Reachability Analysis of Rewriting for Software Verification

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Habilitation à diriger des recherches

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Motivation : proving safety properties

```
①
    n := i ;
②
    while (i>1) do {
③
          n := n*(i-1) ;
④
          i := i-1 ; }
⑤
```

If $i \geq 1$ in ①

then

$n \geq 1$ in ⑤

or

If $i \geq 1$ in ①

then

⑤ with $n = 0$ unreachable

Verification using Model-checking

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①   {i ≥ 1}
    n := i ;
②
    while (i>1) do {
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          n := n*(i-1) ;
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          i := i-1 ; }
⑤   {n ≥ 1}
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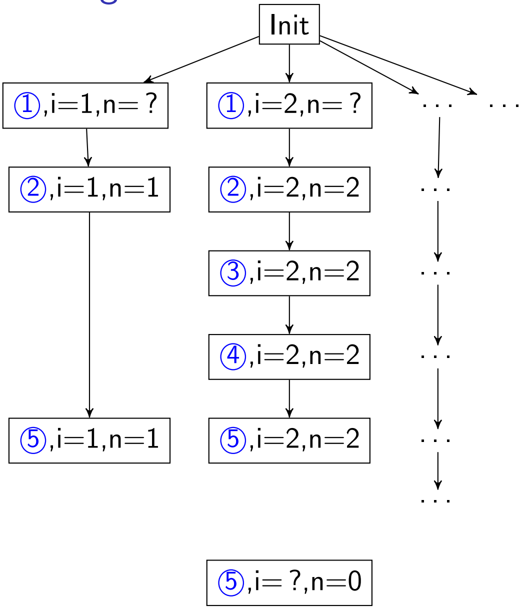

Verification using Static Analysis and Abstract Interpretation

$D = \mathbb{N}$	$D^\#$: intervals on $\mathbb{N}$
① $\{i \geq 1\}$	① $i^\# = [1; +\infty[$, $n^\# = [0; +\infty[$

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```

```
FORALL (i: int):
  i >=1 IMPLIES
    (FORALL (x: int):
      x = i IMPLIES
        (FORALL (i0: int):
           FORALL (x0: int):
             x0 >= 1 IMPLIES
               i0 > 1 IMPLIES
                 (FORALL (x1: int):
                   x1 = x0 * (i0 - 1)
                   IMPLIES x1 >= 1))))
```

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```

```
(skosimp*)
(replace -6 1)
(lemma "both_sides_times_pos_ge1")
(inst -1 "i0!1-1" "x0!1" "1")
(grind)
```

Proving (un)reachability on infinite state systems

- Static analyzers based on abstract interpretation
- Model-checkers adapted to infinite state systems
  - Regular model-checking
  - Abstract model-checking, . . .

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Is there something in between ?

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Our proposition for (un)reachability analysis

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   and bonus :

4. In the end, the same level of confidence as with a Coq proof !

Outline

1. Term rewriting and reachability analysis
2. Regular model-checking of term rewriting systems
3. Defining abstractions for infinite non regular systems
4. Refining abstractions by hand using equations
5. Tools and applications
6. Conclusion and further work

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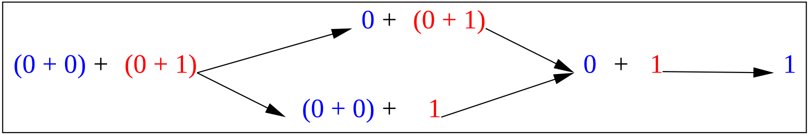
- Set of terms $\mathcal{T}(\mathcal{F}, \mathcal{X}) = \{x,\ 0 + x,\ 1 + 0, \ldots\}$
- Rewrite rules $0 + x \rightarrow x$



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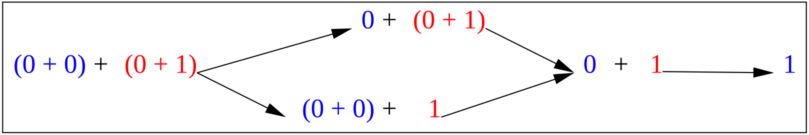

$$\begin{tikzcd}
 & 0 + (0+1) \arrow[dr] & & \\
(0+0) + (0+1) \arrow[ur] \arrow[dr] & & 0 + 1 \arrow[r] & 1 \\
 & (0+0) + 1 \arrow[ur] & &
\end{tikzcd}$$

- Term rewriting system (TRS) = set of rewrite rules

With TRS $\mathcal{R} = \{0 + x \to x\}$:

$$0 + 1 \to_{\mathcal{R}} 1$$

$$(0+0) + (0+1) \to_{\mathcal{R}}^{*} 1$$

TRS as a formal model of programs

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$$\mathcal{F} = \{(\_,\_,\_), 0, s, +, *, \textcircled{1}, \textcircled{2}, \textcircled{3}, \textcircled{4}, \textcircled{5}\}$$
$$\mathcal{X} = \{I, N, X, Y\}$$

$$\begin{array}{ll}
(\textcircled{1}, I, N) & \to (\textcircled{2}, I, I) \\
(\textcircled{2}, s(s(I)), N) & \to (\textcircled{3}, s(s(I)), N) \\
(\textcircled{3}, s(I), N) & \to (\textcircled{4}, s(I), I * N) \\
(\textcircled{4}, s(I), N) & \to (\textcircled{2}, I, N) \\
(\textcircled{2}, 0, N) & \to (\textcircled{5}, 0, N) \\
(\textcircled{2}, s(0), N) & \to (\textcircled{5}, s(0), N) \\
 & \\
0 * X & \to 0 \\
s(X) * Y & \to Y + (X * Y) \\
\ldots &
\end{array}$$

Proving safety by (un)reachability analysis :

$$(\textcircled{1}, i, x) \not\to_{\mathcal{R}}^{*} (\textcircled{5}, y, 0) \qquad \text{with } i \geq 1, x, y \in \mathbb{N}$$

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Reachability analysis of rewriting

Given a TRS $\mathcal{R}$ and $s, t \in \mathcal{T}(\mathcal{F})$, is $s \rightarrow_{\mathcal{R}}^{*} t$?

