# **Decentralized control for secrecy**

joint work with

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## Works in the field of Security

- Many aspects (access control, encryption, authentification, trust, intrusion detection, leaks of information ...)
- Static Verification (Cryptographic Protocols) with rewriting techniques, model checking, information theory...
- Run time Verification (User Requests in Web Services) using type and effect systems, automata...
- Any contribution of SCT?

## This presentation

- A first attempt in this direction
- Partial results, toy applications

#### Thesis

Supervisory Control Theory may help enforcing Security

## **Supervisory Control**

#### **Usual Game**

Control Objective : Safety / Liveness Observers: on the side of the Controller

#### **Other Game**

Control Objective : Secrecy Observers: on the side of the Opponent

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# **Example 1**





UNCONTROLLED BEHAVIOUR L INCLUDED IN (A1+A2+A3) \*

FIND MAXIMAL PERMISSIVE CONTROL K INCLUDED IN L SUCH THAT USERS i +1 AND i +2 MAY NEVER KNOW THAT USER i HAS PERFORMED wi IN Li EVEN THOUGH THEY TALK TO EACH OTHER

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## Formalization

# SECRET SETOPPONENT'S ALPHABET $S_1 = L_1 \parallel (A_2 + A_3)^* \cap L$ $\Sigma_1 = A_2 \cup A_3$ $S_2 = L_2 \parallel (A_1 + A_3)^* \cap L$ $\Sigma_2 = A_1 \cup A_3$ $S_3 = L_3 \parallel (A_1 + A_2)^* \cap L$ $\Sigma_3 = A_1 \cup A_2$

## $\mathcal{S} = \{(S_1, \Sigma_1), (S_2, \Sigma_2), (S_3, \Sigma_3)\}$ is a CONCURRENT SECRET

#### Definition

S is opaque if  $\forall w \in L \ \forall i$  $w \in S_i \Rightarrow \Pi_{\Sigma_i}(w) = \Pi_{\Sigma_i}(w')$  for some  $w' \in L \setminus S_i$ 

opacity is the opposite of normality when  $(\forall i)S_i = \overline{S_i}$  $S_i$  is normal if  $\Pi_{\Sigma_i}(w) = \Pi_{\Sigma_i}(w') \Rightarrow w \in \overline{S_i}$  iff  $w' \in \overline{S_i}$ 

# An earlier definition of opacity

Bryans, Koutny, Mazare, Ryan

## Definition

A predicate  $\phi$  over runs  $\rho$  of the system is opaque w.r.t. the observation function *obs* if, for every run  $\rho \in \phi$ , there is a run  $\rho' \notin \phi$  such that  $obs(\rho) = obs(\rho')$ 

single observer arbitrary observation function (states may be observable) opacity is in general not decidable

opacity is asymmetric

Concurrent opacity is needed in order that the observer neither knows  $\phi$ , nor he knows not  $\phi$ 

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# Safe Kernels

If *L* is prefix closed and all secrets  $S_i$  are regular, one can decide whether the concurrent secret S is opaque w.r.t. *L*. If not, one can compute the safe kernel K(L, S) of *L*.



## Definition

The safe kernel K(L, S) of L is the subset of all words  $w \in L$ such that for every prefix u of w and for every i $\Pi_{\Sigma_i}(u) = \Pi_{\Sigma_i}(u')$  for some  $u' \in L \setminus S_i$ 

# Example 2

But using K(L, S) as a controller does not solve our problem ... because users know the system and the controller!



 $S_1 = \Sigma^* afc(\Sigma \setminus \{c\})^*$  (last *c* follows *af*),  $\Sigma_1 = \{c, f\}$ ,  $S_2 = \Sigma^* deb(\Sigma \setminus \{b\})^*$  (last *b* follows *de*),  $\Sigma_2 = \{b, e\}$ 

$$egin{aligned} & \mathcal{K}(L,\mathcal{S}) = L \setminus \mathsf{af} \, m{c} \Sigma^* \ & \mathcal{K}(\mathcal{K}(L,\mathcal{S}),\mathcal{S}) = \mathcal{K}(L,\mathcal{S}) \setminus \mathsf{afdeb} \Sigma^* \end{aligned}$$

What remains in the end is (afde)\*

# Supremal Safe Sublanguage

 $SupK(\bullet, S)$  is monotone in first argument

#### Definition

Let SupK(L, S) be the greatest fixpoint of the operator  $K(\bullet, S)$  included in *L* 

#### Theorem

SupK(L, S) is the union of all controls enforcing the opacity of concurrent secret S

Sufficient conditions under which SupK(L, S) is regular and computable ?

