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Partial Order Control of Discrete Event Systems modeled as Polynomial Dynamical Systems

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Abstract

In this paper, we address computational methods for the synthesis of controllers for discrete event systems modeled as polynomial dynamical systems over $\mathbb{Z}/_{3\mathbb{Z}}$. The control objectives are specified as order relations to be checked by the system. The control objectives equations are then synthesized using algebraic tools. The applications of these methods to the safety specification of a power transformer station controller is presented.

Keywords : Control Synthesis, Polynomial Dynamical System, Order Relations, Power systems.

1 Introduction

The control theory of discrete event systems (DES) introduced by [1] is based on formal languages and automata. The DES are described in terms of prefix-closed language; each element of the alphabet representing a possible event. The control of the plant is then performed by inhibiting some events (controllable events) while the other events (uncontrollable events) can not be prevented from occuring.

In our framework, the specification of the plant is realized using the synchronous language SIGNAL[2], dedicated to the specification of real-time systems. The formal bases used in the definition of this language allow us to to check properties on the system. To this purpose, the boolean part of the program (*i.e.,* the plant) is translated into a polynomial dynamical system (PDS) over $\mathbb{Z}/_{3\mathbb{Z}}$[3]. Using algebraic methods and a PDS as a formal model, we are also able to synthesize controllers satisfying various kinds of control objectives. The plant is then represented by a PDS while the control of the system is performed by restricting the controllable input values to values suitable for the control goal. This restriction is obtained by incorporating new algebraic equations, called *controller* to the initial system. Traditionally, control objectives are expressed as *invariance*, *reachability* and *attractivity* of the plants [4]. In this paper, we extend the class of control objectives to the class of optimal control objectives expressed as order relation over the states of the plant. We finally consider in this paper the application of this control theory to the specification of the automatic control system of a power transformer station.

The remainder of this paper is organized as follows: Section 2 is dedicated to the presentation of PDS's and to an overview of logical control problems in our framework. In Section 3, the order relation controller synthesis problem is presented. Section 4 deals with the application of these methods to the incremental construction of a power transformer station controller.

2 Control of PDS

The encoding of SIGNAL programs results in dynamical systems (PDS) over the Galois field $\mathbb{Z}/_{3\mathbb{Z}}$ of the general form :

$$S = \begin{cases} X' & = & P(X,Y,U) \\ Q(X,Y,U) & = & 0 \\ Q_0(X) & = & 0 \end{cases} \quad (1)$$

where X, Y, U, X' are vectors of variables in $\mathbb{Z}/_{3\mathbb{Z}}$ and $dim(X) = dim(X') = n$. The components of vectors X and X' are called *state variables* represent the states of the system. Y is a vector of variables in $\mathbb{Z}/_{3\mathbb{Z}}$, called *uncontrollable event variables*, whereas U is a vector of *controllable event variables.* We can consider that the uncontrollable event variables are emitted by the system to the controller, and the controllable event variables are emitted by the controller to the system. The first equation is the *state transition equation* and captures the dynamical aspect of the system; the second equation is called the *constraint equation*, it specifies which event may occur in a given state. The last equation defines the set of initial states. The behavior of such a PDS is the following: at each instant t, given a state x_t and an admissible y_t (*i.e.,* which means that $Q'(x, y) = 0$[1].),

[1] $Q'(X,Y) = \exists elim_U(Q(X,Y,U))$ where the solutions of the polynomial $\exists elim_U(Q(X,Y,U))$ is the set that is equal to

we can choose some u_t that is admissible *i.e.*, such that $Q(x_t, y_t, u_t) = 0$. In this case, the system e-volves into state $x_{t+1} = P(x_t, y_t, u_t)$.

2.1 Control Synthesis of PDS

A PDS S can be controlled by first selecting a particular initial state x_0 and then by choosing suitable values for $u_1, u_2, \ldots, u_n, \ldots$. We will here consider static control policies, *i.e.*, the value of the control u_t is instantaneously computed from the value of x_t and y_t. Such a controller is called a *static controller*. It is a system of two equations: $C(X, Y, U) = 0$ and $C_0(X) = 0$, where the equation $C_0(X) = 0$ determines initial states satisfying the control objectives and the other one describes how to choose the instantaneous controls; when the controlled system is in state x, and when an event y occurs, any value u such that $Q(x, y, u) = 0$ and $C(x, y, u) = 0$ can be chosen.

2.2 Logical control objectives

We now illustrate the use of the present framework for solving a particular traditional control synthesis problem we shall reuse in the sequel. Suppose we want to ensure the *invariance* of a set of states E.