- Undecidable in general (TRS are Turing-complete)

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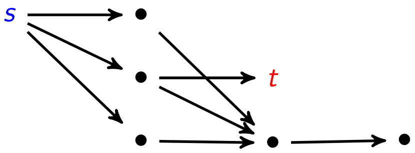

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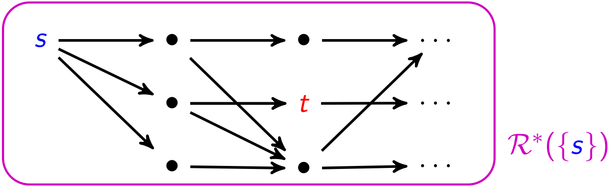

where $\mathcal{R}^*(\mathcal{L}) = \{u \mid s \in \mathcal{L} \wedge s \rightarrow_{\mathcal{R}}^{*} u\}$

- Decidable, if $\mathcal{R}^*(\{s\})$ is finite (≈ finite model-checking)

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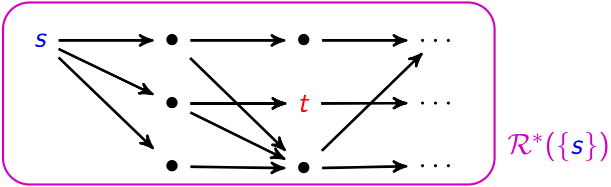

where $\mathcal{R}^*(\mathcal{L}) = \{u \mid s \in \mathcal{L} \wedge s \rightarrow_{\mathcal{R}}{}^* u\}$

- Decidable, if $\mathcal{R}^*(\{s\})$ is finite (≈ finite model-checking)
- Decidable, for classes of $\mathcal{R}$ such that $\mathcal{R}^*(\{s\})$ is regular (≈ regular model-checking)

Reachability analysis of rewriting (extended)

Recall that for verification, the problem we have is :

$$(①, i, x) \not\to_{\mathcal{R}}^{*} (⑤, y, 0) \qquad \text{with } i \geq 1, x, y \in \mathbb{N}$$

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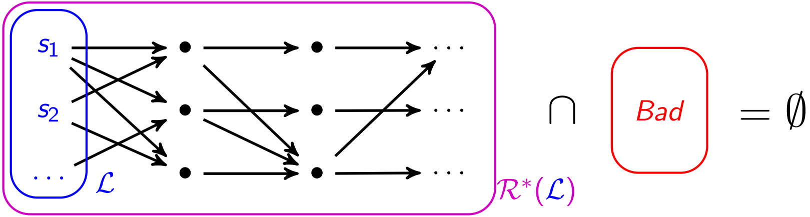
which can be seen as :



The reachability analysis problem becomes :

$$\boxed{\mathcal{R}^*(\mathcal{L}) \cap Bad = \emptyset\,?}$$

Two applications of reachability analysis of rewriting

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- Cryptographic protocol verification [Genet, Klay, 00]
  - $\mathcal{L}=$ protocol initial configurations
  - $\mathcal{R}=$ | specification of protocol exchanged messages
    deduction rules of the intruder

Two applications of reachability analysis of rewriting

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- Java application verification [Boichut, Genet, Jensen, Le Roux, 07]
- Cryptographic protocol verification [Genet, Klay, 00]
  - $\mathcal{L}=$ protocol initial configurations
  - $\mathcal{R}= \left|\begin{array}{l}\text{specification of protocol exchanged messages}\\ \text{deduction rules of the intruder}\end{array}\right.$
  - Properties : secrecy, authentication, freshness
  - Unbounded number of agents, protocol sessions and intruder actions
  - Verification of copy-protection on Thomson's SmartRight protocol [Genet, Tang-Talpin, Viet Triem Tong, 03]

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How to finitely represent $\mathcal{R}^*(\mathcal{L})$?

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Set constraints, Horn clauses, Tree Grammars, Tree automata, ...

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Tree automata are well adapted (they are also based on rewriting)

- Finite Tree Automata (Regular Term Language)
- Tree Automata with constraints
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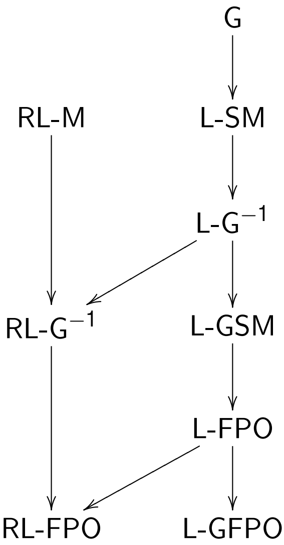
We stick to (Non-Deterministic) Finite Tree Automata because :

We want to decide (efficiently) if $\mathcal{R}^*(\mathcal{L}) \cap Bad = \emptyset$

- The complexity of the algorithm for $\cap$ is quadratic
- The complexity of the algorithm deciding $=^? \emptyset$ is polynomial

$\mathcal{R}$ classes where $\mathcal{L}$ regular $\Rightarrow \mathcal{R}^*(\mathcal{L})$ regular



G Ground [Dauchet, Tison, 90], [Brainerd, 69]

RL-M Right-linear and Monadic [Salomaa, 88]

L-SM Linear and Semi-Monadic [Coquidé et al., 91]

L-G^{-1} Linear and inversely Growing [Jacquemard, 96]

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L-GSM Linear Generalized Semi-Monadic [Gyenizse, Vágvölgyi, 98]

L-FPO, **RL-FPO** (Right)-Linear Finite Path Overlapping [Takai et al. 00]

L-GFPO Linear Generalized Finite Path Overlapping [Takai 04]

$\mathcal{R}$ classes where $\mathcal{L}$ regular $\Rightarrow \mathcal{R}^*(\mathcal{L})$ regular (II)

Plus some classes incomparable with others :

L-IOSLT Linear I/O Separated Layered Transducing (a.k.a. Tree Transducers) [Seki et al. 02]

Constructor Constructor based + constraints on $\mathcal{L}$ [Réty 99]

WOS Well Oriented Systems [Bouajjani, Touili, 02]

$\mathcal{R}$ classes where $\mathcal{L}$ regular $\Rightarrow \mathcal{R}^*(\mathcal{L})$ regular (III)

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Tree automata recognizing **regular** sets of terms

Representation of $f(s^*(a))$ by tree grammar/tree automaton

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$\{f(s^*(a))\}$	Tree grammar G axiom : N_1	
N_1	$:=$	$f(N_2)$
N_2	$:=$	$s(N_2)$
N_2	$:=$	a

$$N_1 \rightarrow_G^* f(s(s(a)))$$

Tree automata recognizing **regular** sets of terms

Representation of $f(s^*(a))$ by tree grammar/tree automaton

Tree grammar G			Tree automaton A		
$\{f(s^*(a))\}$		axiom : N_1	$\{f(s^*(a))\}$		final state : q_1
N_1	$:=$	$f(N_2)$	$f(q_2)$	$\rightarrow$	q_1
N_2	$:=$	$s(N_2)$	$s(q_2)$	$\rightarrow$	q_2
N_2	$:=$	a	a	$\rightarrow$	q_2
$N_1 \rightarrow_G^* f(s(s(a)))$			$f(s(s(a))) \rightarrow_A^* q_1$		

