Choosing Word Occurrences for the Smallest Grammar Problem

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Smallest Grammar Problem

Problem Definition

Given a sequence s, find a context-free grammar $G(s)$ of minimal size that generates exactly this and only this sequence.

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Example

$s =$ "`how much wood would a woodchuck chuck if a woodchuck could chuck wood?`", a possible $G(s)$ (not necessarily minimal) is

$$
\begin{aligned}
S &\rightarrow \text{how much } N_2 \text{ w}N_3 \; N_4 \; N_1 \text{ if } N_4 \text{ c}N_3 \; N_1 \; N_2 \text{ ?} \\
N_1 &\rightarrow \text{chuck} \\
N_2 &\rightarrow \text{wood} \\
N_3 &\rightarrow \text{ould} \\
N_4 &\rightarrow \text{a } N_2 N_1
\end{aligned}
$$

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Applications

- Data Compression
- Sequence Complexity
- Structure Discovery

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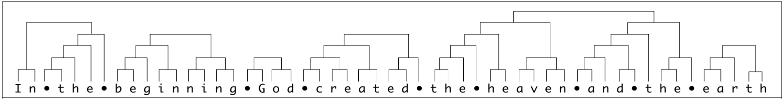

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Remark

Not only S, but any non-terminal of the grammar generates only one sequence of terminal symbols: $cons :: \mathcal{N} \rightarrow \Sigma^*$

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\end{array}
\quad \Rightarrow \quad
\begin{array}{lcl}
\text{cons}(S) & = & s \\
\text{cons}(N_1) & = & \text{chuck} \\
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Given a sequence s, find a context-free grammar $G(s)$ of minimal size that generates exactly this and only this sequence.

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$$|G| = \sum_{N \to \omega \in \mathcal{P}} (|\omega| + 1)$$

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\end{array}$$

$$\Downarrow$$

how much N_2 wN_3 N_4 N_1 if N_4 cN_3 N_1 N_2 | chuck | wood | ould | a N_2 N_1 |

Previous Approaches

1. Practical algorithms: Sequitur (and offline friends). 1996

   "Compression and Explanation Using Hierarchical Grammars". Nevill-Manning & Witten. The Computer Journal. 1997

2. Compression theoretical framework: Grammar Based Code. 2000

   "Grammar-based codes: a new class of universal lossless source codes". Kieffer & Yang. IEEE T on Information Theory. 2000

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Offline algorithms

- Maximal Length (ML): take longest repeat, replace all occurrences with new symbol, iterate.

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$$\Downarrow$$

$$S \rightarrow \text{how\_much\_wood\_would}N_1\text{huck\_if}N_1\text{ould\_chuck\_wood?}$$

$$N_1 \rightarrow \text{\_a\_woodchuck\_c}$$

Offline algorithms

- Maximal Length (ML): take longest repeat, replace all occurrences with new symbol, iterate.

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⇓

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⇓

S → how_much_wood_wouldN_1huck_if_N_1ould_N_2wood?
N_1 → _a_woodN_2c
N_2 → chuck_

Offline algorithms

- Maximal Length (ML): take longest repeat, replace all occurrences with new symbol, iterate.

  Bentley & McIlroy "Data compression using long common strings". DCC. 1999.

  Nakamura, et.al. "Linear-Time Text Compression by Longest-First Substitution". MDPI Algorithms. 1999

- Most Frequent (MF): take most frequent repeat, replace all occurrences with new symbol, iterate

  Larsson & Moffat. "Offline Dictionary-Based Compression". DCC. 1999

- Most Compressive (MC): take repeat that compress the best, replace with new symbol, iterate

  Apostolico & Lonardi. "Off-line compression by greedy textual substitution" Proceedings of IEEE. 2000

A General Framework: IRR

IRR (Iterative Repeat Replacement) framework
Input: a sequence s, a score function f

1. Initialize Grammar by $S \rightarrow s$
2. take repeat ω that maximizes f over G
3. **if** replacing ω would yield a bigger grammar than G **then**
   3.1 **return** G

   else
   3.1 replace all (non-overlapping) occurrences of ω in G by new symbol N
   3.2 add rule $N \rightarrow \omega$ to G
   3.3 goto 2

