Reaching Your Goal Optimally by Playing at Random with no Memory

Julie Parreaux
Benjamin Monmege Pierre-Alain Reynier

Aix–Marseille Université

TickTac meeting
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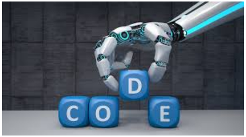
Motivation : game theory for synthesis

Game theory

Interaction between two antagonistic agents : environment and controller



Code synthesis

Correct by construction : synthesis of controller

Classic approach

Check the correctness of a system

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Shortest-Path games

- Combination of a qualitative with a quantitative objective
- Reach a target with a minimum cost

Different sorts of games

Timed games

- Consider timed issues : receive a message...
- Infinite games

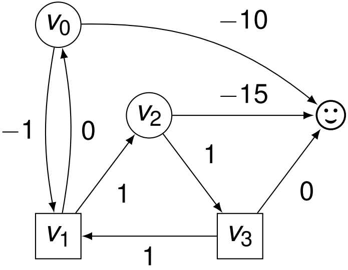
Weighted Timed games

Combination of timed games and shortest path games.

Shortest Path Game

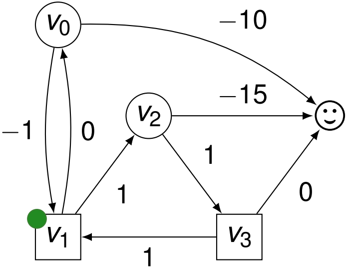
☐ Adam ○ Eve

☺ target (T)



Shortest Path Game

□ Adam ○ Eve

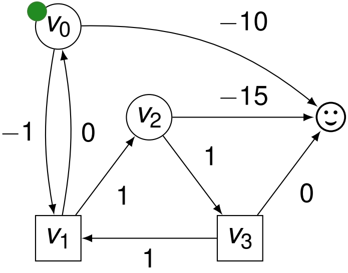


How to play?

Move a token along an edge

$$\pi = v_1$$

Shortest Path Game



How to play?

Move a token along an edge

$$\pi = v_1 v_0$$

Shortest Path Game

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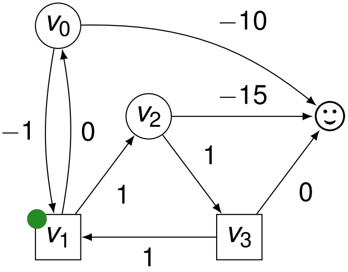

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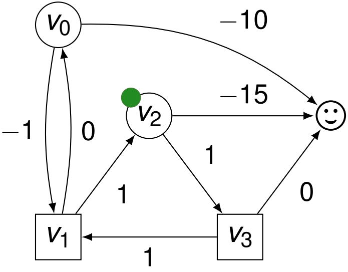

How to play?

Move a token along an edge

$$\pi = v_1 v_0 v_1 v_2$$

Shortest Path Game

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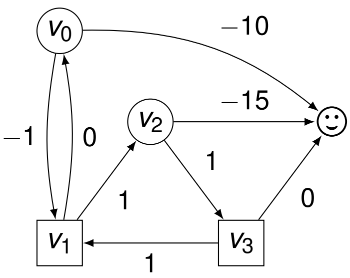


Play

Infinite path or reach the target

$\pi = (v_i)_i \in V^\omega$ $\pi = (v_i)_i$☺

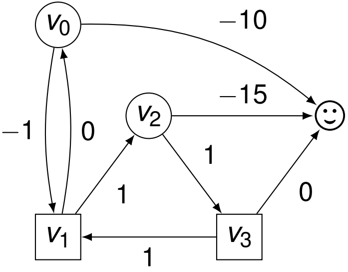
How to play?

Move a token along an edge

$\pi = v_1 v_0 v_1 v_2 v_3$☺

Shortest Path Game

☐ Adam ○ Eve



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Move a token along an edge

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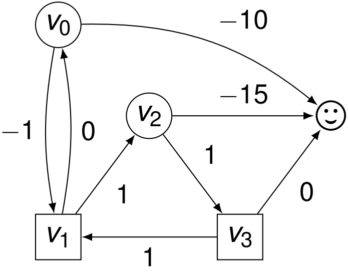
Shortest Path payoff of a play π

$$\mathbf{SP}(\pi) = \begin{cases} \sum_{i=0}^{n-1} w((\pi_i, \pi_{i+1})) & \text{if } \exists n \text{ (the smallest) s.t. } \pi_n = \text{☺} \\ +\infty & \text{if } \pi \text{ does not reach ☺} \end{cases}$$

Shortest Path Game

☐ Adam ○ Eve



Play

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$\pi = (v_i)_i \in V^\omega$ $\pi = (v_i)_i$☺

How to play?

Move a token along an edge

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$\mathbf{SP}(\pi) = 0 + (-1) + 1 + 1 + 0 = 1$

Shortest Path payoff of a play π

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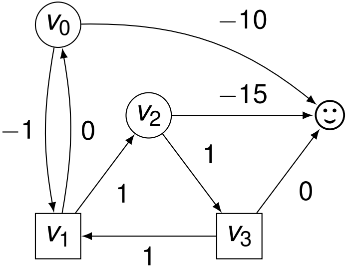

Objectives

Eve	maximise the payoff
Adam	minimise the payoff

Shortest Path payoff of a play π

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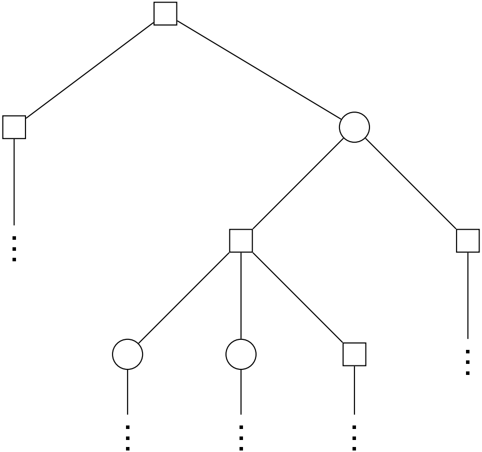
Strategies for Adam



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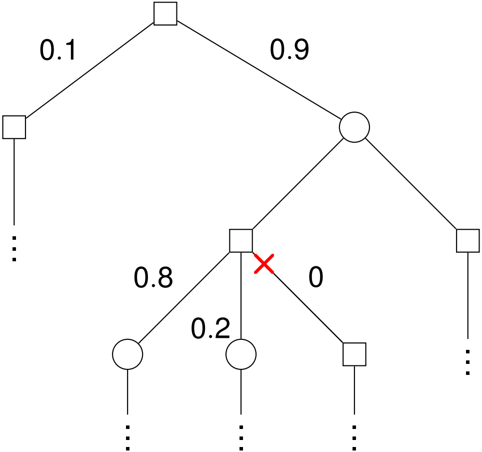


A strategy

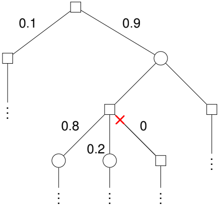
$\sigma : V^* V_{Adam} \rightarrow \Delta(V)$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Strategies for Adam

Infinite memory

$\sigma : V^* V_{Adam} \to \Delta(V)$



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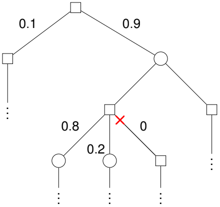
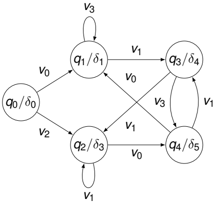


Finite memory

Moore machine



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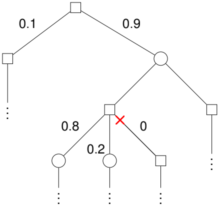

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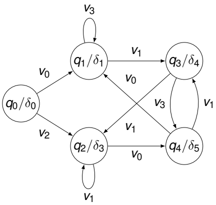
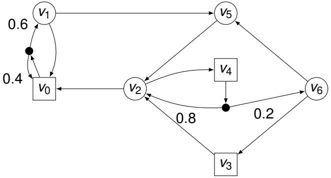


Memoryless

$\sigma : V_{Adam} \to \Delta(V)$



Strategies for Adam

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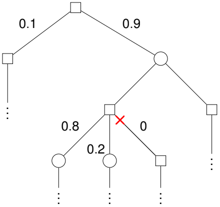

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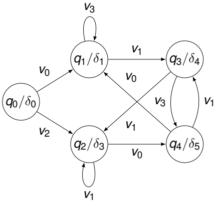

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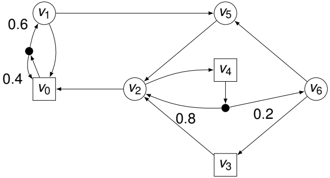
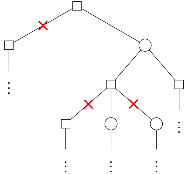


Deterministic

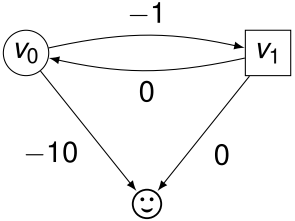
$\sigma : V^* V_{Adam} \to V$



Deterministic Strategies

$\sigma : V^* \times V_{\text{Adam}} \to V$

$\boxed{\sigma}$ Adam $\quad$ $\textcircled{\tau}$ Eve



Value

$$\overline{\text{dVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \textbf{SP}(\text{Play}(v, \sigma, \tau))}_{\text{dVal}^{\sigma}(v)}$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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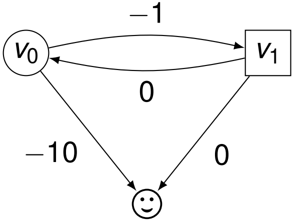

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Determinacy

$$\mathrm{dVal}(v) = \overline{\mathrm{dVal}}(v) = \underline{\mathrm{dVal}}(v)$$

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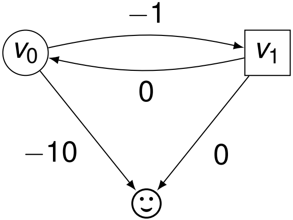

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Bellman equation

$$\mathrm{dVal}(v) = \begin{cases} 0 & \text{if } v = ☺ \\ \max_{v'} \left(w(v, v') + \mathrm{dVal}(v')\right) & \text{if } v \in V_{\text{Eve}} \\ \min_{v'} \left(w(v, v') + \mathrm{dVal}(v')\right) & \text{if } v \in V_{\text{Adam}} \end{cases}$$

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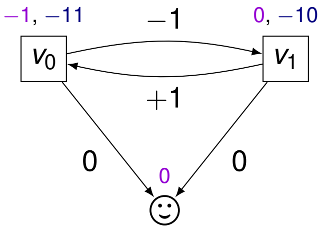

Value

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Unicity

Bellman equation may have many solutions.