Tree automata recognizing **regular** sets of terms

Representation of $f(s^*(a))$ by tree grammar/tree automaton

Tree grammar G			Tree automaton A		
$\{f(s^*(a))\}$		axiom : N_1	$\{f(s^*(a))\}$		final state : q_1
N_1	$:=$	$f(N_2)$	$f(q_2)$	$\rightarrow$	q_1
N_2	$:=$	$s(N_2)$	$s(q_2)$	$\rightarrow$	q_2
N_2	$:=$	a	a	$\rightarrow$	q_2
$N_1 \rightarrow_G^* f(s(s(a)))$			$f(s(s(a))) \rightarrow_A^* q_1$		

$A = \langle \mathcal{F}, \mathcal{Q}, \mathcal{Q}_f, \Delta \rangle$ where

$\mathcal{Q} = \{q_1, q_2\}$, $\mathcal{Q}_f = \{q_1\}$, $\Delta = \{a \rightarrow q_2, s(q_2) \rightarrow q_2, f(q_2) \rightarrow q_1\}$

$f(s(s(a))) \rightarrow_A^* q_1$ and $q_1 \in \mathcal{Q}_f$. Here $\mathcal{L}(A) = \{f(s^*(a))\}$

A unified algorithm to build $\mathcal{R}^*(\mathcal{L})$

First step : an upper bound for $\mathcal{R}^*(\mathcal{L})$ [Genet, 98]

Definition ($\mathcal{R}$-closed tree automaton)

Given a tree automaton $\mathcal{B}$ and a TRS $\mathcal{R}$, $\mathcal{B}$ is $\mathcal{R}$-*closed* if

$\forall l \rightarrow r \in \mathcal{R}, \forall q \in \mathcal{Q}, \forall \sigma : \mathcal{X} \mapsto \mathcal{Q}$:

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Theorem (Upper bound)

Given a left-linear TRS $\mathcal{R}$ and tree automata $\mathcal{A}, \mathcal{B}$.

$$\left.\begin{array}{r} \mathcal{L}(\mathcal{B}) \supseteq \mathcal{L}(\mathcal{A}) \\ \\ \mathcal{B} \textit{ is } \mathcal{R}\textit{-closed} \end{array}\right| \Rightarrow \mathcal{L}(\mathcal{B}) \supseteq \mathcal{R}^*(\mathcal{L}(\mathcal{A}))$$

A unified algorithm to build $\mathcal{R}^*(\mathcal{L})$ (II)

Tree automata completion algorithm

- Input : a TRS $\mathcal{R}$ and a tree automaton $\mathcal{A}$
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$$\begin{tikzcd}
l\sigma \arrow[r, "\mathcal{R}"'] \arrow[d, "\mathcal{A}"', "*"] & r\sigma \\
q &
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$\mathcal{A}$ completed into $\mathcal{A}^*_{\mathcal{R}} \;\Rightarrow\; \mathcal{L}(\mathcal{A}^*_{\mathcal{R}}) \supseteq \mathcal{L}(\mathcal{A})$

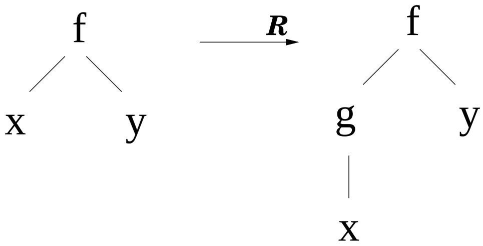
$\mathcal{A}^*_{\mathcal{R}}$ is $\mathcal{R}$-closed

$\Rightarrow \mathcal{L}(\mathcal{A}^*_{\mathcal{R}}) \supseteq \mathcal{R}^*(\mathcal{L}(\mathcal{A}))$

Tree Automata Completion may not terminate

$\mathcal{R} = \{f(x, y) \to f(g(x), y)\}$

$\mathcal{A}^0$		
$f(q_1, q_2) \to q_0$ $a \to q_1$ $b \to q_2$		
$\{f(a, b)\}$		



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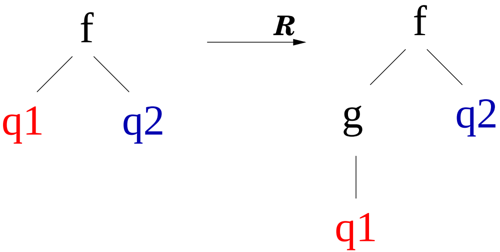

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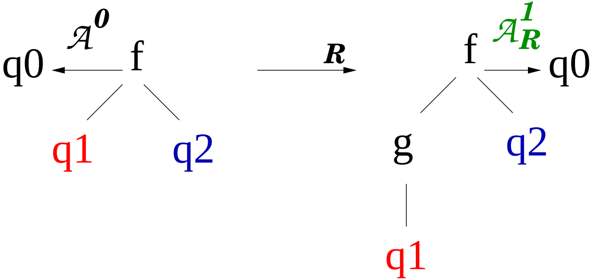

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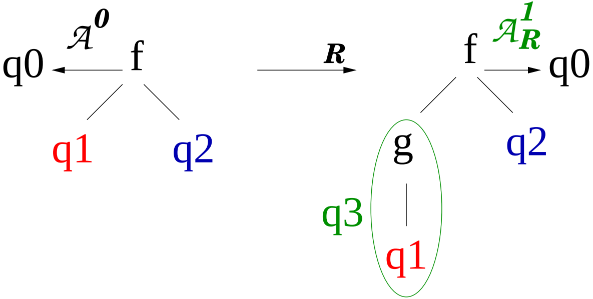
$\{f(a,b)\}$	$\{f(a,b), f(g(a),b)\}$	

Normalization is necessary !