Complexity: $\mathcal{O}(n^3)$

Results on Canterbury Corpus

sequence	Sequitur	IRR-ML	IRR-MF	IRR-MC
alice29.txt	19.9%	37.1%	8.9%	41000
asyoulik.txt	17.7%	37.8%	8.0%	37474
cp.html	22.2%	21.6%	10.4%	8048
fields.c	20.3%	18.6%	16.1%	3416
grammar.lsp	20.2%	20.7%	15.1%	1473
kennedy.xls	4.6%	7.7%	0.3%	166924
lcet10.txt	24.5%	45.0%	8.0%	90099
plrabn12.txt	14.9%	45.2%	5.8%	124198
ptt5	23.4%	26.1%	6.4%	45135
sum	25.6%	15.6%	11.9%	12207
xargs.1	16.1%	16.2%	11.8%	2006
average	*19.0%*	*26.5%*	*9.3%*	

Extends and confirms results of Nevill-Manning & Witten "On-Line and Off-Line Heuristics for Inferring Hierarchies of Repetitions in Sequences". Proc. of the IEEE. vol 80 no 11. November 2000

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Split the Problem

Smallest Grammar Problem → Choice of Constituents

Smallest Grammar Problem → Choice of Occurrences

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   3.3 goto 2

Complexity: $\mathcal{O}(n^3)$

A General Framework: IRRCOO

IRRCOO (Iterative Repeat Replacement with Choice of Occurrence Optimization) framework

Input: a sequence s, a score function f

1. Initialize Grammar by $S \rightarrow s$
2. take repeat ω that maximizes f over G
3. **if** replacing ω would yield a bigger grammar than G **then**
   3.1 **return** G

   else

   3.1 $G \leftarrow mgp(cons(G) \cup cons(\omega))$

   3.2 goto 2

Choice of Occurrences

Minimal Grammar Parsing (MGP) Problem

Given sequences $\Omega = \{s = w_0, w_1, \ldots, w_m\}$, find a context-free grammar of minimal size that has non-terminals $\{S = N_0, N_1, \ldots N_m\}$ such that $\text{cons}(N_i) = w_i$.

Choice of Occurrences: an Example

Given sequences $\Omega = \{ababbababbabaabbabaa, abbaba, bab\}$

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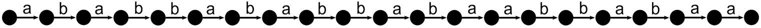
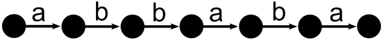
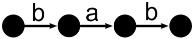
N_0 

N_1 

N_2 

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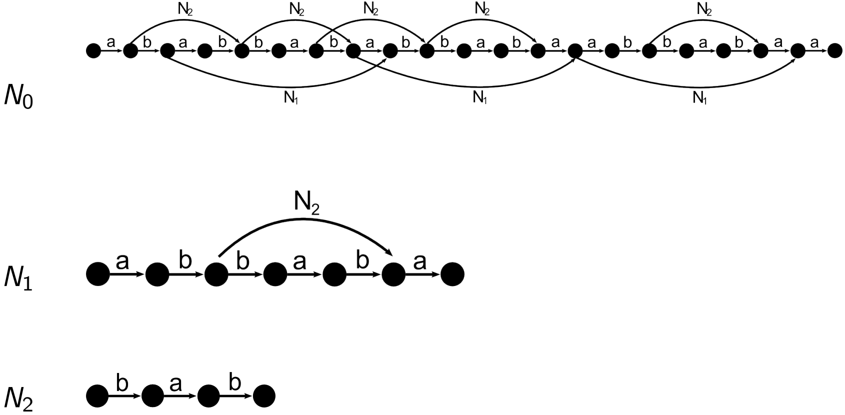

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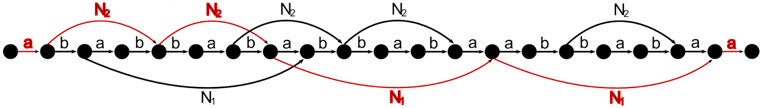

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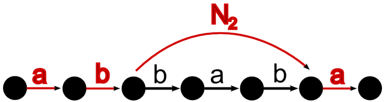
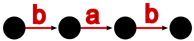