Bellman equation

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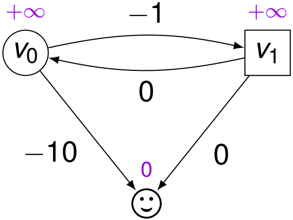

Value

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Value iteration

- Compute dVal as a greatest fixpoint

Bellman equation

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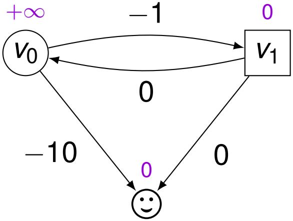

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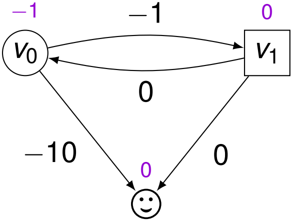

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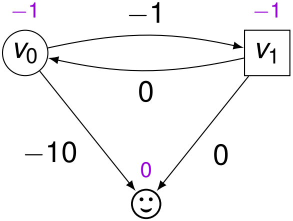

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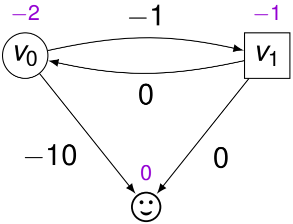

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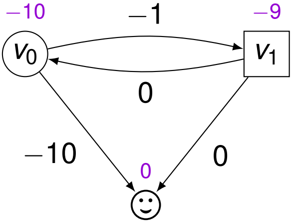

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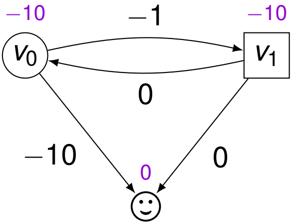

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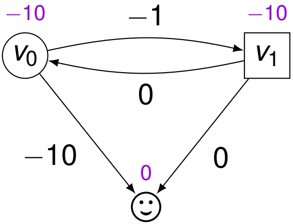

Value

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Value iteration

- Compute dVal as a greatest fixpoint
- Complexity: pseudo-polynomial

Bellman equation

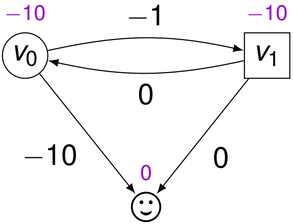
$$\mathrm{dVal}(v) = \begin{cases} 0 & \text{if } v = \text{☺} \\ \max_{v'} \left(w(v, v') + \mathrm{dVal}(v')\right) & \text{if } v \in V_{\text{Eve}} \\ \min_{v'} \left(w(v, v') + \mathrm{dVal}(v')\right) & \text{if } v \in V_{\text{Adam}} \end{cases}$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathrm{Adam}} \to V$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

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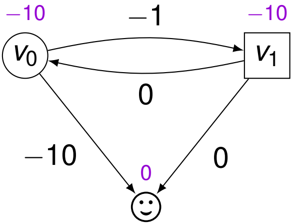

Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v,\sigma,\tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

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Deterministic Strategies

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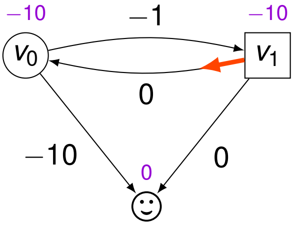
$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle

Optimal strategy

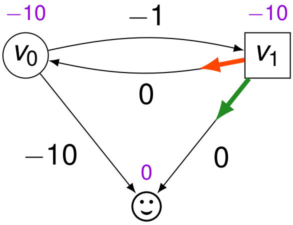
$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathrm{Adam}} \to V$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

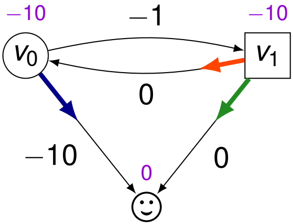
$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathrm{Adam}} \to V$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v,\sigma,\tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Optimal strategy for Eve

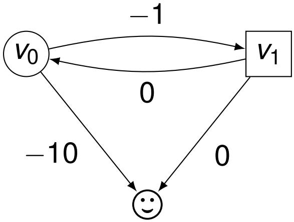
Eve has a memoryless optimal strategy.

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Memoryless strategies

$\boxed{\sigma}$ Adam $\quad$ τ Eve

$$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$$



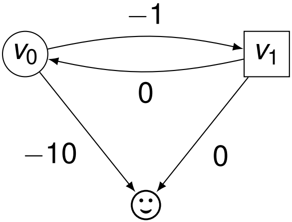
Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_{v}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\boxed{\sigma}$ Adam $\quad$ $\textcircled{\tau}$ Eve

$$\sigma : V_{\text{Adam}} \to \Delta(V)$$



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

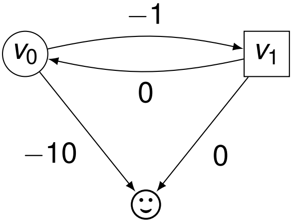
Determinacy

$$\text{mVal}(v) = \overline{\text{mVal}}(v) = \underline{\text{mVal}}(v)$$

Memoryless strategies

$\boxed{\sigma}$ Adam τ Eve

$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$



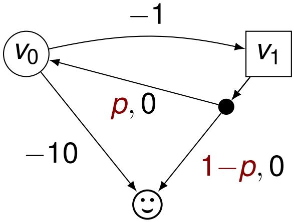
Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_{v}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



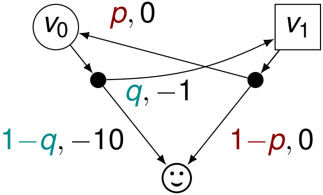
Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\boxed{\sigma}$ Adam ◯τ Eve

$\sigma : V_{\mathsf{Adam}} \to \Delta(V)$



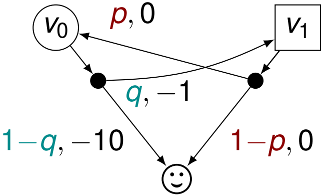
Value

$$\mathsf{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathsf{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\mathsf{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathsf{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathsf{mVal}^{\sigma}(v)}$$

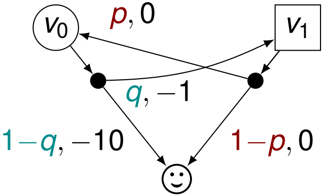
Bellman equation in a Markov Chain

$$\mathsf{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathsf{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve

$$\sigma : V_{\text{Adam}} \to \Delta(V)$$



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Unicity

A unique fixpoint : $\mathrm{mVal}^{\sigma,\tau}$

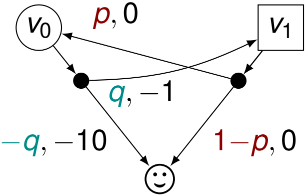
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p \times \mathrm{mVal}^{\sigma,\tau}(v_0)$$
$$\mathrm{mVal}^{\sigma,\tau}(v_0) = q(\mathrm{mVal}^{\sigma,\tau}(v_1) - 1) - 10(1 - q)$$

Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

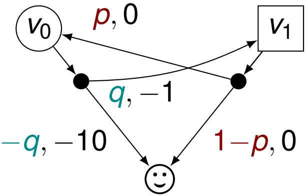
Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$



Compute mVal$^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

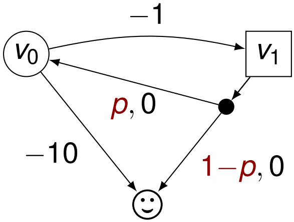
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad \textcircled{\tau}$ Eve



Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
- If $p \geqslant \frac{9}{10}$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

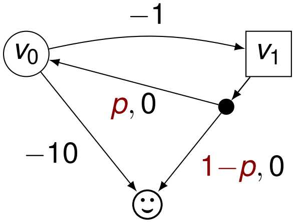
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
- If $p \geqslant \frac{9}{10}$, then $q = 0$:

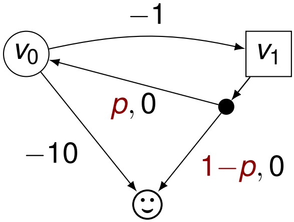
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad \textcircled{\tau}$ Eve



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$

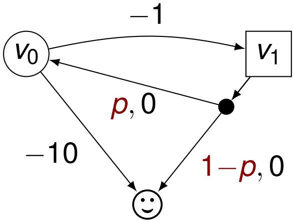
  $\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$
- If $p \geqslant \frac{9}{10}$, then $q = 0$:

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
  $$\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$$
  $$\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$
- If $p \geqslant \frac{9}{10}$, then $q = 0$:

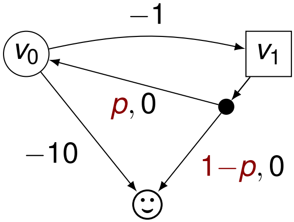
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

$\boxed{\sigma}$ Adam $\textcircled{$\tau$}$ Eve

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
  $$\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$$
  $$\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$
- If $p \geqslant \frac{9}{10}$, then $q = 0$:
  $$\text{mVal}^{\sigma}(v_1) = -10p$$
  $$\text{mVal}^{\sigma}(v_0) = -10$$

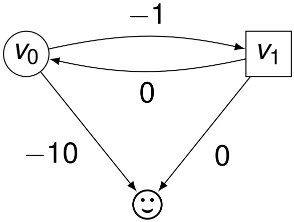
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

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- If $p < \frac{9}{10}$, then $q = 1$:

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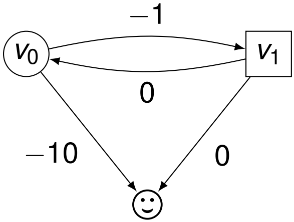
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



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$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $$\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} > -9$$

  $$\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $$\mathrm{mVal}^{\sigma}(v_1) = -10p$$

  $$\mathrm{mVal}^{\sigma}(v_0) = -10$$

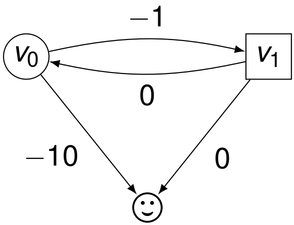
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve

$\sigma : V_{\text{Adam}} \to \Delta(V)$



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Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
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  $$\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p} > -10$$
- If $p \geqslant \frac{9}{10}$, then $q = 0$:
  $$\text{mVal}^{\sigma}(v_1) = -10p$$
  $$\text{mVal}^{\sigma}(v_0) = -10$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

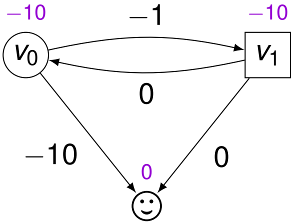
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



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Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} > -9$

  $\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p} > -10$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $\mathrm{mVal}^{\sigma}(v_1) = -10p$

  $\mathrm{mVal}^{\sigma}(v_0) = -10$

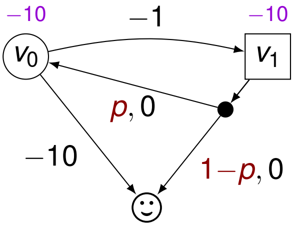
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_{v}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Value in a MDP

Computable in polynomial time

Bellman equation in a Markov Chain

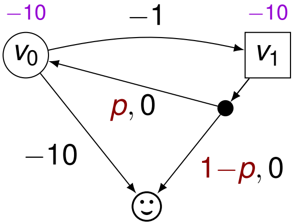
$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Value in a MDP

Computable in polynomial time

ε-optimal strategy

$$\text{mVal}^{\sigma^*}(v) \leqslant \text{mVal}(v) + \varepsilon$$

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Contribution

$$\text{dVal} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

$$\text{dVal} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives

$$\text{dVal} = \text{mVal}$$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

$$\mathrm{dVal} \quad = \quad \mathrm{mVal}$$

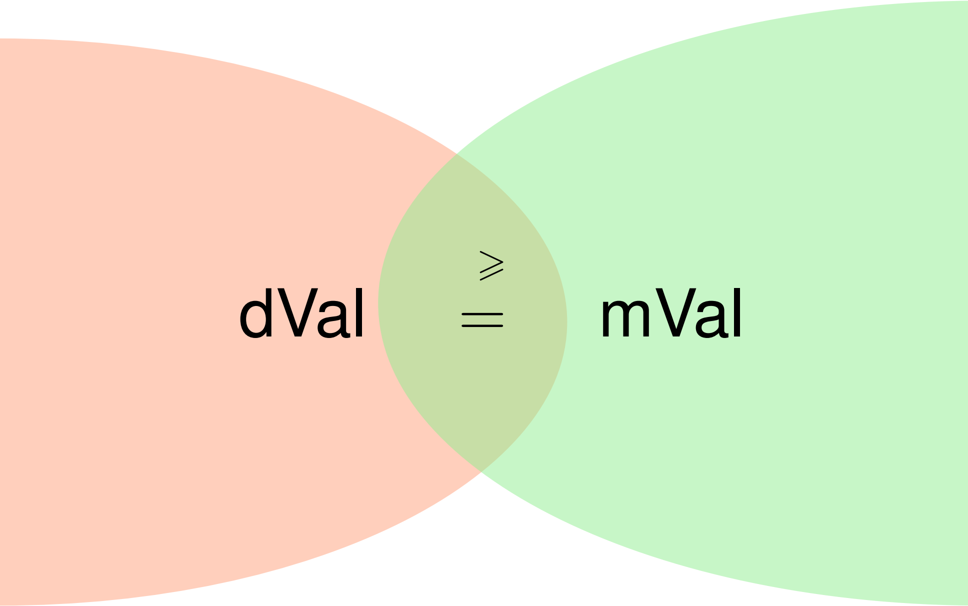
Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Contribution