Let us introduce the operator $\widetilde{pre}$, defined by: for any set of states F,

$$\begin{array}{rcl} \widetilde{pre}\,(F) & = & \{x \in (\mathbb{Z}/3\mathbb{Z})^n \;/\; \forall y \text{ admissible} \\ & & \exists u, Q(x,y,u) = 0 \;\text{ and } P(x,y,u) \in F\} \end{array}$$

Consider now the sequence $(E_i)_{i \in \mathbb{N}}$ defined by:

$$\left\{ \begin{array}{rcl} E_0 & = & E \\ E_{i+1} & = & E_i \cap \widetilde{pre}\,(E) \end{array} \right. \qquad (2)$$

The sequence (2) is decreasing. Since all sets E_i are finite, there exists a j such that $E_{j+1} = E_j$. The set E_j is then the greatest control-invariant subset of E. Let g_j be the polynomial that has E_j as solution, then $C_0(X) = g_j$ and $C(X, Y, U) = P^*(g_j)$[2] is an admissible feed-back controller and the system $S_C : S + (C_0, C)$ verifies the invariance of the set of states E. Using the same methods, we are able to ensure *attractivity*, *reachability* of a set of states (see [3, 4] for more details. After this brief presentation of classical control objectives, the next section will describe some new kinds of control objectives specified by order relations over the states of the systems.

3 Partial order control

Expressing control objectives as partial order relations is motivated by the fact that some control objectives cannot be expressed as logical objectives. These control objectives are more concerned with the way to reach a given logical goal, rather than with the goal to be reached.

3.1 Order relation controller synthesis

We suggest here a method for the synthesis of a controller for a control objective modeled as a partial order relation. To this purpose, let S be a PDS as (1). Let us suppose that the system evolves to a state x, and that y is an admissible event at x. It may exist several controls u such that $Q(x, y, u) = 0$. Let u_1 and u_2 be two controls admissible for the pair (x, y). The system can evolve into two different states x_1 and x_2 such that $x_1 = P(x, y, u_1)$ and $x_2 = P(x, y, u_2)$. The goal of the controller is to choose between u_1 and u_2, in such a way that the system evolves into the state that is optimal for some given order relation $\succeq$. Since the set of states is finite, each order relation can be translated into an equation: $R_{\succeq}(x, x') = 0 \Leftrightarrow x \succeq x'$. A strict order relation between the different states is computed, defined as:

$$x \succ x' \Leftrightarrow \; (x \succeq x') \text{ and } \neg(x' \succeq x). \qquad (3)$$

The translation of (3) into polynomial equations is then given by:

$$R_{\succ}(x, x') = R_{\succeq}(x, x') \oplus (1 - R_{\succeq}(x', x)) = 0 \qquad (4)$$

where $f \oplus f' = (f^2 + f'^2)^2$. A controller can then be computed using the function $R_{\succ}$. The possible initial states are the optimal states (for $R_{\succ}$) among all the solutions of the equation $Q_0(X) = 0$. Let $I = \{x \;/\; Q_0(x) = 0\}$ be the set of initial states, then the optimal states (according to $R_{\succ}$) are obtained by removing, from the set of states I, all states for which there exists a "smaller" state for $R_{\succ}$: $I_{opt} = I \backslash \{x \;/\; \exists x' \in I, x' \succ x\}$. Finally, to force the system to choose the best control, we now introduce the definition:

Definition 1 *A control u_1 is said to be* optimal *compared to a control u_2, if and only if the state $x_1 = P(x, y, u_1)$ is greater than the states $x_2 = P(x, y, u_2)$ for the order relation $R_{\succ}$.*

In other words, the control chooses, for a pair (x, y), an admissible control that makes the state x evolve to the state which is optimal for the relation $R_{\succ}$ (note that there may exist more than one optimal state). The controller of the system is then provided by the following polynomial relation:

$$\begin{array}{c} \{\; R(X, Y, U) \;=\; 0 \;\} \\ \Leftrightarrow \\ \{\forall U' \in (\mathbb{Z}/3\mathbb{Z})^p, (Q(X, Y, U') = 0 \Rightarrow \\ \qquad P(X, Y, U) \succ P(X, Y, U'))\} \end{array} \qquad (5)$$

The controlled system is $S_C : (S + (R, I_{opt}))$ s.t.,

$\{(x, y) \;/\; \exists u,\; (x, y, u) \text{ is solution of } Q\}$

[2] $P^*(g)$ is the polynomial has as solutions the set $\{(x, y, u) \;/\; P(x, y, u) \textit{ is solution of } g\}$

$$S_c = \left\{ \begin{array}{lcc} X' & = & P(X,Y,U) \\ Q(X,Y,U) & = & 0 \\ R(X,U,U) & = & 0 \\ I_{opt}(X_0) & = & 0 \end{array} \right. \quad (6)$$

3.2 Examples of order relations

various kinds of order relations (or partial order relations) can be used to express properties over the states. we present here some of them.