N_2



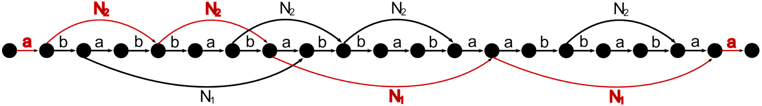
A minimal grammar for Ω is

$$
\begin{aligned}
N_0 &\rightarrow aN_2N_2N_1N_1a \\
N_1 &\rightarrow abN_2a \\
N_2 &\rightarrow bab
\end{aligned}
$$

Choice of Occurrences: an Example

Given sequences $\Omega = \{ababbababbabaabbabaa, abbaba, bab\}$

N_0



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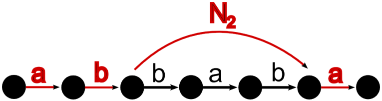

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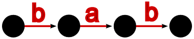


mgp can be computed in $\mathcal{O}(n^3)$

Split the Problem

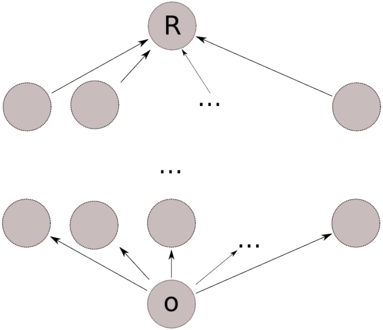
Smallest Grammar Problem

Choice of Constituents

Choice of Occurrences

A Search Space for the SGP

Given s, take the lattice $\langle \mathcal{R}(s), \subseteq \rangle$ and associate a score to each node η: the size of the grammar $mgp(\eta \cup \{s\})$. A smallest grammar will have associated a node with minimal score.



A Search Space for the SGP

Lattice is a good search space

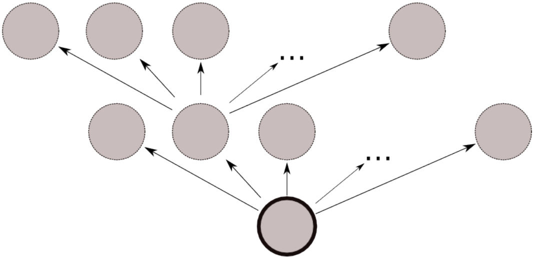
For every sequence s, there is a node η in $\langle \mathcal{R}(s), \subseteq \rangle$ such that $mgp(\eta \cup \{s\})$ is a smallest grammar.

Not the case for IRR search space

But, there exists a sequence s such that for any score function f, $IRR(s, f)$ does not return a smallest grammar ▸ Proof

The ZZ Algorithm

bottom-up phase: given node η, compute scores of nodes $\eta \cup \{w_i\}$ and take node with smallest score.



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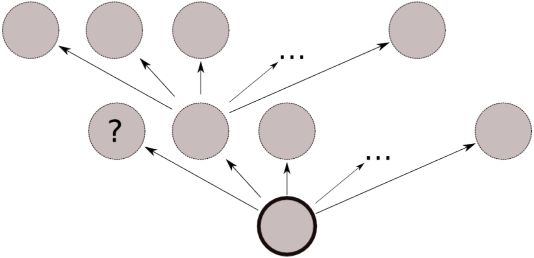

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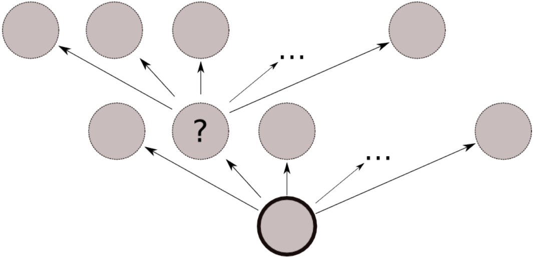

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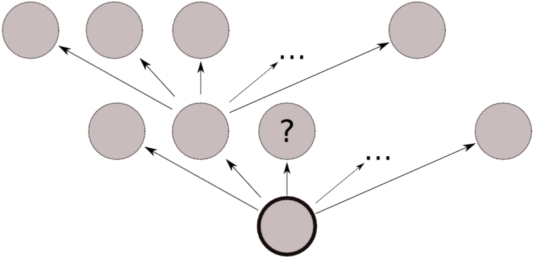