dVal = mVal

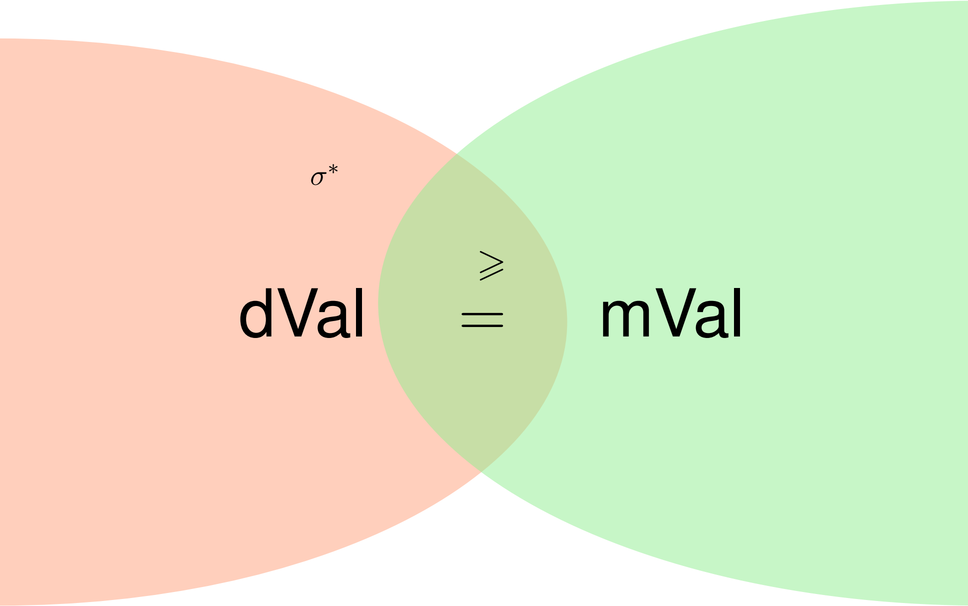
Contribution



$\geqslant$

dVal $=$ mVal

Contribution

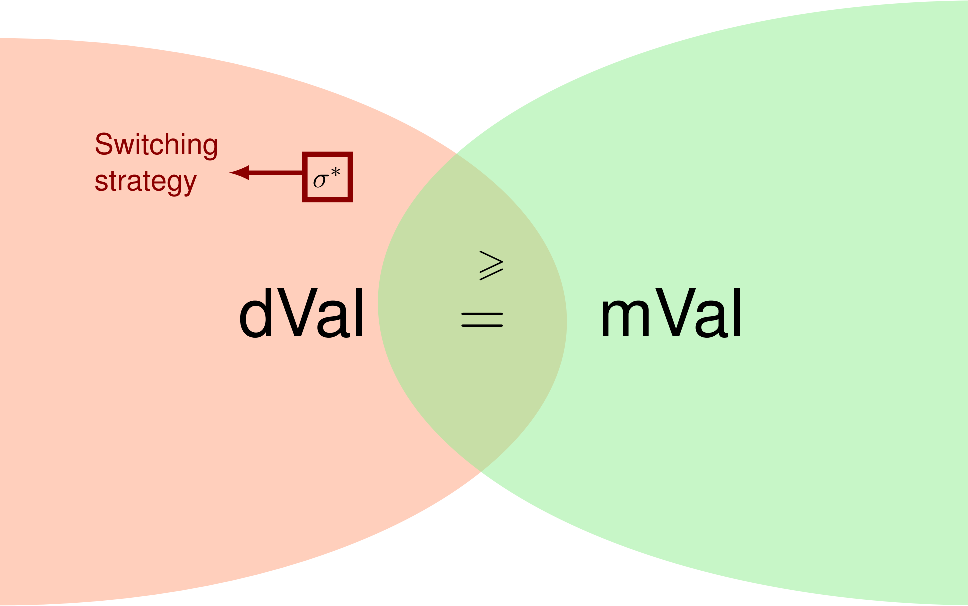


σ^*

$\geqslant$

dVal $=$ mVal

Contribution



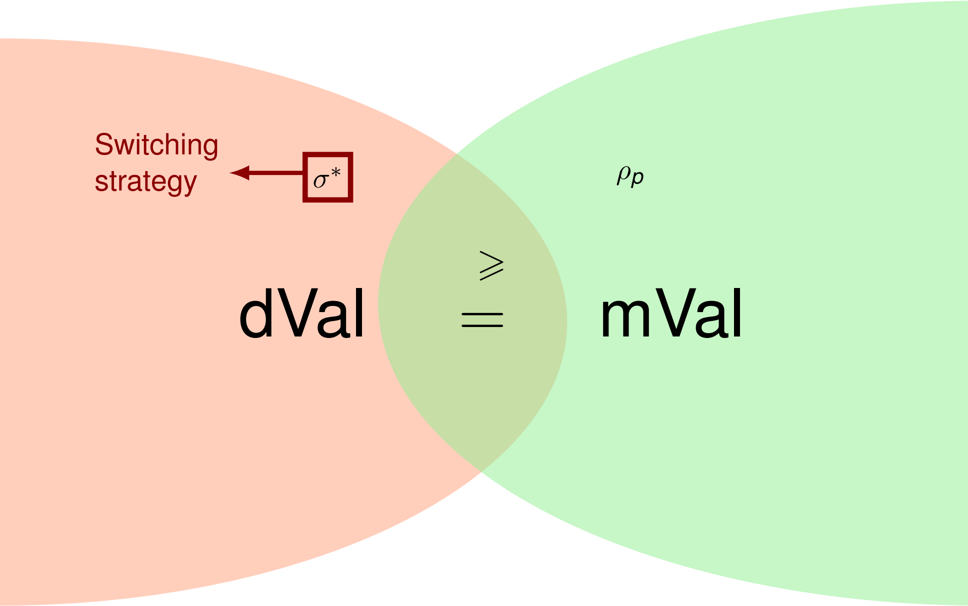
Switching strategy

$\boxed{\sigma^*}$

$\geqslant$

dVal $=$ mVal

Contribution



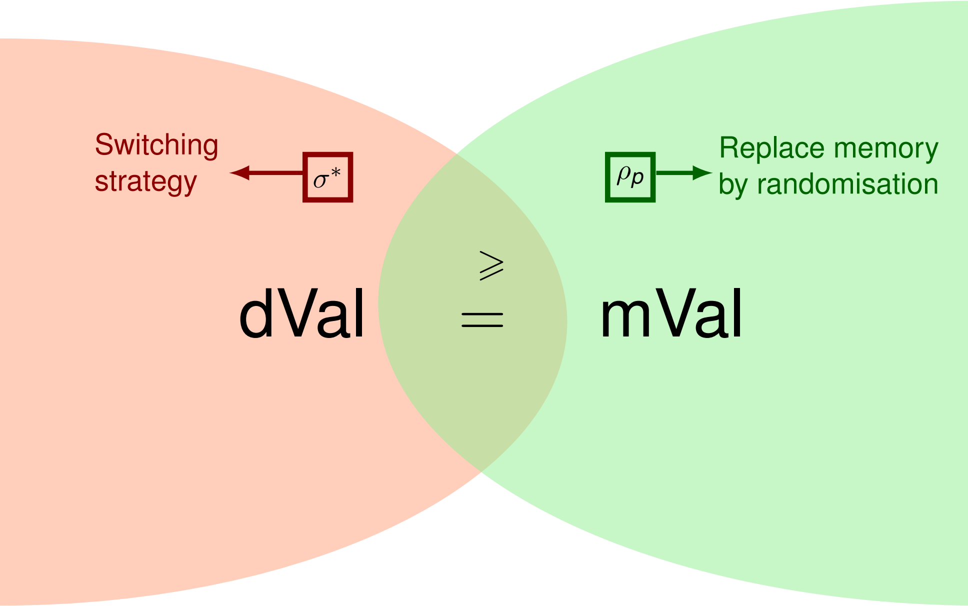
Switching strategy ← $\boxed{\sigma^*}$

ρ_p

$\geqslant$

dVal $=$ mVal

Contribution

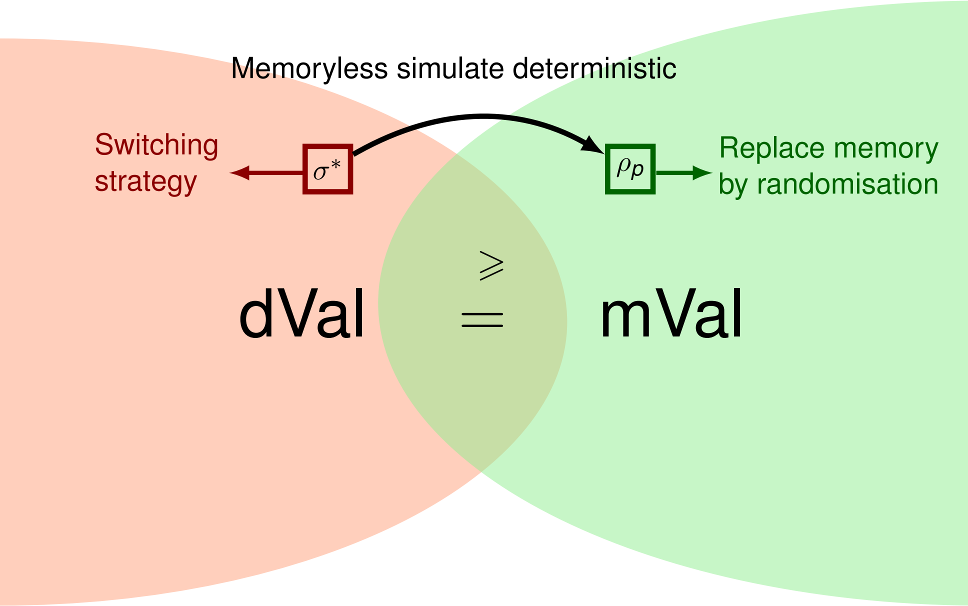


Switching strategy ← $\boxed{\sigma^*}$

$\boxed{\rho_p}$ → Replace memory by randomisation

$$\mathrm{dVal} \overset{\geqslant}{=} \mathrm{mVal}$$

Contribution

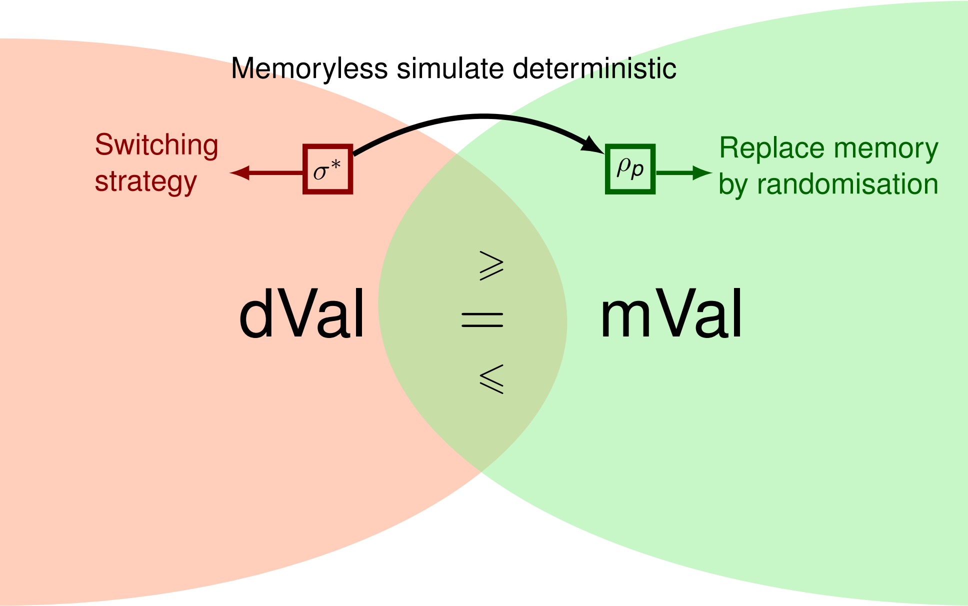


Memoryless simulate deterministic

Switching strategy ← $\boxed{\sigma^*}$ → $\boxed{\rho_p}$ → Replace memory by randomisation