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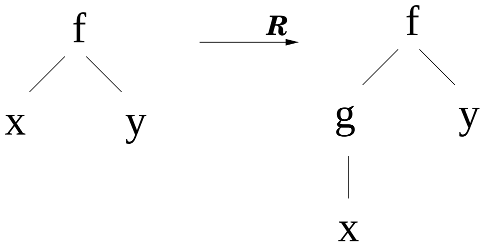

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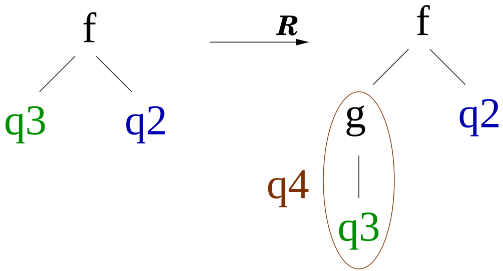

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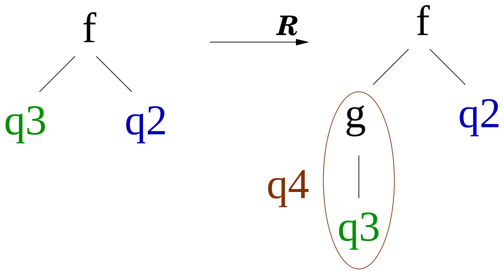

Exact Normalization Strategy

[Feuillade, Genet, Viet Triem Tong, 04]

Principle of Exact Normalization Strategy

Normalize new transitions added to $\mathcal{A}$ using $\mathcal{A}$ when possible, use new states otherwise.

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Given a linear TRS $\mathcal{R}$ and a tree automaton $\mathcal{A}$, if tree automata completion with exact normalization strategy terminates *on $\mathcal{A}^*_{\mathcal{R}}$, then*

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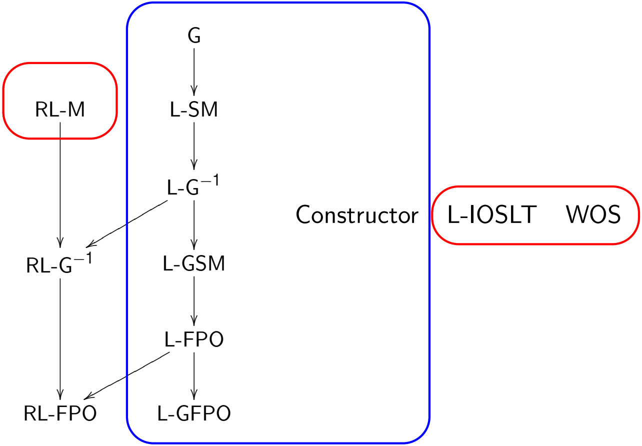
Theorem

Tree automata completion with exact normalization strategy terminates for TRS in classes : **G**, **L-SM**, **L-G^{-1}**, **L-GSM**, **L-FPO** *and* **L-GFPO**.

Regular classes covered by tree automata completion



- with exact normalization strategy
- with other normalization strategies
- it also covers TRS and tree automata outside of those classes !

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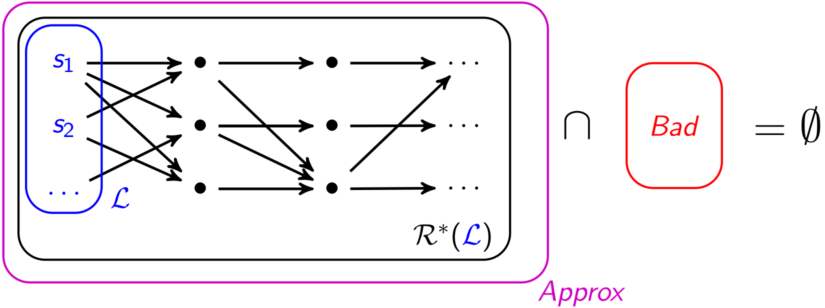
Outside of the regular classes

- This is *generally* the case when the TRS models a program
- We can use over-approximations, i.e.



$$Approx \cap Bad = \emptyset \quad \Rightarrow \quad \mathcal{R}^*(\mathcal{L}) \cap Bad = \emptyset$$

Building approximations using normalization rules

[Genet and Viet Triem Tong 2001]

$\mathcal{R} = \{f(x, y) \to f(g(x), g(y))\}$

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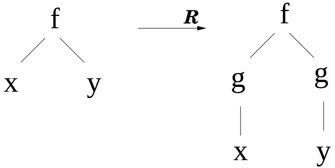

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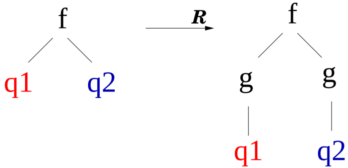

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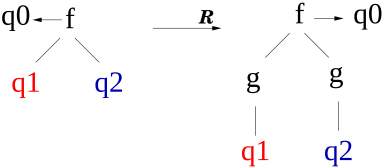

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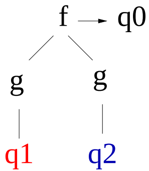

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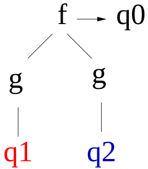

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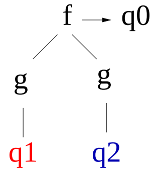

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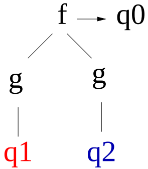

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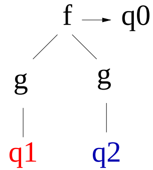

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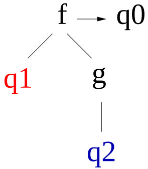

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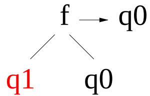

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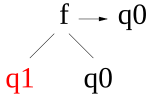

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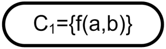
Intuition behind equational over-approximations

$$\mathcal{R} = \begin{cases} (1)\ f(x,y) \rightarrow f(g(x),y) \\ (2)\ f(x,y) \rightarrow f(x,h(y)) \end{cases}$$

prove that $f(a,b) \not\rightarrow_{\mathcal{R}}{}^{*} f(a,h(g(b)))$?