3.2.1 Minimally restrictive constraints on uncontrollable events: Let us assume that the system S is in x and receives the event y; then the system can choose any control such that $Q(x, y, u) = 0$. Let u_1 and u_2 be two possible controls, and x_1 and x_2 the corresponding successor states. Then a minimally restrictive control can be synthesized by adopting the following strategy: let Ad_1 and Ad_2 bet the sets of admissible y events in, respectively, x_1 and x_2. Using the polynomial of the constraints equations over the uncontrollable event variables denoted $Q'(X, Y)$ (*i.e.*, the projection of Q on the X, Y components), we get

$$\left\{ \begin{array}{lcl} Ad_1 & = & \{y \in (\mathbb{Z}/_{3\mathbb{Z}})^m \ / \ Q'(x_1, y) = 0\} \\ Ad_2 & = & \{y \in (\mathbb{Z}/_{3\mathbb{Z}})^m \ / \ Q'(x_2, y) = 0\} \end{array} \right.$$

We now specify our order relation depending on the following three different cases:

1. $Ad_1 \subset Ad_2$: there are more spontaneous evolutions in x_2 than in x_1. The controller must choose the control u_2 rather than u_1.
2. $Ad_2 \subset Ad_1$: there are more spontaneous evolutions in x_1 than in x_2. The controller must choose the control u_1 rather than u_2.
3. u_1 and u_2 are not comparable. The controller can choose either u_1 or u_2.

We will now translate the above strategy into an order relation. The result is a polynomial function $R_{\succ}$ such that $x \succeq x'$ if and only if every admissible event in the state x' is also admissible in the states x :

$$\forall y \in (\mathbb{Z}/_{3\mathbb{Z}})^m, \ Q'(x', y) = 0 \quad \Rightarrow \quad Q'(x, y) = 0$$

and, $x \succeq x' \Leftrightarrow R_{\succeq}(X, X') = 0$, with $R_{\succeq}(X, X') = \forall elim_Y((1 - Q'(X', Y))Q'(X, Y))$ [3]. By applying the methods described in section 3.1 (equation (5), we are then able to synthesize a controller such that the controlled system respects the control strategy of minimally restrictive constraints on uncontrollable events.

[3] The solutions of the polynomial $\forall elim_{X'}(P(X, X'))$ is the set that is equal to $\{x \ / \ \forall x', \ (x, x') \text{ is solution of } P\}$.

3.2.2 Maximization of the number of state variables equal to 1: Let $(X_1, \ldots, X_k)$ be a subset of the set of state variables X; d_1 and d_2 be two tuples of this subset of $(\mathbb{Z}/_{3\mathbb{Z}})^k$, $k \leq n$, where the integer n represents the number of state variables.

$$d_1 = (x_1^1, \ldots, x_k^1) \text{ and } d_2 = (x_1^2, \ldots, x_k^2)$$

Definition 2 *We say $d_1 \sqsupseteq d_2$ if and only if*

$$\forall i \in [1..k], \ x_i^2 = 1 \Rightarrow x_i^1 = 1 \ ,$$

To express $\sqsupseteq$, we introduce the polynomial function δ from $(\mathbb{Z}/_{3\mathbb{Z}}) \times (\mathbb{Z}/_{3\mathbb{Z}})$ to $\mathbb{Z}/_{3\mathbb{Z}}$ such that $\delta(x, y) = (x(x+1)(1-y))^2$. We can check that $\delta(x, y) = 0 \Leftrightarrow \{(x = 1) \Rightarrow (y = 1)\}$. The partial order relation defined in definition (2), can then be expressed in polynomial terms: $d_1 \sqsupseteq d_2 \Leftrightarrow T(d_1, d_2) = 0$, with