dVal $\overset{\geqslant}{=}$ mVal

Contribution



Memoryless simulate deterministic

Switching strategy

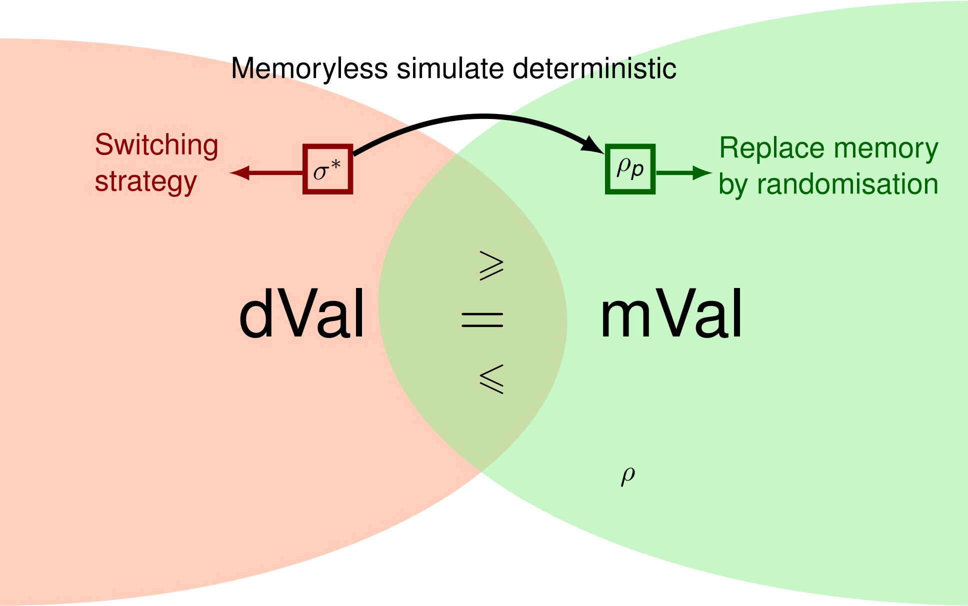
σ^*

ρ_p

Replace memory by randomisation

$$\mathrm{dVal} \overset{\geqslant}{\underset{\leqslant}{=}} \mathrm{mVal}$$

Contribution



Memoryless simulate deterministic

Switching strategy

σ^*

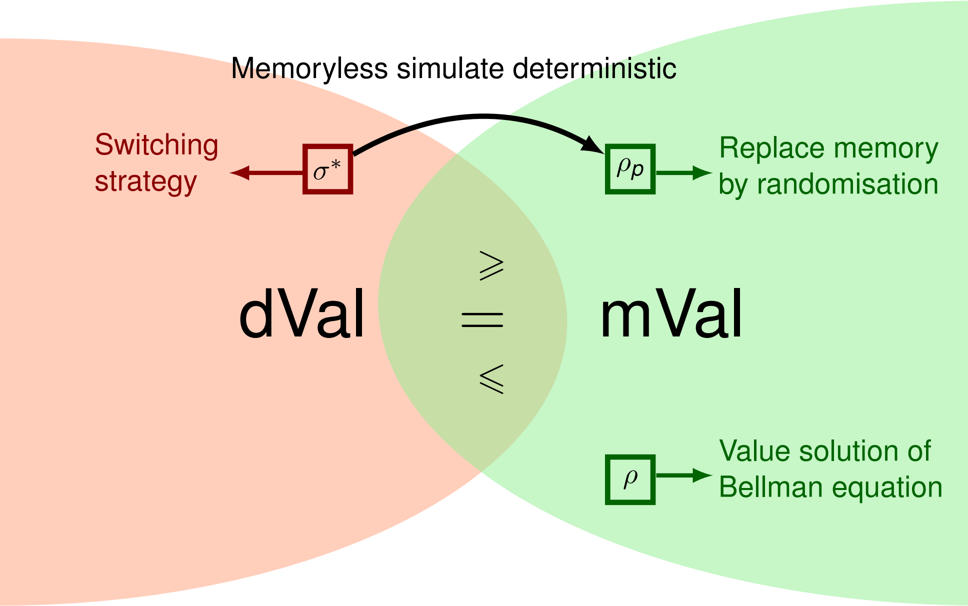
ρ_p

Replace memory by randomisation

dVal $\geqslant$ $=$ $\leqslant$ mVal

ρ

Contribution



Memoryless simulate deterministic

Switching strategy

σ^*

ρ_p

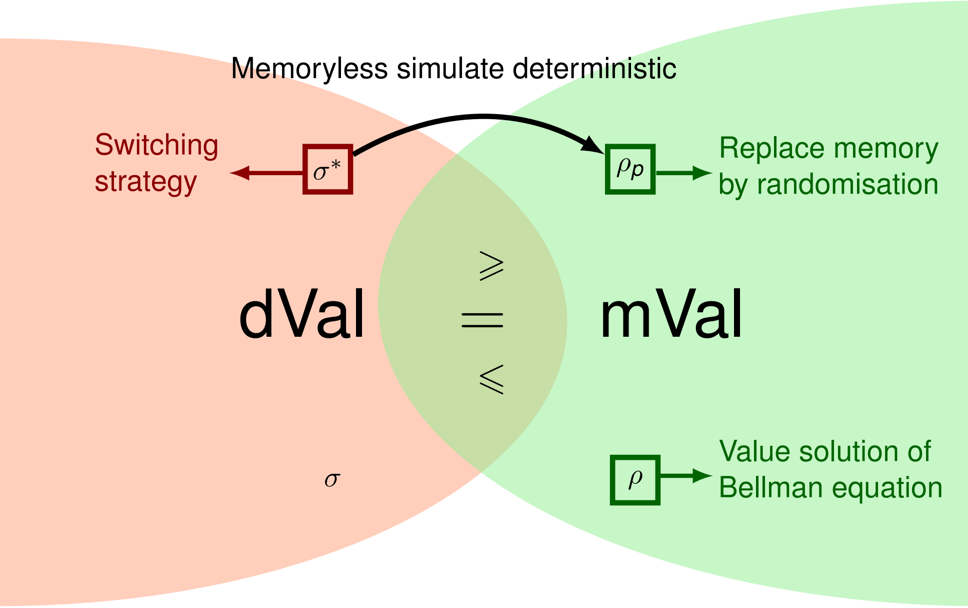
Replace memory by randomisation

$$\mathrm{dVal} \quad \begin{matrix} \geqslant \\ = \\ \leqslant \end{matrix} \quad \mathrm{mVal}$$

ρ

Value solution of Bellman equation

Contribution



Memoryless simulate deterministic

Switching strategy

σ^*

ρ_p

Replace memory by randomisation

$\geqslant$

dVal $=$ mVal

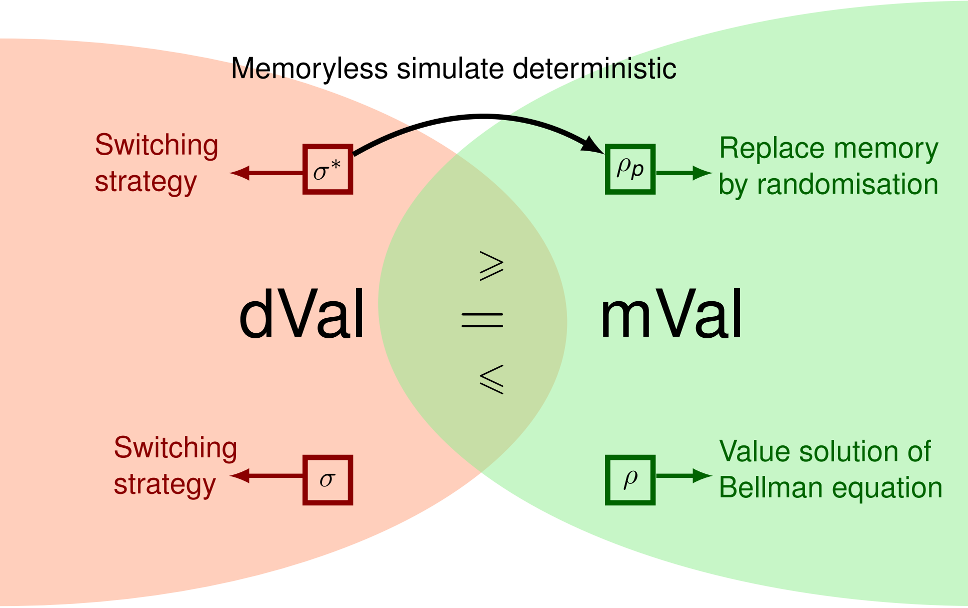
$\leqslant$

σ

ρ

Value solution of Bellman equation

Contribution



Memoryless simulate deterministic

Switching strategy ← σ^*

ρ_p → Replace memory by randomisation

$\geqslant$

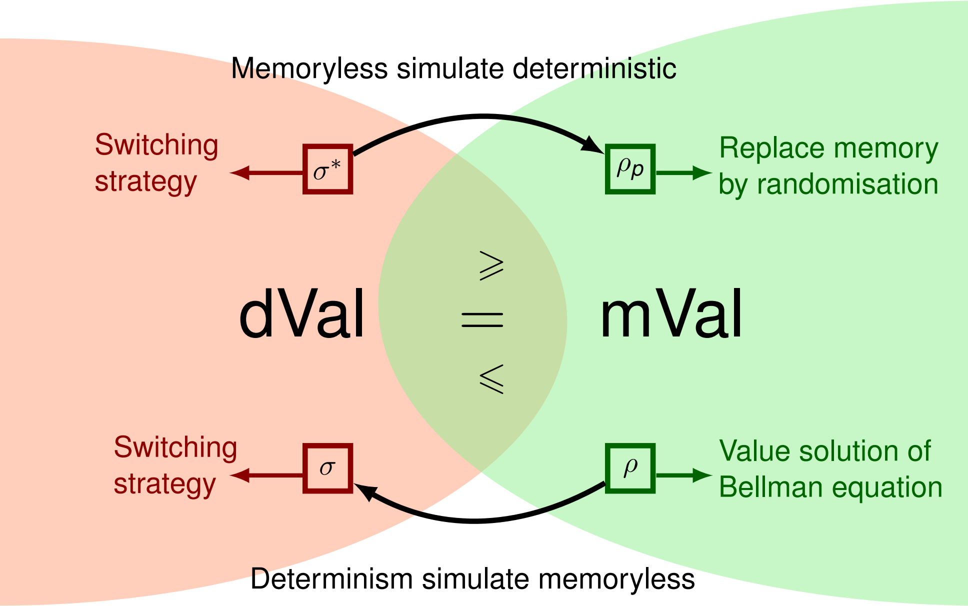
dVal $=$ mVal

$\leqslant$

Switching strategy ← σ

ρ → Value solution of Bellman equation

Contribution



Memoryless simulate deterministic

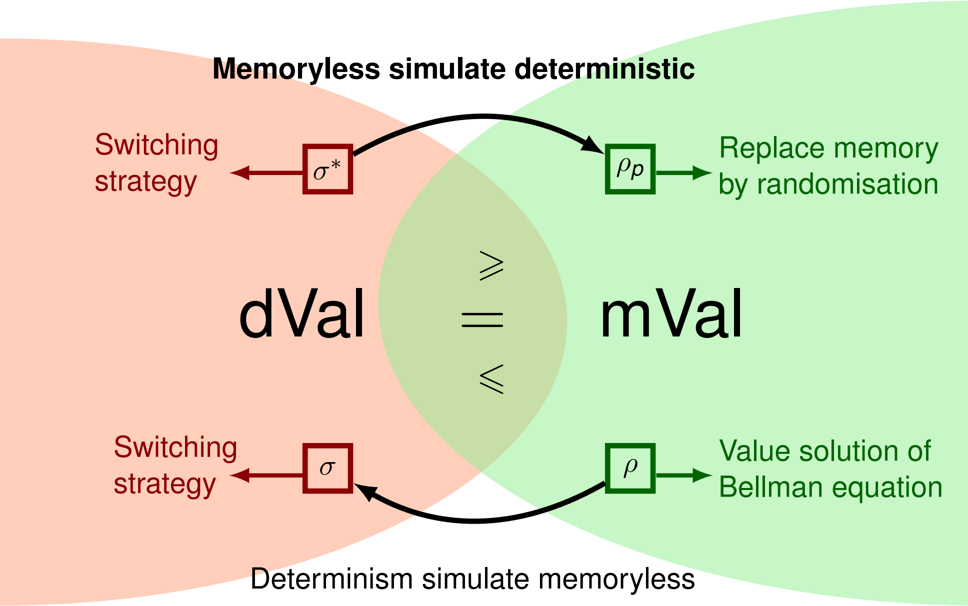
Switching strategy ← σ^* → ρ_p → Replace memory by randomisation

$$\mathrm{dVal} \overset{\geqslant}{\underset{\leqslant}{=}} \mathrm{mVal}$$

Switching strategy ← σ ← ρ → Value solution of Bellman equation

Determinism simulate memoryless

Contribution



Memoryless simulate deterministic

Switching strategy ← σ^* → ρ_p → Replace memory by randomisation

dVal $\geqslant$ $=$ $\leqslant$ mVal

Switching strategy ← σ ← ρ → Value solution of Bellman equation

Determinism simulate memoryless

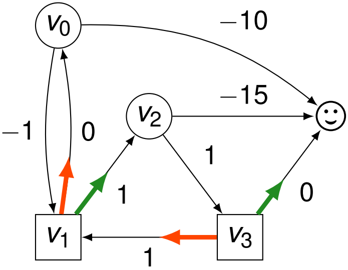
Memoryless simulate deterministic

Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Memoryless simulate deterministic