C_1={f(a,b)}

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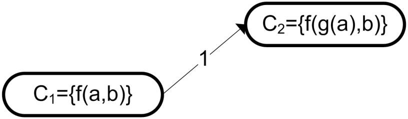

Intuition behind equational over-approximations

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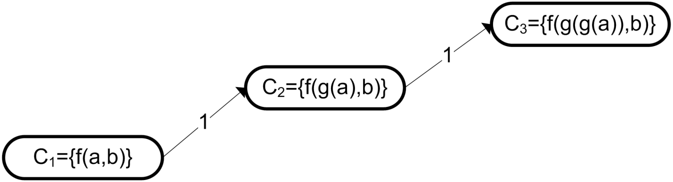

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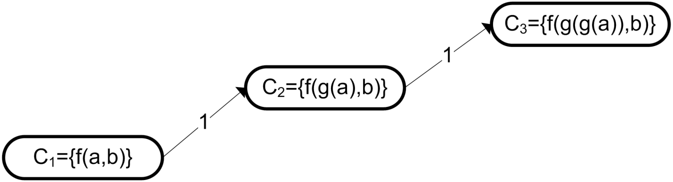

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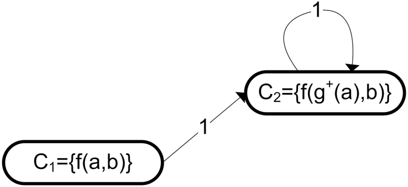

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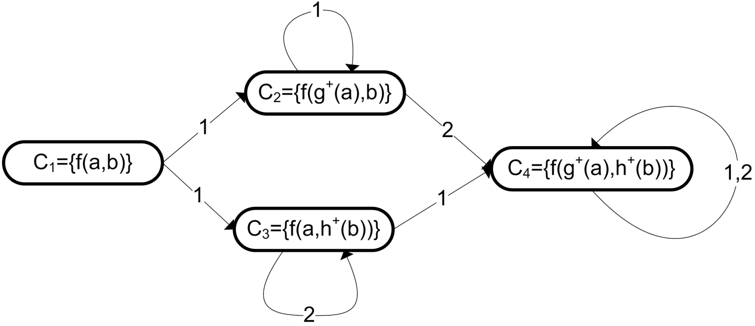

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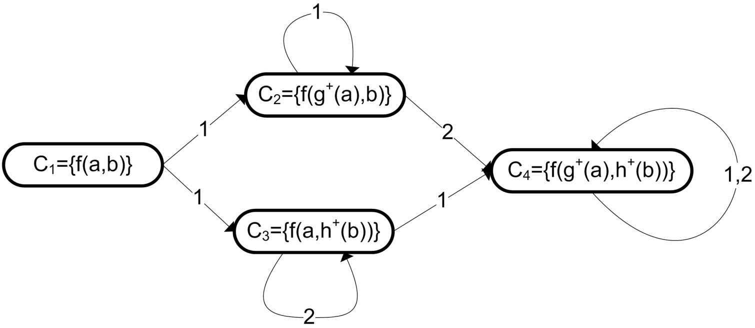

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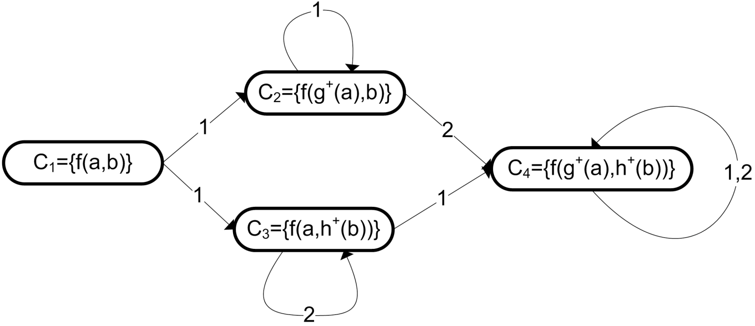

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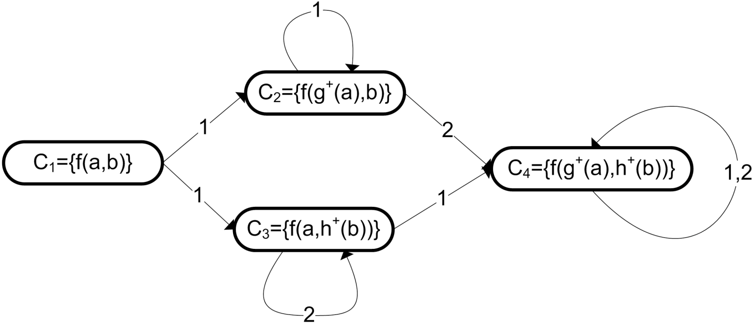

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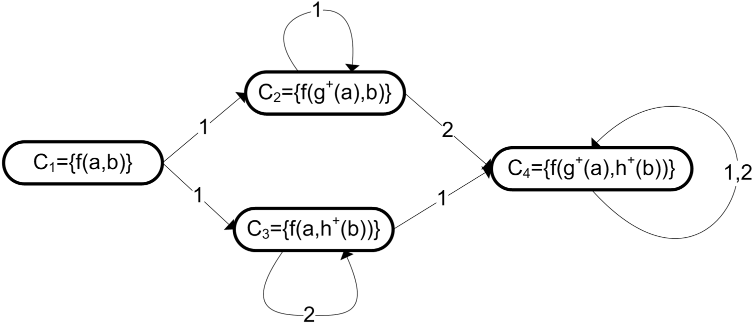

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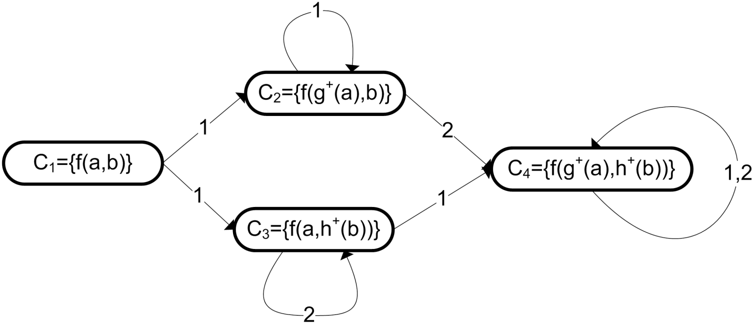

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$f(a,b) \not\rightarrow^{*}_{\mathcal{R}/E} f(a,h(g(b)))$ $\Rightarrow$ $f(a,b) \not\rightarrow^{*}_{\mathcal{R}} f(a,h(g(b)))$

[Meseguer, Palomino, Marti-Oliet, 03] [Takai, 04]

Equations for tree automata approximation

[Genet, Rusu, 09]

Simplification relation $\mathcal{A} \rightsquigarrow_E \mathcal{A}'$

Given $(u = v) \in E$ and a tree automaton $\mathcal{A}$

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After completion step i, we propagate E on $\mathcal{A}^i_{\mathcal{R}}$ using $\leadsto_E$ up to a fixpoint

Equations for tree automata approximation

$\mathcal{R} = \{f(x, y) \rightarrow f(s(x), s(y))\}$ and $E = \{s(s(x)) = s(x)\}$