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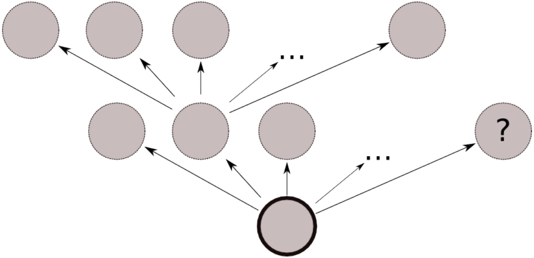

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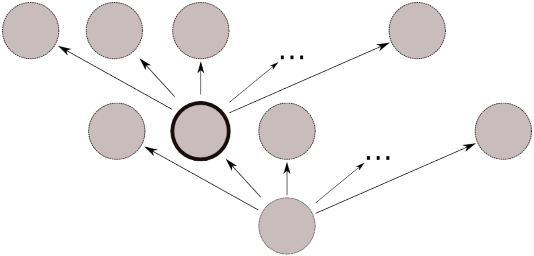

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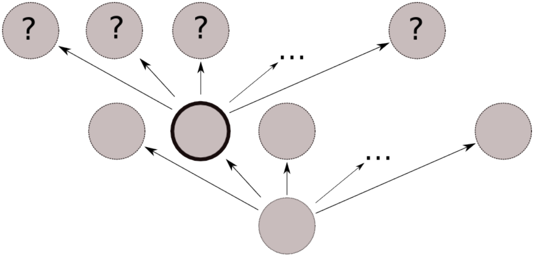
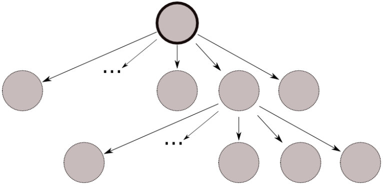


The ZZ Algorithm

bottom-up phase: given node η, compute scores of nodes $\eta \cup \{w_i\}$ and take node with smallest score.

top-down phase: given node η, compute scores of nodes $\eta \setminus \{w_i\}$ and take node with smallest score.



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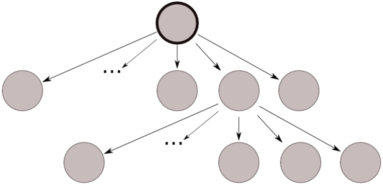

ZZ: succession of both phases. Is in $\mathcal{O}(n^7)$

Results on Canterbury Corpus

sequence	IRRCOO-MC	ZZ	IRR-MC
alice29.txt	-4.3%	-8.0%	41000
asyoulik.txt	-2.9%	-6.6%	37474
cp.html	-1.3%	-3.5%	8048
fields.c	-1.3%	-3.1%	3416
grammar.lsp	-0.1%	-0.5%	1473
kennedy.xls	-0.1%	-0.1%	166924
lcet10.txt	-1.7%	–	90099
plrabn12.txt	-5.5%	–	124198
ptt5	-2.6%	–	45135
sum	-0.8%	-1.5%	12207
xargs.1	-0.8%	-1.7%	2006
average	*-2.0%*	*-3.1%*	

New Results

Classi-fication	sequence name	length	IRRMGP*	size im-provement
Virus	P. lambda	48 Knt	13061	-4.25%
Bacterium	E. coli	4.6 Mnt	741435	-8.82%
Protist	T. pseudonana chrI	3 Mnt	509203	-8.15%
Fungus	S. cerevisiae	12.1 Mnt	1742489	-9.68%
Alga	O. tauri	12.5 Mnt	1801936	-8.78%

Back to Structure

How similar are the structures returned by the different algorithms?