□ Adam ○ Eve



Claim

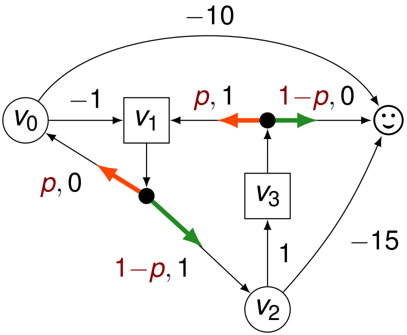
For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy,

Memoryless simulate deterministic

□ Adam ○ Eve



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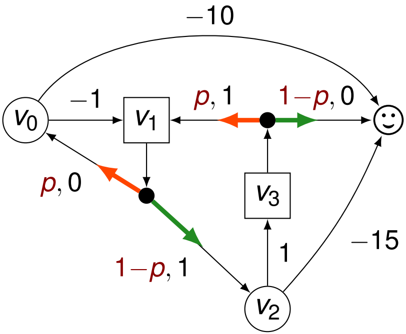
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Memoryless simulate deterministic

□ Adam ○ Eve



Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Properties of ρ_p

- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond ☺) = 1$

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy, for all $p \in (0, 1)$,

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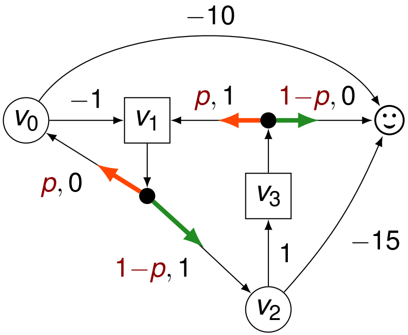
Memoryless simulate deterministic

☐ Adam ○ Eve



Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Properties of ρ_p

- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond☺) = 1$
- Eve has an optimal memoryless deterministic strategy.

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy, for all $p \in (0, 1)$,

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An analysis of stochastic shortest path problems, D. Bertsekas and J. Tsitsiklis, 1991, Mathematics of Operations Research.

Memoryless simulate deterministic

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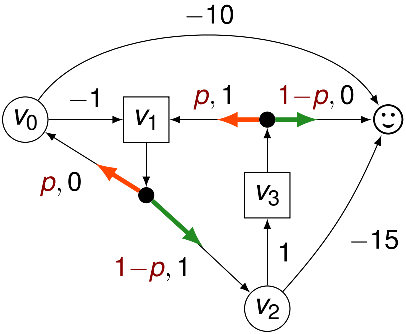

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Problem

Presence of non-negative cycles

Strategy ρ_p

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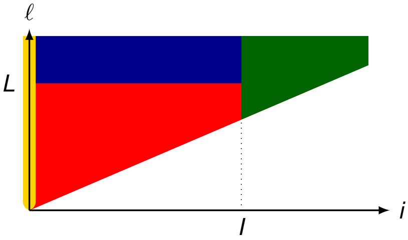
Memoryless simulate deterministic

□ Adam ○ Eve



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Problem

Presence of non-negative cycles

Tool for the proof

Control the non-negative cycles with a partition of plays

Strategy ρ_p

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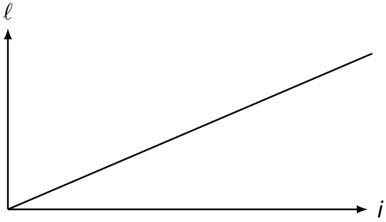
$$\rho_p = p \times \sigma_1 + (1 - p) \times \sigma_2$$

Focus on the partition of plays

ℓ size of play reaching the target

i number of non-negative cycles

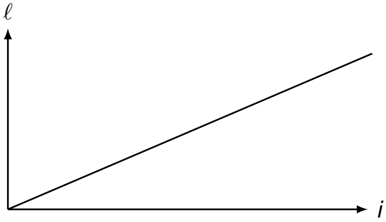


Focus on the partition of plays

ℓ size of play reaching the target

i number of non-negative cycles

Fix a deterministic strategy for Eve



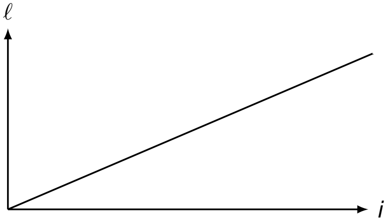
Focus on the partition of plays

Fix a deterministic strategy for Eve

ℓ size of play reaching the target
i number of non-negative cycles

Good zones

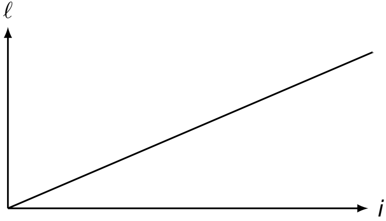
SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}($**SP**$) \leqslant$ dVal



Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

Fix a deterministic strategy for Eve



Good zones

SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}($**SP**$) \leqslant$ dVal

Zones to control

$\mathbb{E}($**SP**$) \leqslant \varepsilon$

Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

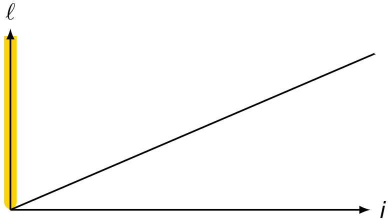
Fix a deterministic strategy for Eve

Yellow zone

All plays conforming to σ_1

Good zones

$\mathbf{SP} \leqslant \mathrm{dVal}$
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \mathrm{dVal}$



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Fix a deterministic strategy for Eve

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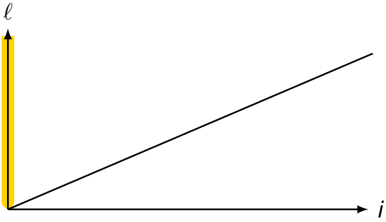
All plays conforming to σ_1

Weight of each play is $\leqslant$ dVal

Good zones

SP $\leqslant$ dVal

$\Rightarrow \mathbb{E}($**SP**$) \leqslant$ dVal



Zones to control

$\mathbb{E}($**SP**$) \leqslant \varepsilon$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Focus on the partition of plays

Fix a deterministic strategy for Eve

ℓ size of play reaching the target

i number of non-negative cycles

Yellow zone

All plays conforming to σ_1

Weight of each play is $\leqslant$ dVal

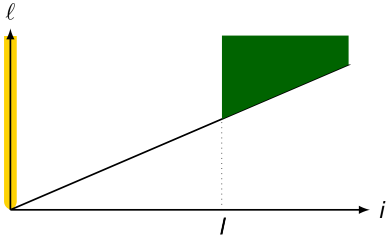
Green zone

Plays contain many non-negative cycles

Good zones

$\mathbf{SP} \leqslant \mathrm{dVal}$

$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \mathrm{dVal}$



Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

Fix a deterministic strategy for Eve

Yellow zone

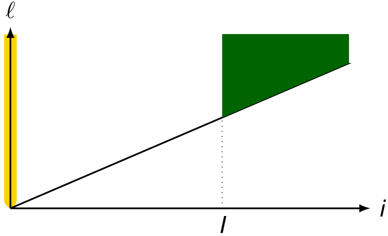
All plays conforming to σ_1
Weight of each play is $\leqslant$ dVal

Green zone

Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Good zones

SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}(\textbf{SP}) \leqslant$ dVal



Zones to control

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ℓ size of play reaching the target
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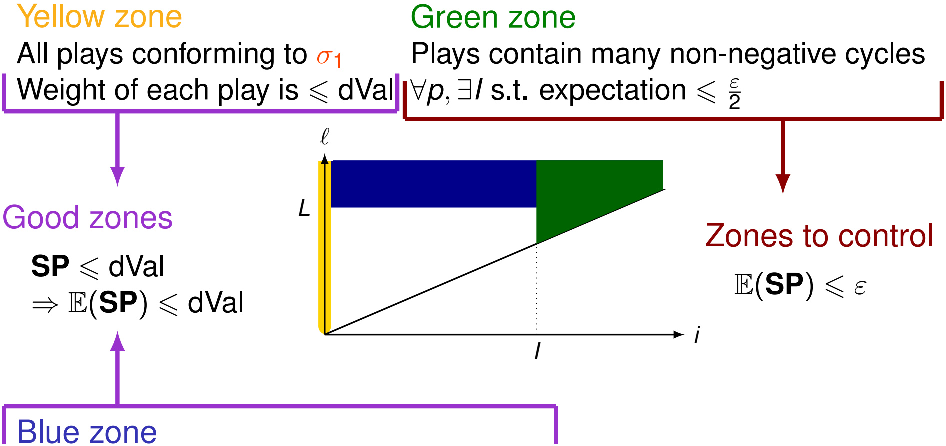
Fix a deterministic strategy for Eve



Yellow zone

All plays conforming to σ_1
Weight of each play is $\leqslant$ dVal

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Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Good zones

$\mathbf{SP} \leqslant \mathrm{dVal}$
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \mathrm{dVal}$

Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

Blue zone

Plays with many negative cycles and few non-negative cycles

Focus on the partition of plays

ℓ size of play reaching the target
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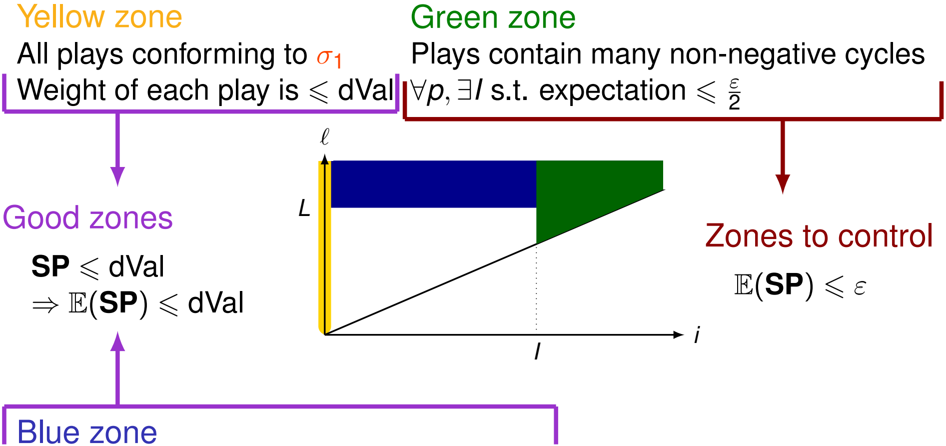
Fix a deterministic strategy for Eve



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Green zone

Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Good zones

SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}(\textbf{SP}) \leqslant$ dVal

Zones to control

$\mathbb{E}(\textbf{SP}) \leqslant \varepsilon$

Blue zone

Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Focus on the partition of plays

ℓ size of play reaching the target
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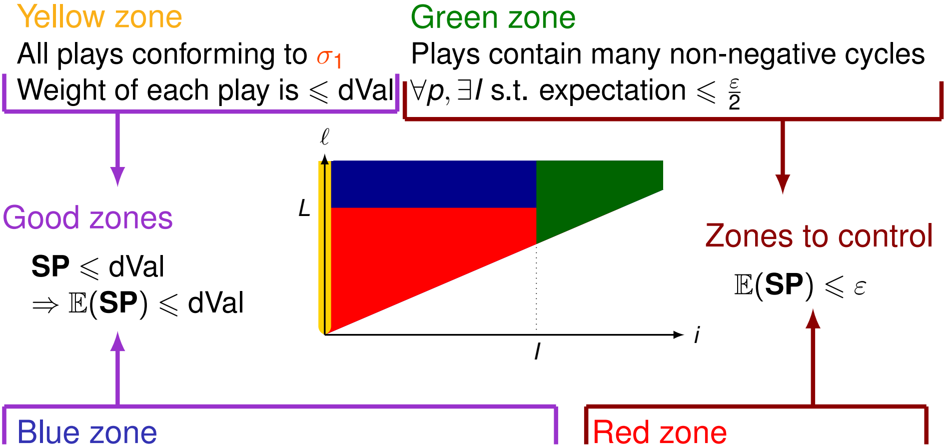
Fix a deterministic strategy for Eve



Yellow zone

All plays conforming to σ_1
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Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Good zones

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$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant$ dVal

Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

Blue zone

Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone

Rest of plays

Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

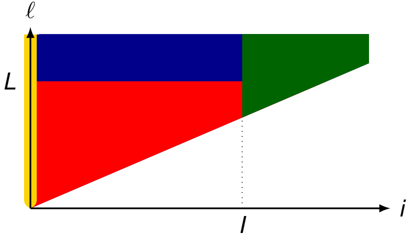
Fix a deterministic strategy for Eve

Yellow zone
All plays conforming to σ_1
Weight of each play is $\leqslant$ dVal

Green zone
Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Good zones

$\mathbf{SP} \leqslant$ dVal
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant$ dVal



Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

Blue zone
Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone
Rest of plays
$\exists p$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Results on shortest path games

Contributions

1. Adam has the same hope using memory or randomness.
2. Existence of an optimal memoryless strategy for Adam is testable in polynomial time.