$$T(d_1, d_2) = \bigoplus_{i=1}^{k} \delta(X_i^2, X_i^1)$$

To extend this partial order relation to all states of the system, let us consider two states of the system x_1 and x_2, such that $x_1 = (x_1^1, d_1)$ and $x_2 = (x_2^1, d_2)$, where d_1 and d_2 belong to $(\mathbb{Z}/_{3\mathbb{Z}})^k$. Thus, $x_1 \sqsupseteq x_2$ if and only if $d_1 \sqsupseteq d_2$. Finally $R_{\sqsupseteq}(x_1, x_2) = 0 \Leftrightarrow x_1 \sqsupseteq x_2$, with $R_{\sqsupseteq}(x_1, x_2) = T(\exists elim_{[X_{k+1} \ldots X_n]}(x_1), \exists elim_{[X_{k+1} \ldots X_n]}(x_2))$. By applying the construction described in section 3.1, we synthesize a controller that chooses, in a state x, one of the best controls for the relation $R_{\sqsupseteq}$.

Though it is always possible to express priorities over the states using algebraic order relations, it is sometimes more useful to express directly the priorities using numerical cost functions, we investigate this next.

3.2.3 Numerical order relations: In this section, we use cost functions over the states or the events to express order relations. Let $X = (x_1, \ldots, X_n)$ be the state variables of the system. A cost function is a map from $(\mathbb{Z}/_{3\mathbb{Z}})^n$ to $\mathbb{N}$, that associates to each x of $(\mathbb{Z}/_{3\mathbb{Z}})^n$ some integer k. Consider a PDS S, we assume some cost function f is given. We introduce definition:

Definition 3 *A state x_1 is said to be f-optimal than a state x_2 for the PDS S (noted $x_1 \succeq_f x_2$), if and only if, $f(x_2) \geq f(x_1)$.*

Using algebraic methods relying on the ADD developed by [5], it is possible to re-express $\succeq_f$ as a polynomial relation $R_{\succeq_f}$ ($x \succeq_f x' \Leftrightarrow R_{\succeq_f}(X, X') = 0$) (See [6] for the algorithms details). As this order relation is now expressed as a polynomial relation, we are able to use the method described previously to synthesize the corresponding controller.

4 The power transformer station controller

4.1 Brief description

The purpose of an electric power transformer station is to lower the voltage of power so that it can be distributed in urban centers. The kind of transformer we are interested in receives high voltage lines, and several medium voltage lines come out of it and distribute power to end-users. For each high voltage line, a transformer lowers the voltage. In the course of exploitation of this system, several kinds of electrical defects can occur, due to causes internal or external to the station. In order to protect the device and the

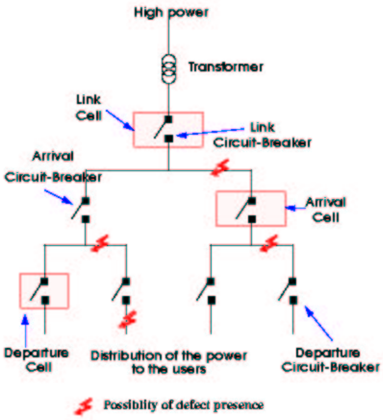

Figure 1: The power transformer station topology.

environment, several circuit breakers are placed in different parts of the station. These circuit breakers are informed about the possible presence of defects by sensors at different locations on the lines and are controlled by local control systems called *cells*.

The controller of the power transformer station can be divided into two parts. The first part concerns the local controllers (*i.e.*, the cells). We chose to specify each local controller in SIGNAL, because they merge logical and numerical aspects. A description of the behavior of the different cells can be found in [7]. The other part concerns more general requirements to be checked by the global controller of the power transformer station and is now described in the sequel.

One of the most significant problems concerns the appearance of two defects at two differents departure level, at the same time. Double defect is very dangerous, because it implies high defective currents. At the place of the defect, this results in a dangerous path voltage that can electrocute people and other living creatures. The detection of these two defects must be performed as fast as possible as well as the handling of one of the defects. Another important aspect is to know which of the circuit breakers must be opened. If the defect appears on the departure line, it is possible to open the circuit breaker at departure level, or at link level, or at arrival level. Obviously, it is the interest of users that the circuit be broken at departure level, and not at a higher level, so that the fewest users are deprived of power. We also have to take into account the importance of the circuit-breakers. Assume that some departure line, involved in the double defect problem, supplies with electricity an hospital. Then, if the double defect problem occurs, the controller should not open this circuit-breaker, since electricity must always delivered to an hospital. To take into account these requirements, with the purpose of obtaining an optimal controller, we choose to rely on our optimal control theory. We will now describe how such controllers can be synthesized.

4.2 Specification of controller

We have seen in the previous section, that one of the most critical requirements concerns the double defect problem. Assume that we have already a polynomial dynamical system coming from the logical abstraction of the plant specified in SIGNAL. This one is composed by four departure circuit-breakers, two arrival circuit-breakers and one link circuit-breaker[4].