$\mathcal{A}^0$		
$f(q_a, q_b) \rightarrow q_0$ $a \rightarrow q_a$ $b \rightarrow q_b$		
$\mathcal{L}(\mathcal{A}^0) = \{f(a, b)\}$		

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$\mathcal{A}^0$	$\mathcal{A}^1_{\mathcal{R}}$	
$f(q_a,q_b) \rightarrow q_0$ $a \rightarrow q_a$ $b \rightarrow q_b$	$f(q_1,q_2) \rightarrow q_0$ $s(q_a) \rightarrow q_1$ $s(q_b) \rightarrow q_2$	
$\mathcal{L}(\mathcal{A}^0) = \{f(a,b)\}$	$\mathcal{L}(\mathcal{A}^1) = \{f(a,b),$ $f(s(a),s(b))\}$	

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$\mathcal{A}^0$	$\mathcal{A}^1_{\mathcal{R}}$	$\mathcal{A}^2_{\mathcal{R}}$
$f(q_a, q_b) \rightarrow q_0$	$f(q_1, q_2) \rightarrow q_0$	$f(q_3, q_4) \rightarrow q_0$
$a \rightarrow q_a$	$s(q_a) \rightarrow q_1$	$s(q_1) \rightarrow q_3$
$b \rightarrow q_b$	$s(q_b) \rightarrow q_2$	$s(q_2) \rightarrow q_4$
$\mathcal{L}(\mathcal{A}^0) = \{f(a,b)\}$	$\mathcal{L}(\mathcal{A}^1) = \{f(a,b),$	$\mathcal{L}(\mathcal{A}^2_{\mathcal{R}}) = \{f(a,b),$
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$$\begin{tikzcd}
s(s(q_a)) \arrow[d, "*"', "\mathcal{A}^2_\mathcal{R}"] & =_E & s(q_a) \arrow[d, "*", "\mathcal{A}^2_\mathcal{R}"'] \\
q_3 & & q_1
\end{tikzcd}$$

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$$\begin{tikzcd}
s(s(q_a)) \arrow[d, "*" near start, "\mathcal{A}^2_{\mathcal{R}}"] & =_E & s(q_a) \arrow[d, "*"' near start, "\mathcal{A}^2_{\mathcal{R}}"'] \\
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\end{tikzcd}
\qquad
\begin{tikzcd}
s(s(q_b)) \arrow[d, "*" near start, "\mathcal{A}^2_{\mathcal{R}}"] & =_E & s(q_b) \arrow[d, "*"' near start, "\mathcal{A}^2_{\mathcal{R}}"'] \\
q_4 & & q_2
\end{tikzcd}$$

Equations for tree automata approximation

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q_3 \arrow[r, phantom, "="] & q_1
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\begin{tikzcd}
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s(s(q_a)) \arrow[r, phantom, "=_E"] \arrow[d, "*"] & s(q_a) \arrow[d, "*"] & s(s(q_b)) \arrow[r, phantom, "=_E"] \arrow[d, "*"] & s(q_b) \arrow[d, "*"] \\
q_3 \arrow[r, phantom, "\mathcal{A}^2_{\mathcal{R}} \quad = \quad \mathcal{A}^2_{\mathcal{R}}"] & q_1 & q_4 \arrow[r, phantom, "\mathcal{A}^2_{\mathcal{R}} \quad = \quad \mathcal{A}^2_{\mathcal{R}}"] & q_2
\end{tikzcd}$$

Properties of $\rightsquigarrow_E$

The simplification relation $\rightsquigarrow_E$ enjoys the following properties

- If $\mathcal{A} \rightsquigarrow_E \mathcal{A}'$ then $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{A}')$

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$\Rightarrow$ equations of E can be used in any order for $\rightsquigarrow_E^!$

New completion algorithm : from $\mathcal{A}^{i}_{\mathcal{R},E}$ to $\mathcal{A}^{i+1}_{\mathcal{R},E}$

i-th Completion step

$$\begin{tikzcd}
l\sigma \arrow[r, "\mathcal{R}"'] \arrow[d, "\mathcal{A}^{i}_{\mathcal{R}}"'] & r\sigma \arrow[d, "\mathcal{A}^{i+1}_{\mathcal{R}}"] \\
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- Normalize $r\sigma \rightarrow q'$ using exact norm. strat. or new states

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Simplification

- Find instances of an equation $u = v$ of E in $\mathcal{A}^{i+1}_{\mathcal{R}}$

$$\begin{tikzcd}
u\sigma \arrow[r, equal, "E"'] \arrow[d, "{\mathcal{A}^{i+1}_{\mathcal{R}}, \not\epsilon}"', "*"] & v\sigma \arrow[d, "*"', "{\mathcal{A}^{i+1}_{\mathcal{R}}, \not\epsilon}"] \\
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\end{tikzcd}$$

- Rename q_2 by q_1 in $\mathcal{A}^{i+1}_{\mathcal{R}}$
- Repeat until a fixpoint is reached

Theorems

Theorem (Upper bound)

Let $\mathcal{R}$ be a left-linear TRS, $\mathcal{A}$ be a tree automaton and E be a set of linear equations. If completion terminates on $\mathcal{A}^*_{\mathcal{R},E}$ then

$$\mathcal{L}(\mathcal{A}^*_{\mathcal{R},E}) \supseteq \mathcal{R}^*(\mathcal{L}(\mathcal{A}))$$

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Theorem (Lower bound)

Let $\mathcal{R}$ be a left-linear TRS, E a set of linear equations and $\mathcal{A}$ a $\mathcal{R}/E$-coherent tree automaton. For any $i \in \mathbb{N}$:

$$\mathcal{R}^*_{/E}(\mathcal{L}(\mathcal{A})) \supseteq \mathcal{L}(\mathcal{A}^i_{\mathcal{R},E})$$

and $\mathcal{A}^i_{\mathcal{R},E}$ is $\mathcal{R}/E$-coherent.