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## Two sources of problems

• The closure ordinal of  $K(\bullet, S)$  may be transfinite

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•  $SupK(\bullet, S)$  may be not regular

# $K(\bullet, S)$ has a transfinite closure ordinal



 $S_1 = \Sigma^* afc(\Sigma \setminus \{c\})^*$  (last *c* follows *af*),  $\Sigma_1 = \{c, f\}$ ,  $S_2 = \Sigma^* deb(\Sigma \setminus \{b\})^*$  (last *b* follows *de*),  $\Sigma_2 = \{b, e\}$  $S_3 = L \setminus (\Sigma^* c \Sigma^*)$  (there is no *c*)

 $S_3$  safe w.r.t. any  $L' \subseteq L$  with at least one word with c

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$$\lim_{i\to\omega} K^i(L,S) = Pref((afde)^{\omega})$$

 $K^{\omega+1}(L,\mathcal{S})=\emptyset$ 

# $SupK(\bullet, S)$ is not regular



$$\begin{split} \Sigma_1 &= \{a, b\}, \ \ \mathbb{C}S_1 = \varepsilon + (ax)^* ab(yb)^* + \{a, x, y\}^* \\ \Sigma_2 &= \{x, y\}, \ \ \mathbb{C}S_2 = (ax)^* (yb)^* \\ \Sigma_3 &= \{a, b, x, y\}, \ \ \mathbb{C}S_3 = \varepsilon + a\Sigma^* \end{split}$$

$$S_1 = \rightarrow$$
  
 $S_2 = \rightarrow \rightarrow$ 

 $SupK(L, S) = Pref(\cup_{n \in \mathbb{N}} (ax)^n (\varepsilon + ab) (yb)^n)$ 

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$$(ax)^{n} (\varepsilon) (yb)^{m}$$

$$\downarrow_{\{a,b\}}$$

$$(ax)^{n-1} (ab) (yb)^{m-1}$$

$$\downarrow_{\{x,y\}}$$

$$(ax)^{n-1} (\varepsilon) (yb)^{m-1}$$

$$\downarrow_{\{a,b\}}$$

$$(ax)^{n-2} (ab) (yb)^{m-2}$$

$$\cdots$$

hence n = m ( ) ( ) ( ) ( )

## Some sufficient conditions

language theoretic conditions (i) and (ii)

i) system language L closed under prefix

ii) secrets closed under suffix  $(S_i \Sigma^* \subseteq S_i)$ 

structural conditions (iii) or (iv) or (v)	
iii) $\Sigma_1 \subseteq \Sigma_2 \ldots \subseteq \Sigma_n$	chain of alphabets
$iv) \; \boldsymbol{\mathcal{S}}_1 \subseteq \boldsymbol{\mathcal{S}}_2 \ldots \subseteq \boldsymbol{\mathcal{S}}_n$	chain of secrets
v) $(\forall i \neq j) (\forall w, w' \in L)$	observers $\perp$ secrets
$\Pi_{\Sigma_j}(w) = \Pi_{\Sigma_j}(w') \; \Rightarrow \; w \in S_i \text{ iff } w' \in S_i$	true in Example 1

#### do not hold for Example 2!

# **Example 1**





UNCONTROLLED BEHAVIOUR L INCLUDED IN (A1+A2+A3) \*

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# Orthogonality in Example 1

## SECRET SET

$$S_1 = L_1 \parallel (A_2 + A_3)^* \cap L S_2 = L_2 \parallel (A_1 + A_3)^* \cap L S_3 = L_3 \parallel (A_1 + A_2)^* \cap L$$

## **OPPONENT'S ALPHABET**

$$\begin{split} \Sigma_1 &= A_2 \cup A_3 \\ \Sigma_2 &= A_1 \cup A_3 \\ \Sigma_3 &= A_1 \cup A_2 \end{split}$$

# $L_1 \subseteq A_1^* \ L_2 \subseteq A_2^* \ L_3 \subseteq A_3^*$



Equivalence Classes w.r.t. Observer 3

$$\begin{array}{l} \Pi_{\Sigma_3}(u) = \Pi_{\Sigma_3}(u') \ \Rightarrow \ \Pi_{A_1}(u) = \Pi_{A_1}(u') \\ \text{hence } u \in S_1 \text{ if and only if } u' \in S_1 \end{array}$$

## $S_1 \subseteq S_2$ $\Sigma_3 \subseteq \Sigma_2$ $Obs_1 \perp S_3$ (a mixed case)



Finite pattern of proofs for  $w \in SupK(L, S)$ 

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## $S_1 \subseteq S_2$ $\Sigma_3 \subseteq \Sigma_2$ $Obs_1 \perp S_3$



## the missing edges

#### Theorem

If there exists a finite number of patterns of proof for all  $w \in SupK(L, S)$ , then SupK(L, S) is a regular language

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## Constructing an automaton from a pattern



## Synchronized moves



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$$egin{aligned} t_{ij} \in (\Sigma_k) ext{ or } t_{ij} = arepsilon \ t_{ij} \in (\Sigma_k) ext{ and } t_{ijk} \in (\Sigma_k) \Rightarrow tt_{ij} = t_{ijk} \end{aligned}$$

## Compute the projection on topmost nodes

## A case where finite patterns are not enough

 $S_1 \subseteq S_2$   $\Sigma_2 \subseteq \Sigma_3$   $Obs_1 \perp S_3$ 



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## The four rules



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#### Theorem

If the complete n-ary tree rewrites to some finite graph, the spanning tree of this graph is a uniform pattern of proofs for all  $w \in SupK(L, S)$ 

#### Theorem

It is decidable whether some finite graph may be derived from the complete n-ary tree, and such graphs may be computed when they exist

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One can then construct a finite automaton accepting SupK(L, S)

## **Decentralized control**

#### Theorem

Let  $w \in L$ . If, for all  $i \in \{1, ..., n\}$ ,  $\pi_i(w) = \pi_i(w_i)$  for some  $w_i \in SupK(L, S)$  then  $w \in SupK(L, S)$ 



BY DEFINITION OF THE GREATEST FIXED POINT

# **Example 1**



SECRETS Li IN Ai\*

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# **Example 1**