We assume here that the circuit-breakers are ideal, i.e. they immediately react to actuators. With this assumption, the double defect problem can be rephrased as follows: *if two defects are picked up at the same time by two different departure cells, then at the next instant, one of the two defects (or both) must have disappeared.*

The double defect requirement: in order to synthesize the controller, we assume that the only controllable events involve the opening and closing demands of the different circuit-breakers. The other events concern the appearance of the defects and can not be considered as controllable.

The specification of the control objective is performed as follows. After translating the plant into a PDS S, we introduce a polynomial *double_ defect* (X) that is equal to 1 when two defects are present at the same time, and -1 otherwise. We are then able to compute the set of states, where two defects are present at two consecutive instants. This is performed by computing the following polynomial: $Error(X_t, X_{t-1}) = double\_ defect(X_t) \text{ and } double\_ defect(X_{t-1})$. The solutions of this polynomial (*i.e.*, when $Error(X_t, X_{t-1}) = 0$) are the set of states where two defects are present at two consecutive instants. The problem of controller synthesis is now to ensure the invariance of the set of states *no_ double_ defect*, which is the complementary

[4] The obtained PDS S is represented by more than 60 states variables, 14 controllable events variables and 21 uncontrollable events variables.

of the set of states *Error*. Applying the algorithm, described in section 2.2, we are able to synthesize the controller, given by the pair (C_1, C_0), that ensures the invariance of the set of states *no_ double_ defect* for the controlled system $S_{C_1} = S + (C_1, C_0)$. **The qualitative requirements**: however, even if the double defect problem is solved, different requirements had not been taken into account. The first one is induced by the obtained controller itself. Indeed, several solutions are available at each instant. For example, when two defects appear at a given instant, the controller can choose to open all the circuit-breakers, or at least the link circuit-breaker. This kind of solutions is not admissible and must not be considered. The second requirements concerns the importance of the lines. The first controller (C_1, C_0) does not look at this kind of problems and can force the system to open the bad circuit-breaker.

To encode the importance of the line, we use a cost function (in fact both requirements will be taken into account by this cost function). We simply have to encode the fact that the more important is the circuit-breaker, the more important is the cost allocated to the state variable which encodes the circuit-breaker. The picture (2) summarize the way we allocate the cost.

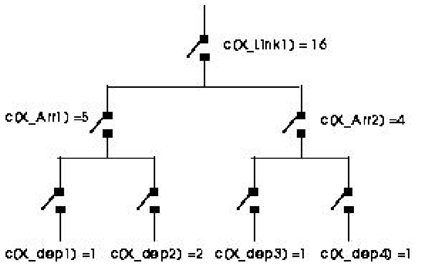

Figure 2: Cost allocated to each circuit breaker.

The cost allocated to each state variable corresponds to the cost when the corresponding circuit-breaker is opened. When it is closed, the cost is equal to 0. The cost of a global state is simply obtained by adding all the circuit-breaker costs. With this cost function, it is always more expensive to open a circuit-breaker at a certain level than to open all the downstream circuit-breakers. Finally, the cost allocated to the state variable that encodes the second departure circuit-breaker (encoded by the state variable X_{dep2})) is bigger than the others because the corresponding line supplies with electricity an hospital (for example). Using methods described in sections 3.2.3 and 3.1, we then obtain a controller (C_2, C_0) which is sufficient to solve our problem.

5 conclusion

In this paper, we have shown the usefulness of control theory concepts for the class of polynomial dynamical systems over $\mathbb{Z}/_{3\mathbb{Z}}$. As this model results from the translation of a SIGNAL program (not presented here)[2], we have a powerful environment to describe the model for real-time data-flow system. Even if classical control can be performed, we showed that using the same algebraic framework, optimal control synthesis problem can also be performed. The order relation controller synthesis problem covers different areas of control. It can first be used to synthesize control objectives which relate more to the way to get to a logical goal, than to the goal to be reached, but can also be used to obtain explicit control laws for the controllable events (by using a strict order relation for example), considering a previous classical control synthesis problem. These methods have finally been successfully applied to the incremental construction of a power transformer station controller. For more details, the reader could refer to [8].

The theory of PDSs deserves much more research. One issue is the control under partial observations or in a slightly different domain the control of implicit non-deterministic PDSs. Another point of interest concerns the optimal control problem (see [8] for more details). Some other perspectives concern the synthesis of fault tolerance controllers.

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