Worst-case efficient multiple string matching in the RAM model

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Single string matching problem

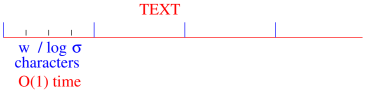
- A single pattern of length m characters from an alphabet of size σ.
- Goal: preprocess the pattern to answer to queries.
- Queries: text T of length $n \longrightarrow$ all occurrences of the pattern.
- Classical solution: *KMP* automaton $\longrightarrow O(m)$ space and $O(n + occ)$ query time for reporting occ occurrences.

Multiple string matching problem

- Set of d patterns (strings): $S = \{s_1, s_2, \ldots, s_d\}$.
- $\sum_{i=1}^{d} |s_i| = m$ characters from an alphabet of size σ.
- Queries: text $T \longrightarrow$ occurrences of patterns in S.
- Classical solution AC automaton $\longrightarrow O(m)$ space and $O(n + occ)$ time for reporting occ occurrences.

WORD-RAM model

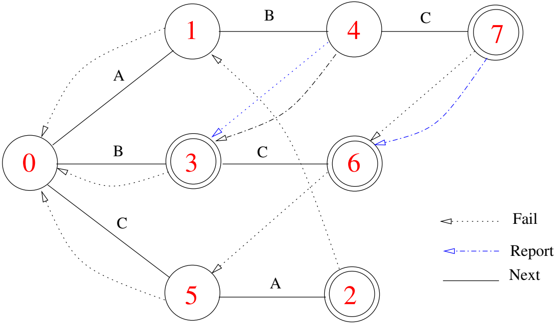
- The computer operates on words of size w (usually $w = 32$ or $w = 64$ bits).
- Usual arithmetic and logic operations take $O(1)$ time.
- Each character of the text is encoded using $\log \sigma$ bits.
- We can read the $\Theta(w/\log \sigma)$ characters at a time.
- The best possible query time $O(n \log \sigma / w + occ)$. $\longrightarrow$ a factor $\Theta(w)$ faster than *AC* and *KMP* for constant sized alphabets (e.g DNA).
- Can we achieve that query time?



New result

- Single string matching:
  - Space usage $O(m)$ (same as KMP).
  - Query time $O(n(1/m + \frac{\log \sigma}{w}) + occ)$.
  - For $m \geq w/\log \sigma$ the query time is optimal $O(n \log \sigma / w + occ)$.
- Multiple string matching:
  - Space usage $O(m)$ (same as AC).
  - Query time $O(n((\log d + \log y + \log \log m)/y + \frac{\log \sigma}{w}) + occ)$.
  - Where y is the length of the shortest pattern.
  - For $y \geq w(\log w + \log d)/\log \sigma$ the query time is optimal $O(n \log \sigma / w + occ)$.

AC automaton (Multiple string matching)



- *P*: the set of all prefixes of strings in *S*.
- states in the automaton correspond to a prefixes in *P*. Number of states is $m = |P|$.
- Three kinds of transitions: *next*, *failure* and *report*.

New result (Old idea used in CPM 2010)

- Order P according to suffix-lexicographic order $\longrightarrow$ strings compared right-to-left instead of left-to-right.
- Give each state a unique number $\longrightarrow$ suffix-lexicographic order of the prefix corresponding to that state in the set P.
- Example: for the set $S = \{''ABC'','' B'','' BC'','' CA''\}$.
- We have $P = \{'' '' = 0,\ ''A'' = 1,\ ''CA'' = 2,\ ''B'' = 3,\ ''AB'' = 4,\ ''C'' = 5,\ ''BC'' = 6,\ ''ABC'' = 7\}$.

New result (Basic string tools)

- A fixed set P of n strings and a query string x.
- Longest prefix matching $\longrightarrow$ find the longest $y \in P$ such that y is prefix of x.
- Longest suffix matching $\longrightarrow$ find the longest $y \in P$ such that y is suffix of x.
- Same solution for both problems $\longrightarrow$ Combination of string B-tree, suffix array and LCP array.
- Query time $O(|x| \log \sigma / w + \log n)$.

New result (Basic Geometric tools)

- 1D stabbing problem $\longrightarrow$ A query point $x \in U$ and (fixed) set of n intervals $S \subset U$, return the tightest interval $y \in S$ such that $x \in y$.
- 2D stabbing problem : A query for a point $x \in U^2$ over a fixed set of n rectangles $\longrightarrow$ all occ rectangles enclosing x.
- 1D stabbing $\longrightarrow$ y-fast trie in time $O(\log \log |U|)$.
- 2D stabbing problem $\longrightarrow$ Chazelle's solution (1986) $O(\log n + occ)$ time and $O(n)$ space.

New result (Basic Geometric tools)

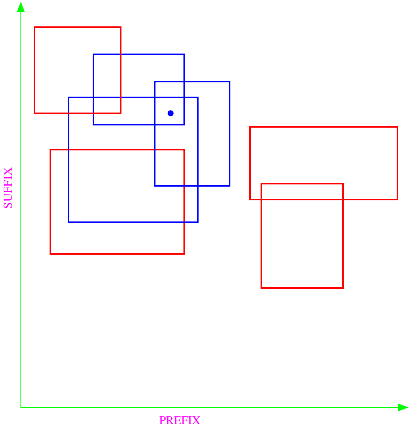


Figure: 2D stabbing: a query for the blue point reports the blue rectangles

New result (Main ideas)

- Read the text T in blocks of $b = w/\log\sigma$ characters.
- At step i read $q = T[ib, (i+1)b]$.
- Current state represented by a number x.
- Report occurrences using 2D stabbing and longest suffix matching.
- Do transitions on b characters $\longrightarrow$ minimal perfect hashing, 1D stabbing and longest prefix matching.

New result (Report occurrences)

- Do a longest suffix matching query on string q relative to set P.
- Convert the returned suffix into a number y.
- Finally do a 2D stabbing query on the on the point $(x, y) \longrightarrow$ all occurrences ending at positions in on $[ib, (i+1)b]$.

New result (transitions)

- Convert string q into a number y using MPHF.
- Concatenate the bits of y with bits of x (y as MSB).
- Do a 1D stabbing query for the number xy.
- Query successful $\longrightarrow$ next state corresponding to a prefix of length $\geq b$.
- Query failed $\longrightarrow$ do a longest prefix matching$\longrightarrow$ next state corresponding to a prefix of length $< b$.

Conclusion

- We have proposed a new solution for single and multiple string matching problems in the RAM model.
- Get the optimal query time $O(m\frac{\log\sigma}{w} + occ)$ when pattern(s) sufficiently long.
- Open question : can we achieve the optimal query time for any pattern length?