Results on shortest path games

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Perspectives

- Extension to probabilistic value (memory and randomisation)
- Extension to weighted timed games
- A polynomial-time algorithm to compute the value

Results on shortest path games

Contributions

1. Adam has the same hope using memory or randomness.
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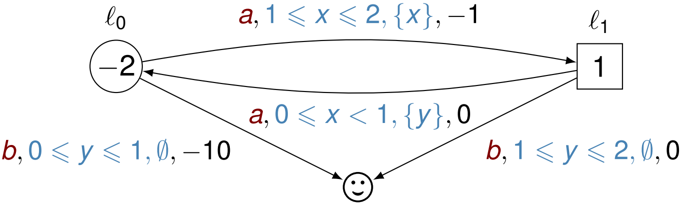
Perspectives

- Extension to probabilistic value (memory and randomisation)
- **Extension to weighted timed games**
- A polynomial-time algorithm to compute the value

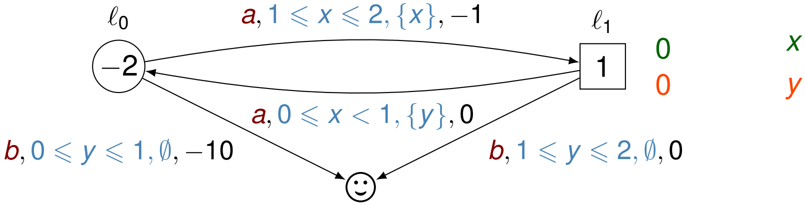
Weighted timed games

□ Adam ○ Eve



Weighted timed games

☐ Adam ○ Eve

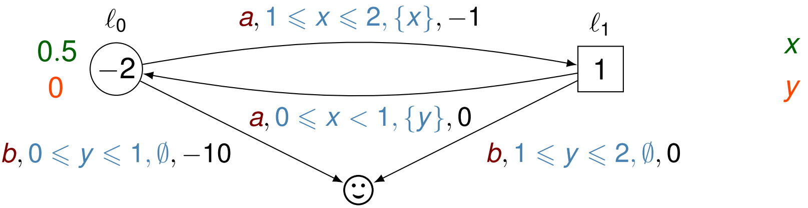


Play

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix})$$

Weighted timed games

☐ Adam ○ Eve

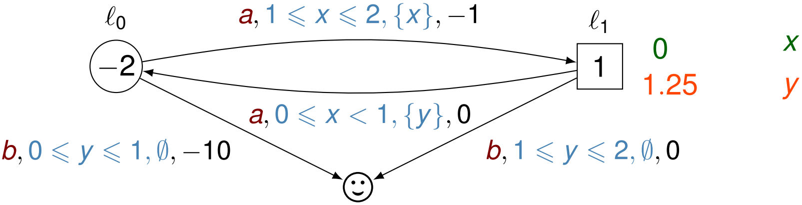


Play

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix})$$

$$1 \times 0.5 + 0$$

Weighted timed games

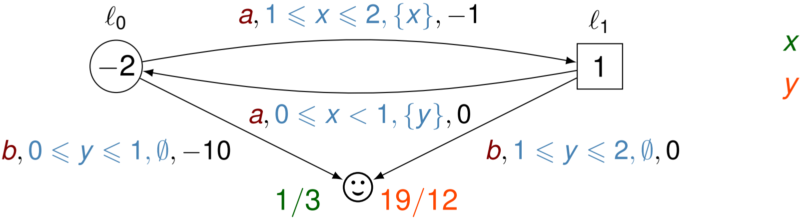


Play

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow[1 \times 0.5 + 0]{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow[-2 \times 1.25 - 1]{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix})$$

Weighted timed games

☐ Adam ○ Eve



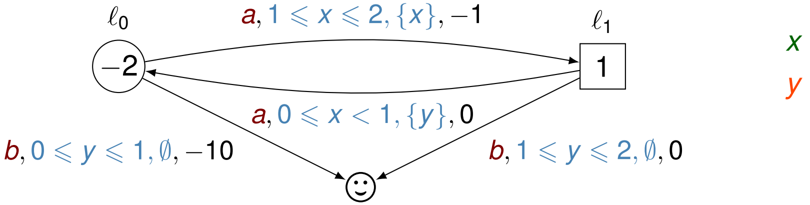
Play

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (\smiley, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \frac{1}{3} + 0$$

Weighted timed games

□ Adam ○ Eve



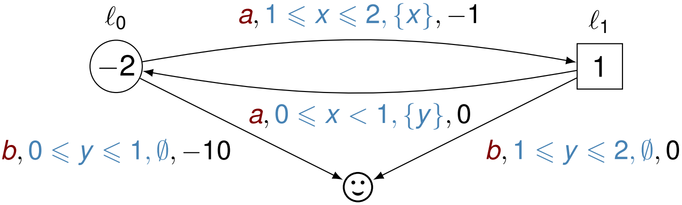
Play

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (\text{☺}, \begin{pmatrix}1/3\\19/12\end{pmatrix}) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \frac{1}{3} + 0$$

Weighted timed games

☐ Adam ○ Eve



Value problem

The value problem, i.e. deciding if $\mathrm{dVal}(\ell_0, \nu_0) \leqslant c$ with $c \in \mathbb{Z}$, is undecidable.

On Optimal Timed Strategies, T.Brihaye, V.Bruyère and J.-F. Raskin, 2005, FORMATS.

On the value problem in weighted timed games, P. Bouyer, S. Jaziri, and N. Markey, 2015, CONCUR.

Our objective

$$\mathrm{dVal} \stackrel{?}{=} \mathrm{mVal}$$

Our objective

Define probabilistic strategy

$$\mathrm{dVal} \overset{?}{=} \mathrm{mVal}$$

Our objective

Define probabilistic strategy

$$\mathrm{dVal} \overset{?}{=} \mathrm{mVal}$$

Fixpoint of a Bellman equation

Our objective

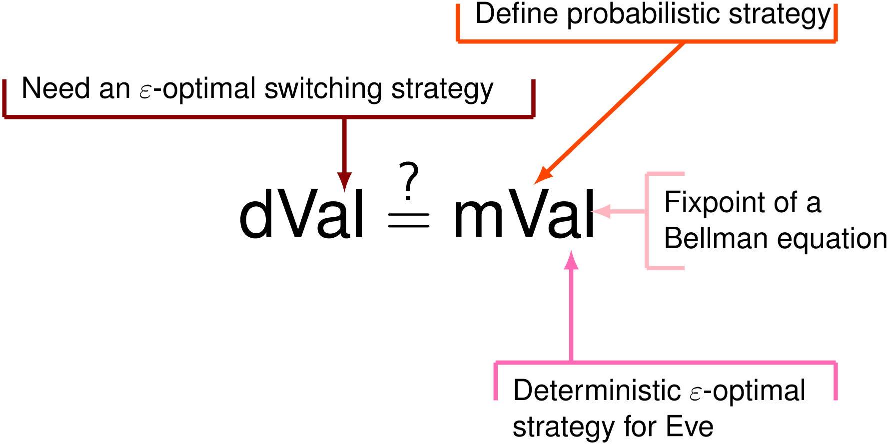
Define probabilistic strategy

$$\text{dVal} \overset{?}{=} \text{mVal}$$

Fixpoint of a
Bellman equation

Deterministic ε-optimal
strategy for Eve

Our objective



Define probabilistic strategy

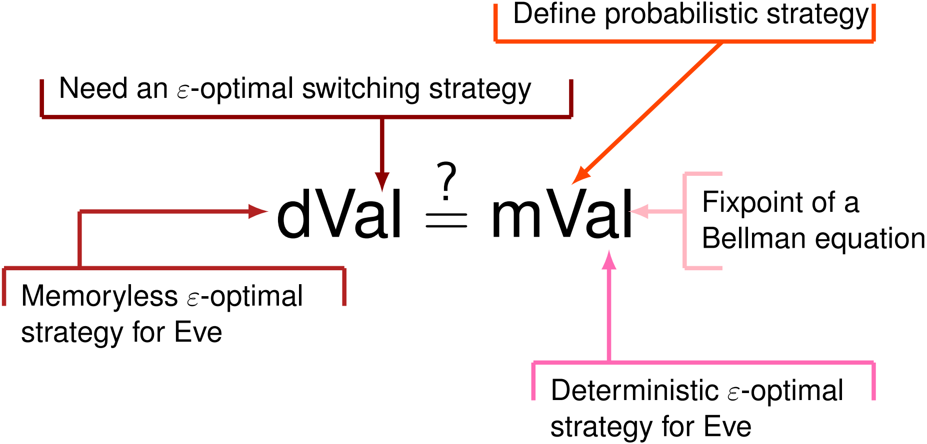
Need an ε-optimal switching strategy

$$\text{dVal} \stackrel{?}{=} \text{mVal}$$

Fixpoint of a
Bellman equation

Deterministic ε-optimal
strategy for Eve

Our objective



Define probabilistic strategy

Need an ε-optimal switching strategy

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Memoryless ε-optimal
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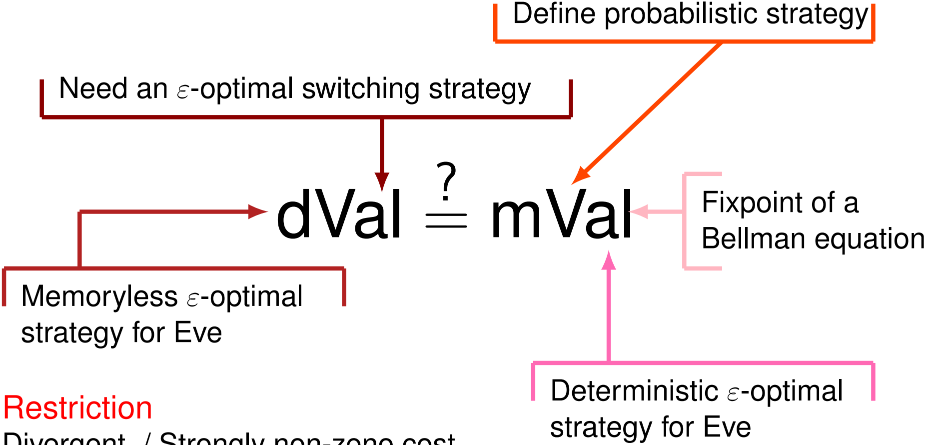

Define probabilistic strategy

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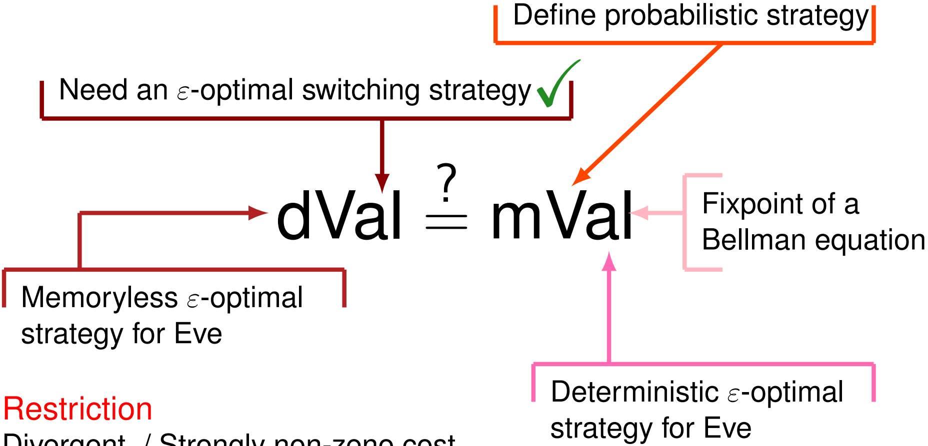
Restriction

