powerful for a linear array; however they can not be directly (if it is possible) extended to a general shape.

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