Outline

1. Term rewriting and reachability analysis
2. Regular model-checking of term rewriting systems
3. Defining abstractions for infinite non regular systems
4. Refining abstractions by hand using equations
5. Tools and applications
6. Conclusion and further work

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[Genet, Viet Triem Tong, Boichut, Boyer]
(Around 13000 lines of Ocaml)

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  - Exact computation of (covered) regular classes
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  - Exact computation of (covered) regular classes
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- Tree automata completion checker

  Given a left-linear TRS $\mathcal{R}$ and tree automata $\mathcal{A}$, $\mathcal{B}$:

$$\texttt{checker}(\mathcal{A}, \mathcal{R}, \mathcal{B}) = \texttt{true} \quad \Rightarrow \quad \mathcal{L}(\mathcal{B}) \supseteq \mathcal{R}^*(\mathcal{L}(\mathcal{A}))$$

The **Timbuk** library

[Genet, Viet Triem Tong, Boichut, Boyer]
(Around 13000 lines of Ocaml)

Timbuk provides

- Tree automata implementation with $\cap, \cup, =^? \emptyset, \subseteq, \ldots$
- Tree automata completion
  - Exact computation of (covered) regular classes
  - Approximations with normalization rules/equations

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`checker` extracted from a Coq spec. [Boyer, Genet, Jensen, 08]

Applications : Java bytecode verification

[Boichut, Genet, Jensen, Le Roux, 07]

$$\boxed{\mathcal{R}^*(\mathcal{L}) \cap \mathit{Bad} = \emptyset}$$

- $\mathcal{R}= \left|\begin{array}{l}\text{A Java byte code program } P\\ \text{Java Virtual Machine (JVM) semantics}\end{array}\right.$
- $\mathcal{L}=$ Java Virtual Machine (JVM) initial state

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- $\mathcal{R}=\left|\begin{array}{l}\text{A Java byte code program } P\\ \text{Java Virtual Machine (JVM) semantics}\end{array}\right.$
- $\mathcal{L}=$ Java Virtual Machine (JVM) initial state
- $\mathcal{R}^*(\mathcal{L})=$ all JVM states reachable while executing P
- $\mathit{Bad}=$ set of forbidden states (*e.g.* bad control flow, data races, etc.)

Encoding JVM semantics and bytecode into rewriting

Copster tool [Barré, Hubert, Le Roux, Genet]

- Translates `.class` into a *left-linear* TRS

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- Translates `.class` into a *left-linear* TRS
- **Copster** covers the following Java aspects :
  - Class and inheritance
  - Object allocation, initialization, access and modification of fields
  - Virtual method invocation
  - Integer, boolean, characters and string types
  - Basic arithmetic and comparisons
  - Basic standard library methods (strings, I/O)
  - Basic thread operations (creation, synchronization, join)

An example of verification performed on a Java program

```
class T1 extends java.lang.Thread{
  private int l;

  public T1(int l){this.l=l;}

  public void run(){
    while (true){
      synchronized(Top.lock){
         System.out.println(Top.f);
         Top.f=l;
         System.out.println(Top.f);
         Top.f=0;
      }}}}
```

```
class Top{
  public static Object lock;
  public static int f;
  public static void main(String[]
    int i=1;
    lock = new Object();
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    while (i<=2){
      T1 t1 = new T1(i++);
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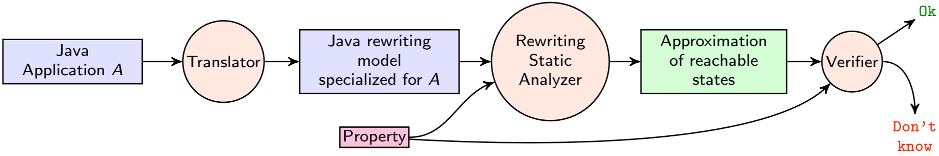
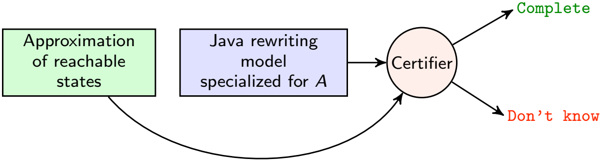
- Subsequences of the form $\ldots, i, i, \ldots$ with $i \geq 1$ should not occur
- One equation is enough : `outstack(x,outstack(y,z))=z`

The RAVAJ Java verification chain

- RAVAJ is an ANR Project between
  LORIA (Nancy), LIFC (Besançon), France Telecom and IRISA
- Certified reachability analysis chain for Java bytecode programs


Java Application A
Translator
Java rewriting model specialized for A
Rewriting Static Analyzer
Approximation of reachable states
Verifier
Ok
Don't know
Property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Outline

1. Term rewriting and reachability analysis
2. Regular model-checking of term rewriting systems
3. Defining abstractions for infinite non regular systems
4. Refining abstractions by hand using equations
5. Tools and applications
6. Conclusion and further work

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- Comparison between Tree Tranducers and TRS is difficult

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≈ Equations could be used on TT, and predicate abstraction on TRS

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+ A *unique* checker for certifying all approximations

Comparison with other verification techniques

- Classes of $\mathcal{R}$ for which $\mathcal{R}^*(\mathcal{L})$ is regular
  - − only left and right linear TRS
  - − only free (*e.g.* no AC) ranked (*e.g.* no hedge) TRS
  - + A *common* algorithm and an optimized tool for all the covered classes

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  - − Completion is more expensive than a pure rewriting approach
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  - − Generate equations automatically (in some cases)
  - + In practice, strong restrictions on equations (syntactical/coherence)

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- Other techniques based on rewriting
  - – Limited to ≪ regular ≫ properties (*e.g.* no induction !)
  - + Simpler properties ⇒ needs less interaction
  - + No need for termination or confluence of the TRS

To sum-up

From the initial (theoretical) idea of tree automata completion, we have shown that this technique

1. covers many regular classes of the litterature
2. deals with automatic/guided approximations
3. is feasible in practice
4. scales up to verify real software
5. can be certified using an external proof assistant

Further Research

- Now : extend the verification capabilities of tree automata completion

  $\rightsquigarrow$ lift-up to temporal properties

  [Boyer, Genet, 09]

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- Within 3 years : certification of distant computation (a.k.a. result certification)

Further Research (II)

- Extend (word) lattice automata to trees

  with T. Legall

- Improve automatic approximations for crypto. protocols

  with Y. Boichut

- Other applications of $\mathcal{R}^*(\mathcal{L})$
  - Checking transformations of SQL query
  - Checking transformations of UML model
  - Javascript programs verification

Further Research (II)

Since the new completion algorithm is based on :

$$\begin{tikzcd}
l\sigma \arrow[r, "\mathcal{R}"'] \arrow[d, "\mathcal{A}_{\mathcal{R}}^{i}"'] & r\sigma \arrow[d, "\mathcal{A}_{\mathcal{R}}^{i+1}"] \\
q & q' \arrow[l, "\epsilon", "\mathcal{A}_{\mathcal{R}}^{i+1}"']
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$$\mathcal{R} = \{f(x,y) \to f(g(x),y),$$
$$f(x,y) \to f(x,h(y))\}$$

$$E = \{g(g(x)) = g(x),$$
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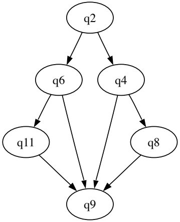

$\mathcal{R}/E$-Coherent tree automata

In the tree automata we distinguish between

- Transitions $f(q_1, \ldots, q_n) \rightarrow q$ recognizing ≪ equivalence classes ≫
- Epsilon transitions $q \xrightarrow{\epsilon} q'$ representing rewriting between classes

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$\mathcal{R} = \{s \rightarrow t, u \rightarrow v\}$
$E = \{s = u\}$

New completion

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 & q_2 \\
q_1 & \\
 & q_3 \\
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 & q_2 \arrow[dl, "\epsilon"'] \\
q_1 & \\
 & q_3 \arrow[ul, "\epsilon"] \\
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s \arrow[r, "\mathcal{R}"'] \arrow[d, "\mathcal{A}"', "\not\epsilon"] \arrow[dd, equal, bend right=50, "E"'] & t \arrow[dl, "\not\epsilon"', "\mathcal{A}"] \\
q_1 & \\
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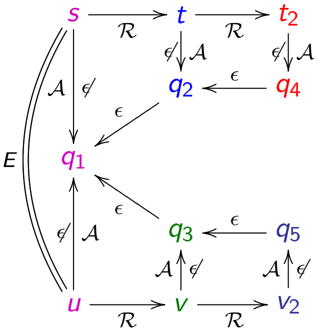

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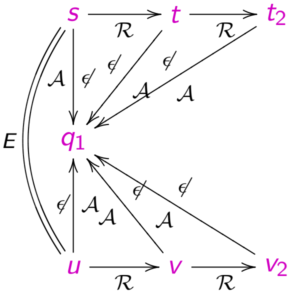

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Definition ($\mathcal{R}/E$-coherent automaton)

Let $\mathcal{A} = \langle \mathcal{F}, \mathcal{Q}, \mathcal{Q}_f, \Delta \rangle$ be a tree automaton, $\mathcal{R}$ a TRS and E a set of equations. The automaton $\mathcal{A}$ is said to be *$\mathcal{R}/E$-coherent* if
$\forall q \in \mathcal{Q} : \exists s \in \mathcal{T}(\mathcal{F}) :$

$$s \rightarrow_{\mathcal{A}}^{\not\epsilon *} q \wedge [\forall t \in \mathcal{T}(\mathcal{F}) : (t \rightarrow_{\mathcal{A}}^{\not\epsilon *} q \Rightarrow s =_E t) \wedge (t \rightarrow_{\mathcal{A}}{}^{*} q \Rightarrow s \rightarrow_{\mathcal{R}/E}^{*} t)]$$

Benchmarks

	Combinatory	NSPK	View-Only	Java prog. 1	Java prog. 2
TRS nb of rules	1	13	15	279	303
Initial Aut. size	43 / 23	14 / 4	21 / 18	26 / 49	33 / 33
Timbuk 2.2 :					
Final Aut. size	8043 / 23	151 / 16	730 / 74	1127 / 334	751 / 335
Time (secs)	**51.1**	**19.7**	**6420**	**25266**	**37387**
Timbuk 3.0 :					
Final Aut. size	8043 / 23	259 / 104	353 / 100		
Time (secs)	**60.1**	**3.1**	**2452**		
Tom-based :					
Final Aut. size	8043 / 23	171 / 21	938 / 89	1974 / 637	1611 / 672
Time (secs)	**5.9**	**5.9**	**150**	**360**	**303**
Bddbddb-based :					
Final Aut. size	? / 25	? / 183	? / 97		
Time (secs)	**0.008**	**2.9**	**3.3**		

Applications : Java bytecode verification (II)

Proving safety properties on Java bytecode using reachability analysis

Java Source .java