Divergent / Strongly non-zeno cost

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury and K. Larsen, 2004, FSTTCS

Our objective



Define probabilistic strategy

Need an ε-optimal switching strategy ✓

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Fixpoint of a
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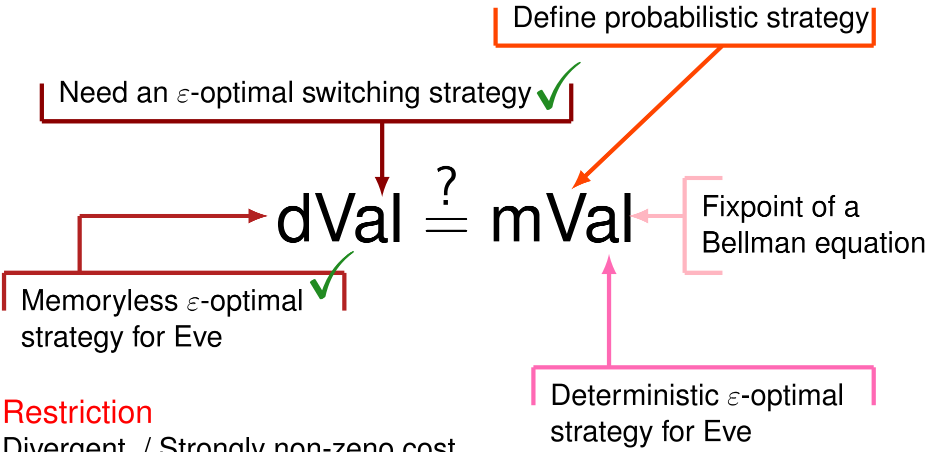

Define probabilistic strategy

Need an ε-optimal switching strategy ✓

$$\text{dVal} \overset{?}{=} \text{mVal}$$

Fixpoint of a
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Memoryless ε-optimal ✓
strategy for Eve

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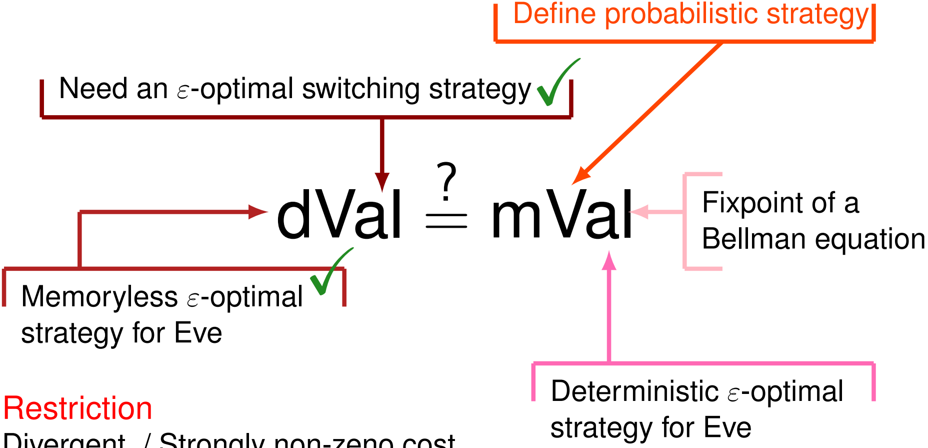

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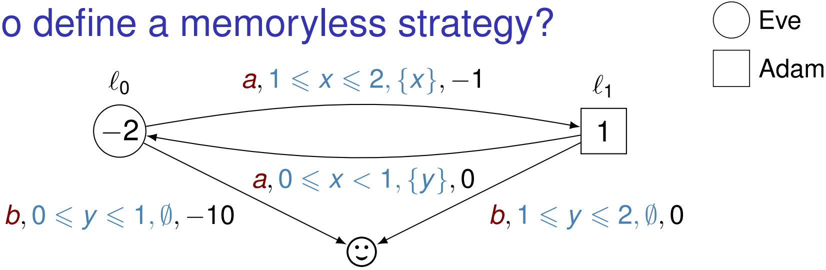
How to define a memoryless strategy?



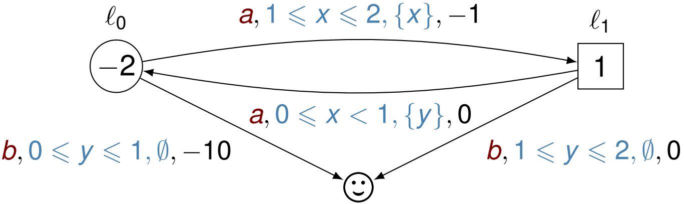
Deterministic strategy

Choose a transition and a delay

How to define a memoryless strategy?

○ Eve

□ Adam



Deterministic strategy

Choose a transition and a delay

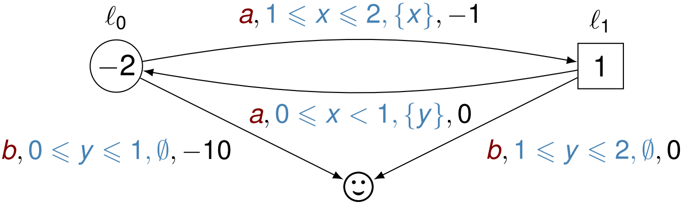
In $(\ell_1, (0, 0))$

Choose between *a* or *b*

How to define a memoryless strategy?

○ Eve

□ Adam



Deterministic strategy

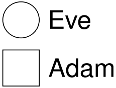
Choose a transition and a delay

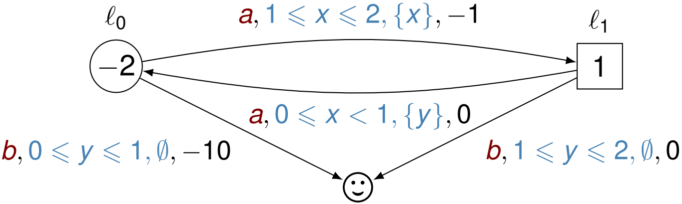
In $(\ell_1, (0, 0))$

Choose between *a* or *b*

- *a* : choose *t* with $t < 1$

How to define a memoryless strategy?





Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between *a* or *b*

- *a* : choose *t* with $t < 1$
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How to define a memoryless strategy?

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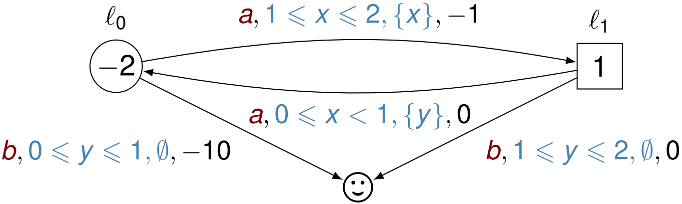


Probabilistic strategy

Distribution over possible choices

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0,0))$

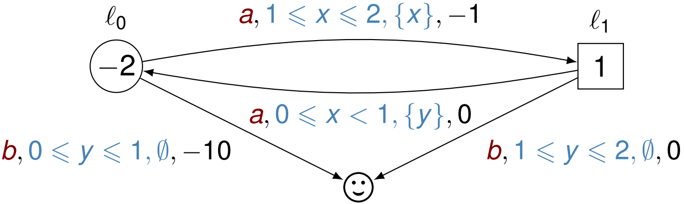
Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?

○ Eve

□ Adam



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

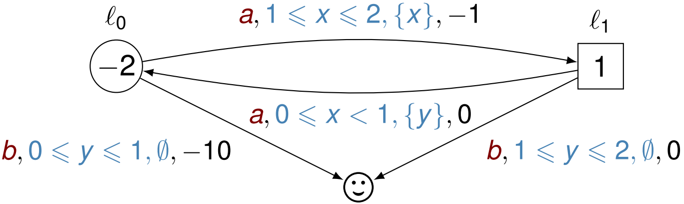
Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?

○ Eve

□ Adam



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

Deterministic strategy

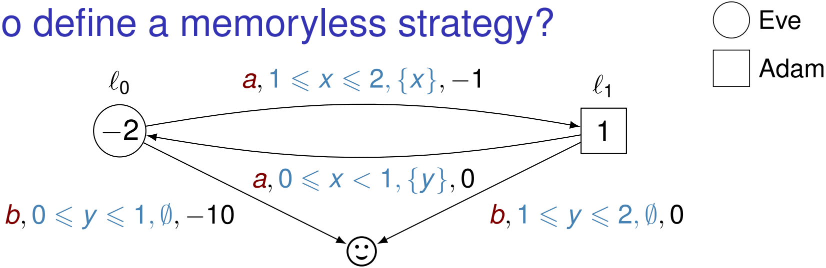
Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition *a* : finite distribution
2. Delay for *a* : infinite distribution

In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(p)$

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between *a* or *b*

- *a* : choose *t* with $t < 1$
- *b* : choose *t* with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?

○ Eve

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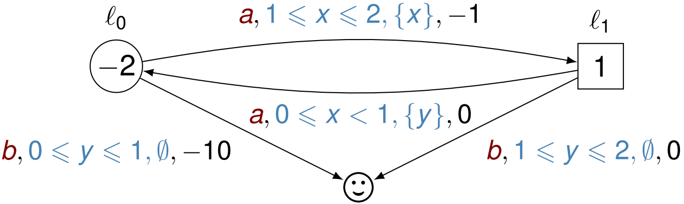

Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

- a : choose t with $\mathcal{U}([0, 1[)$

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?

○ Eve

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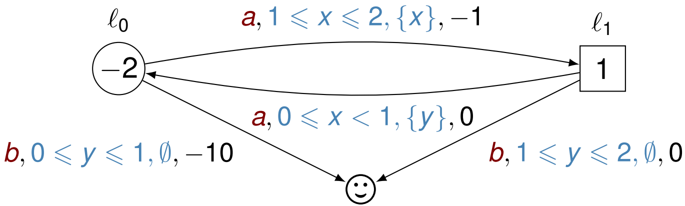


Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

- a : choose t with $\mathcal{U}([0, 1[)$
- b : choose t with $\delta_{1.5}$

Deterministic strategy

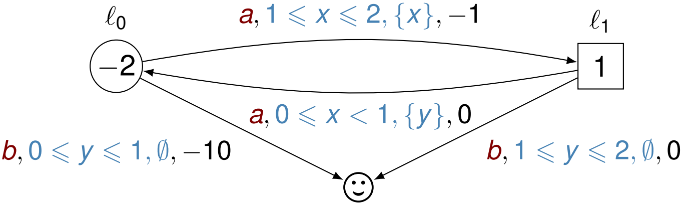
Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition *a* : finite distribution
2. Delay for *a* : infinite distribution

Deterministic strategy

Choose a transition and a delay

Close related work

Stochastic timed automata

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Memoryless value

$$\overline{\mathsf{mVal}}(\ell, \nu) = \inf_{\sigma} \sup_{\tau} \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$$

Memoryless value

$$\overline{\mathsf{mVal}}(\ell,\nu) = \inf_{\sigma}\sup_{\tau} \underbrace{\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})}_{\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})}$$

Memoryless value

$$\overline{\mathsf{mVal}}(\ell, \nu) = \inf_{\sigma} \sup_{\tau} \underbrace{\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})}_{\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})}$$

Cylinder

$\mathcal{C} = ((\ell, \nu), e_1 \ldots e_n) =$
$\{(t_1, \ldots, t_n) \mid (\ell, \nu) \xrightarrow{t_1, e_1} \ldots \xrightarrow{t_n, e_n}\}$

Memoryless value

○ Eve

□ Adam

$$\overline{\mathsf{mVal}}(\ell,\nu) = \inf_{\sigma} \sup_{\tau} \underbrace{\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})}$$

$$\sum_{\boxed{\mathcal{C}}} \mathbb{P}(\mathcal{C})\ \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Cylinder

$\mathcal{C} = ((\ell,\nu), e_1 \dots e_n) =$

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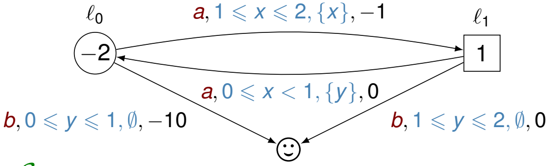