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Standard measure to compare parse trees:

- Unlabeled Precision and Recall (F-measure)
- Unlabeled Non Crossing Precision and Recall (F-measure)

Dan Klein. "The Unsupervised Learning of Natural Language Structure". Phd Thesis. U Stanford. 2005

Similarity of Structure

sequence	algorithm vs IRR-MC	size gain	U_F	UNC_F
fields.c	ZZ	3.1 %	77.8	85.3
	IRRCOO-MC	1.3 %	84.1	88.7
cp.html	ZZ	3.5 %	66.3	75.0
	IRRCOO-MC	1.3 %	81.4	84.8
alice.txt	ZZ	8.0 %	36.6	38.6
	IRRCOO-MC	4.3 %	63.9	66.0
asyoulike.txt	ZZ	6.6 %	34.6	35.8
	IRRCOO-MC	2.9 %	55.1	56.9

Conclusions and Perspectives

- Split SGP into two complementary problems: choice of constituents and choice of occurrences
- Definition of a search space that contains a solution....
- ... and to define algorithms which find smaller grammars than state-of-the-art.

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- ⋆ Split SGP into two complementary problems: choice of constituents and choice of occurrences
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- Promising results on DNA sequences (whole genomes)

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- ⋆ Definition of a search space that contains a solution....
- ⋆ ... and to define algorithms which find smaller grammars than state-of-the-art.
- Promising results on DNA sequences (whole genomes)
- Focus on the structure. Meaning of (dis)similarity.

The End

S → thDkAforBr_attenC._DoAhave_Dy_quesCs?
A → B_
B → _you
C → tion
D → an

Parse Tree Similarity Measures

$$UNC_P(P_1, P_2) = \frac{|\{b \in \text{brackets}(P_1) : b \text{ does not cross brackets}(P_2)|}{|\text{brackets}(P_1)|}$$

$$UNC_R(P_1, P_2) = \frac{|\{b \in \text{brackets}(P_2) : b \text{ does not cross brackets}(P_1)|}{|\text{brackets}(P_2)|}$$

$$UNC_F(P_1, P_2) = \frac{2}{UNC_P(P_1, P_2)^{-1} + UNC_R(P_1, P_2)^{-1}}$$

Parse Tree Similarity Measures

$$U_P(P_1, P_2) = \frac{|\text{brackets}(P_1) \cap \text{brackets}(P_2)|}{|\text{brackets}(P_1)|}$$

$$U_R(P_1, P_2) = \frac{|\text{brackets}(P_1) \cap \text{brackets}(P_2)|}{|\text{brackets}(P_2)|}$$

$$U_F(P_1, P_2) = \frac{2}{U_P(P_1, P_2)^{-1} + U_R(P_1, P_2)^{-1}}$$

Problems of IRR-like algorithms

Example

$xaxbxcx|_1 xbxcxax|_2 xcxaxbx|_3 xaxcxbx|_4 xbxaxcx|_5 xcxbxax|_6 xax|_7 xbx|_8 xcx$

Problems of IRR-like algorithms

Example

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A smallest grammar is:

$$\begin{aligned} S &\rightarrow AbC|_1 BcA|_2 CaB|_3 AcB|_4 BaC|_5 CbA|_6 A|_7 B|_8 C \\ A &\rightarrow xax \\ B &\rightarrow xbx \\ C &\rightarrow xcx \end{aligned}$$

Problems of IRR-like algorithms

Example

$xaxbxcx|_1 xbxcxax|_2 xcxaxbx|_3 xaxcxbx|_4 xbxaxcx|_5 xcxbxax|_6 xax|_7 xbx|_8 xcx$

But what IRR can do is like:

$$
\begin{array}{lcl}
S & \rightarrow & Abxcx|_1 xbxcA|_2 xcAbx|_3 Acxbx|_4 xbAcx|_5 xcxbA|_6 A|_7 xbx|_8 xcx \\
A & \rightarrow & xax \\
 & \Downarrow & \\
S & \rightarrow & Abxcx|_1 BcA|_2 xcAbx|_3 AcB|_4 xbAcx|_5 xcxbA|_6 A|_7 B|_8 xcx \\
A & \rightarrow & xax \\
B & \rightarrow & xbx \\
 & \Downarrow & \\
S & \rightarrow & AbC|_1 BcA|_2 xcAbx|_3 AcB|_4 xbAcx|_5 CbA|_6 A|_7 B|_8 C \\
A & \rightarrow & xax \\
B & \rightarrow & xbx \\
C & \rightarrow & xcx
\end{array}
$$