```
class TestList{
  public static void main(String[] argv){
    List lpos=null;
    InvList lneg=null;
    int x;
    boolean pos;
    pos= true;
    try {x=System.in.read()};}
    catch(java.io.IOException e){x=0;}
    while (x != -1){
      if (pos) {lpos= new List(x, lpos);
                pos=false;}
      else {lneg= new InvList(x, lneg);
            pos=true;}
      try {x=System.in.read();}
      catch(java.io.IOException e){x=0;}
    }
  }
}
```

Java Byte Code .class

```
public static void main(java.l
  Code:
   0:   aconst_null
   1:   astore_1
   2:   aconst_null
   3:   astore_2
   4:   iconst_1
   5:   istore  4
   7:   getstatic       #2; //
   10:  invokevirtual   #3; //
   13:  istore_3
   ...
   47:  new
   50:  dup
   51:  iload_3
```

Encoding JVM semantics and bytecode into rewriting (II)

Encoding of an add bytecode

$$\boxed{\text{add} : \frac{(m, pc, x :: y :: s, l)}{(m, pc+1, x+y :: s, l)}}$$

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$$\boxed{\text{add} : \frac{(m, pc, x :: y :: s, l)}{(m, pc+1, x+y :: s, l)}}$$

1. Associate add bytecode to m, pc

```
public static void foo(...)
   ...
   11 :   add
```

```
frame(foo,11,s,l) -> xframe(add,foo,11,s,l)
```

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```

2. Pop x and y, start evaluation of $(x + y)$

```
xframe(add,m,pc,stack(y,stack(x,s)),l) -> xframe(xadd(x,y),m,pc,s,l)
```

Encoding JVM semantics and bytecode into rewriting (II)

Encoding of an add bytecode

$$\boxed{\text{add} : \frac{(m, pc, x :: y :: s, l)}{(m, pc+1, x+y :: s, l)}}$$

1. Associate add bytecode to m, pc

```
public static void foo(...)
    ...
    11 :    add
```

```
frame(foo,11,s,l) -> xframe(add,foo,11,s,l)
```

2. Pop x and y, start evaluation of $(x+y)$

```
xframe(add,m,pc,stack(y,stack(x,s)),l) -> xframe(xadd(x,y),m,pc,s,l)
```

3. Compute $(x+y)$

```
xadd(...) -> ...
... -> result(x)
```

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xframe(add,m,pc,stack(y,stack(x,s)),l) -> xframe(xadd(x,y),m,pc,s,l)
```

3. Compute $(x + y)$

```
xadd(...) -> ...
... -> result(x)
```

4. Push the result on top of s and move to next pc

```
xframe(result(x),m,pc,s,l) -> frame(m,next(pc),stack(x,s),l)
```