Example of cylinder $\mathcal{C}$

$$\mathcal{C} \quad = \quad ((\ell_1,(0,0)), a\ b) \quad =$$

Memoryless value

○ Eve

□ Adam

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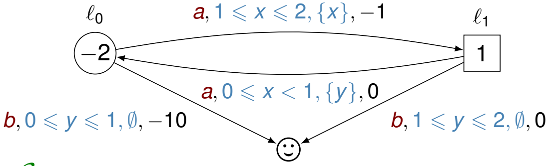
$$\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\ \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Cylinder

$$\mathcal{C} = ((\ell, \nu), e_1 \dots e_n) = \{(t_1, \dots, t_n) \mid (\ell, \nu) \xrightarrow{t_1, e_1} \dots \xrightarrow{t_n, e_n}\}$$



Example of cylinder $\mathcal{C}$

$$\mathcal{C} = ((\ell_1, (0,0)), a\ b) = \{(t_1, t_2) \mid (\ell_1, (0,0)) \xrightarrow{t_1, a} \xrightarrow{t_2, b}\}$$

Memoryless value

○ Eve

□ Adam

$$\overline{\mathrm{mVal}}(\ell,\nu) = \inf_{\sigma} \sup_{\tau} \underbrace{\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})}_{\downarrow}$$

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Cylinder

$\mathcal{C} = ((\ell,\nu), e_1 \dots e_n) =$
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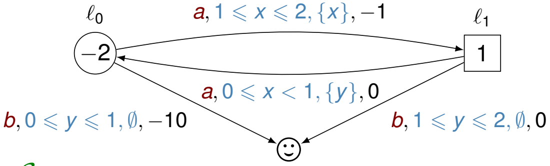

Example of cylinder $\mathcal{C}$

$$\begin{aligned}\mathcal{C} &= ((\ell_1,(0,0)), a\, b) = \{(t_1,t_2) \mid (\ell_1,(0,0)) \xrightarrow{t_1,a}\xrightarrow{t_2,b}\} \\ &= \{(\ell_1, \underset{\substack{\downarrow \\ (0,0)}}{\nu_0}) \xrightarrow{t_1,a} (\ell_0, \underset{\substack{\downarrow \\ (t_1,0)}}{\nu_1}) \xrightarrow{t_2,b} (☺, \underset{\substack{\downarrow \\ (t_1+t_2,t_2)}}{\nu_2}) \mid t_1 < 1, t_2 \leqslant 1\}\end{aligned}$$

Memoryless value

$$\overline{\mathsf{mVal}}(\ell, \nu) = \inf_{\sigma} \sup_{\tau} \underbrace{\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})}$$

Cylinder

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$$\sum_{\mathcal{C}} \underbrace{\mathbb{P}(\mathcal{C})}\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu), e_1)} p_{(\ell,\nu)}(e_1)\, \mathbb{P}(\mathcal{C}_1)\, d\mu_{(\ell,\nu),e_1}(t_1)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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- Probability to choose e_1 in (ℓ, ν)

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- Probability to choose e_1 in (ℓ,ν)
- Interval of possible delays to pass through e_1 from (ℓ,ν)

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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$$\sum_{\mathcal{C}} \underbrace{\mathbb{P}(\mathcal{C})}\,\mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu),e_1)} p_{(\ell,\nu)}(e_1)\, \mathbb{P}(\mathcal{C}_1)\, d\mu_{(\ell,\nu),e_1}(t_1)$$

- Probability to choose e_1 in (ℓ,ν)
- Interval of possible delays to pass through e_1 from (ℓ,ν)
- Probability to choose a delay for e_1 from (ℓ,ν)

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$$\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Probability of a cylinder

$$((\ell_1, \nu_1), e_2 \dots e_n)$$

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu),e_1)} p_{(\ell,\nu)}(e_1)\, \mathbb{P}(\mathcal{C}_1)\, d\mu_{(\ell,\nu),e_1}(t_1)$$

- Probability to choose e_1 in (ℓ, ν)
- Interval of possible delays to pass through e_1 from (ℓ, ν)
- Probability to choose a delay for e_1 from (ℓ, ν)

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Memoryless value

$$\overline{\mathsf{mVal}}(\ell,\nu) = \inf_{\sigma} \sup_{\tau} \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$$

Cylinder

$$\mathcal{C} = ((\ell,\nu), e_1 \dots e_n) = \{(t_1,\dots,t_n) \mid (\ell,\nu) \xrightarrow{t_1,e_1} \dots \xrightarrow{t_n,e_n}\}$$

$$\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Probability of a cylinder

$$((\ell_1,\nu_1), e_2 \dots e_n)$$

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu),e_1)} p_{(\ell,\nu)}(e_1)\, \mathbb{P}(\mathcal{C}_1)\, d\mu_{(\ell,\nu),e_1}(t_1)$$

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- $(\ell,\nu) \xrightarrow{t_1,e_1} (\ell_1,\nu_1)$

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$\{(t_1,\ldots,t_n) \mid (\ell,\nu) \xrightarrow{t_1,e_1} \ldots \xrightarrow{t_n,e_n}\}$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu),e_1)} p_{(\ell,\nu)}(e_1)\,\overbrace{\mathbb{P}(\mathcal{C}_1)}^{((\ell_1,\nu_1),e_2\ldots e_n)}\,d\mu_{(\ell,\nu),e_1}(t_1)$$

- Probability to choose e_1 in (ℓ,ν)
- Interval of possible delays to pass through e_1 from (ℓ,ν)
- Probability to choose a delay for e_1 from (ℓ,ν)
- $(\ell,\nu) \xrightarrow{t_1,e_1} (\ell_1,\nu_1)$
- n-dimensional integral

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1) \overbrace{\left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \dots\right)}^{\mathbb{P}(\mathcal{C}_1)} d\mu_{(\ell,\nu),e_1}(t_1)$$

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Conditional expectation

$$\mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)(w(\ell)t_1 + w(e_1) + \mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}_1)) d\mu_{(\ell,\nu),e_1}(t_1)$$

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Existence and integrability ?

Conditional expectation

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Existence and integrability ?

No restriction on Eve's strategies

Conditional expectation

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Cylinder

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Existence and integrability ?

No restriction on Eve's strategies
Restriction on Adam's strategies

Convergence ?

- $\mathbb{P}(\mathcal{C}) \sim \alpha^{-n}$
- $\mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C}) \sim kn$

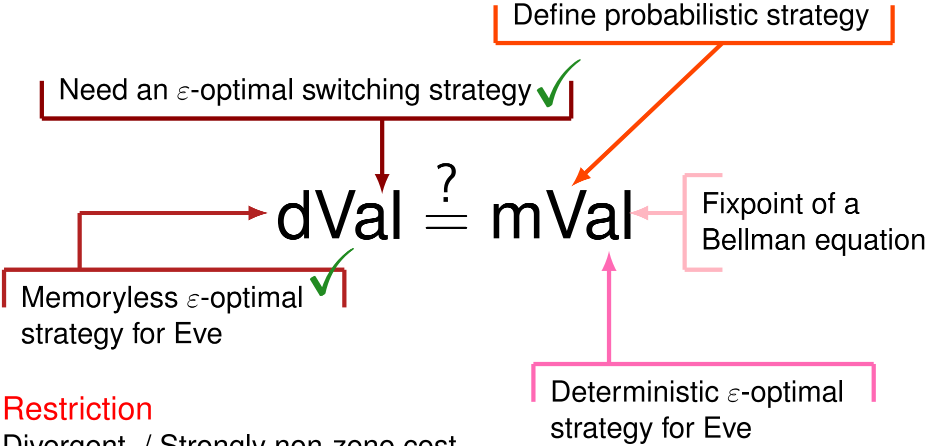
Conditional expectation

$$\mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)(w(\ell)t_1 + w(e_1) + \mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}_1))d\mu_{(\ell,\nu),e_1}(t_1)$$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1) \overbrace{\left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \ldots\right)}^{\mathbb{P}(\mathcal{C}_1)} d\mu_{(\ell,\nu),e_1}(t_1)$$

To conclude



Define probabilistic strategy

Need an ε-optimal switching strategy ✓

$$\mathrm{dVal} \overset{?}{=} \mathrm{mVal}$$

Fixpoint of a
Bellman equation

Memoryless ε-optimal ✓
strategy for Eve

Deterministic ε-optimal
strategy for Eve

Restriction

Divergent / Strongly non-zeno cost

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury and K. Larsen, 2004, FSTTCS

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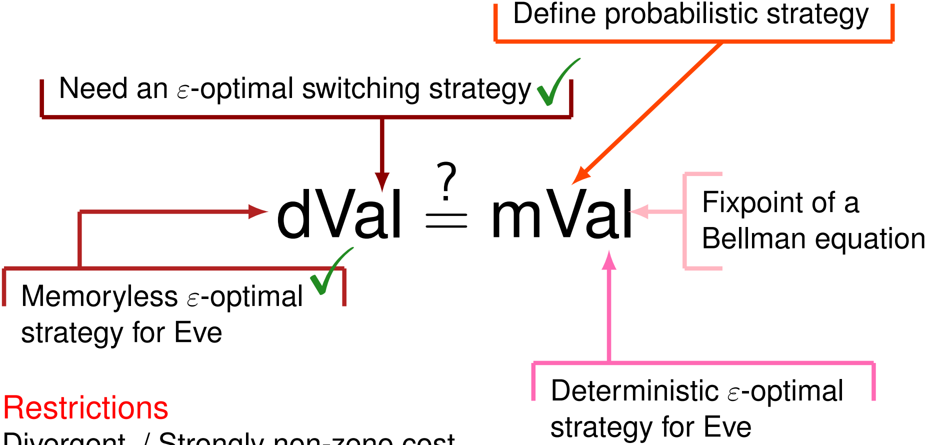

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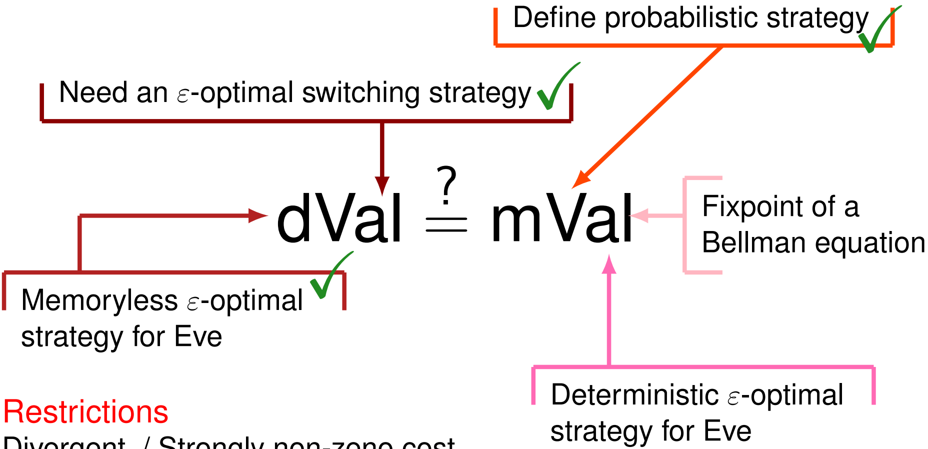

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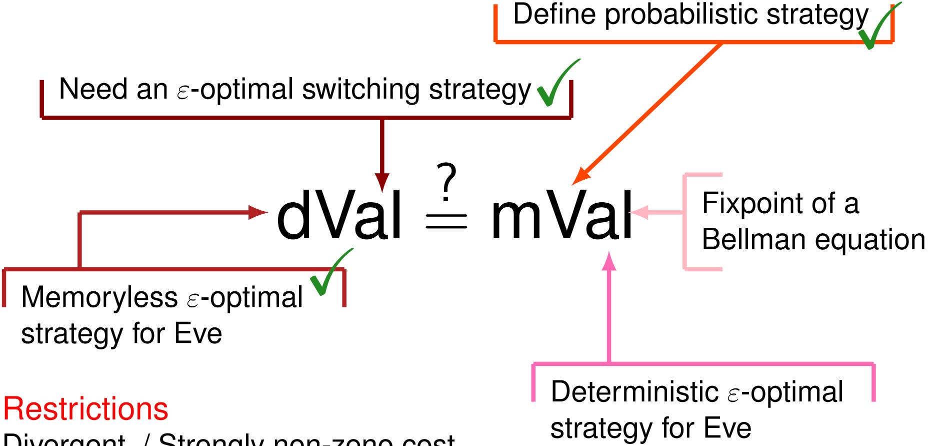

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Thank you! Questions?