Automatic modal analysis: reality or myth?

Jenny Lau, Jeroen Lanslots, Bart Peeters, Herman Van der Auweraer

LMS International, Interleuvenlaan 68, B-3001 Leuven, Belgium, bart.peeters@lms.be

ABSTRACT

The increasing use of modal analysis as a standard tool means that both experienced and inexperienced analysts are faced with new challenges: uncertainty about the accuracy of results, inconsistency between estimates of different operators, the tedious task of selecting obvious poles in a stabilization diagram and the time-consuming iterations required to validate a modal model. Therefore, it is no surprise that considerable research efforts are spent to overcome these difficulties. A few years ago, the PolyMAX modal parameter estimation method was introduced which makes the modal parameter estimation process much easier by better discriminating spurious from physical poles. Nevertheless, the route to automation still requires discrimination methods to distinguish physical from mathematical poles, in particular in the case of high-order and/or highly damped structures. This paper discusses an approach for automating the modal parameter estimation process and its industrial validation.

1 INTRODUCTION: THE CHALLENGE OF INDUSTRIAL MODAL ANALYSIS

The vibration and acoustical behavior of a mechanical structure is determined by its dynamic characteristics. This dynamic behavior is typically described with a linear system model. The inputs to the system are forces ("loads"), the outputs the resulting displacements or accelerations. System poles usually occur in complex conjugate pairs, corresponding to structural vibration "modes". The pole's imaginary part relates to the resonance frequency and the real part to the damping. Structural damping is typically very low (a few percent of the critical damping). The system's eigenvectors, expressed in the basis of the structural coordinates correspond to characteristic vibration patterns or "mode shapes". System identification from input-output measurements yields the modal model parameters [1][2]. This approach is now a standard part of the mechanical product engineering process.

Several constraints however make the system identification process for structural dynamics largely different from this in electrical engineering or process control. A review of the specific challenges of system identification for structural dynamics is given in [3]. A key issue is the difficulty of selecting the correct model order and the corresponding validation of the obtained system poles. First of all, a continuous structure has an infinite number of modes. In practice, the analyst is interested only in a limited number of these, up to a certain frequency or only in a certain frequency band. Still, model orders over 100 are no exception. Furthermore, while some of the modes are separated in resonance frequency, others may be very close leading to highly overlapping responses. The standard approach of selecting a model order and then deriving the corresponding poles is in general not applicable and over-specification of the model order is needed. Finally, the size of the problem often requires more than 1000 responses to be processed (e.g. a car body is discretized by over 500 nodes, measured in 3 directions), and this using large data segments to reduce the measurement noise. The consequence of these constraints is that classical system identification approaches, extracting the parameters of a discrete-time state-space model or of an ARMA model directly from the sampled input-output data, are often not practical or not feasible. Specific procedures are hence needed for modal analysis.

Modal analysis users are faced with following challenges:

• the ever increasing complexity of the tested structures: e.g. fully assembled vehicles instead of components, in-situ instead of laboratory measurements,

- the changing role of testing in the product development cycle [4] implying a reduction of time available for testing and analysis and a demand for increased accuracy adequate for use with hybrid or FE applications,
- specific for the modal parameter estimation process itself: inconsistency between estimates of different
 operators, the tedious task of selecting obvious poles in a stabilization diagram and the time-consuming
 iterations required to validate a modal model.

A key requirement for Experimental Modal Analysis (EMA) hence is that a reliable analysis of complex datasets should be possible with minimal, or even excluding, user interaction. This is the context of the methodology developed in the present paper. Section 2 discusses the use of a stabilization diagram to solve the order determination problem. In section 3, an automatic procedure is elaborated heavily relying on the stabilization diagram concept. The methodology is validated using industrial examples in Section 4.

2 THE STABILIZATION DIAGRAM

The key difficulty in applying system identification for EMA of large-scale structures is the selection of the model order and of the corresponding system poles. In EMA, measured Frequency Response Functions (FRFs) are curve-fitted by a modal model [1]:

$$\left[H(\omega)\right] = \sum_{i=1}^{n} \frac{\{v_i\} < l_i^T >}{j\omega - \lambda_i} + \frac{\{v_i^*\} < l_i^H >}{j\omega - \lambda_i^*}$$
(1)

where $[H(\omega)] \in \mathbb{C}^{l \times m}$ is the FRF matrix, containing the FRFs between all m inputs and all l outputs; n is the number of modes; \bullet^{H} denotes complex conjugate transpose of a matrix; $\{v_i\} \in \mathbb{C}^{l}$ are the mode shapes; $< l_i^T > \in \mathbb{C}^m$ are the modal participation factors and λ_i are the poles. The poles occur in complex conjugated pairs and are related to the eigenfrequencies ω_i [rad/s] and damping ratios ξ_i [-] (\bullet^* denotes complex conjugate):

$$\lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2} \omega_i$$
⁽²⁾

So, in EMA the model order determination problem boils down to deciding how many modes n to use for fitting the FRFs.

It should be noted that in classical system identification literature, many formal procedures exist to solve the model order determination problem. Models of different order are identified and compared according to quality criteria such as Akaike's Final Prediction Error or Rissanen's Minimum Description Length criterion [5]. Most of these techniques were developed in a control theory context where it is the aim to identify optimal low-order models. However, in structural dynamics, the order of the models is typically chosen much higher to reduce the bias on the estimates and to capture all relevant characteristics of the structure, even in the presence of large amounts of measurement noise. As a consequence of order over-specification, the physically meaningful poles are completed with a set of "mathematical" poles, modelling model, data and process noise but without having a relation to the structural problem.

To address this problem, the concept of "stabilization diagram" is introduced. The basic idea is that several runs of the complete pole identification process are made, by using models of increasing order. Experience on a very large range of problems shows that in such analysis, the pole values of the "physical" eigenmodes always appear at a nearly identical frequency, while mathematical poles tend to scatter around the frequency range. The pole values from all these analyses at different orders can be combined in one single diagram, with as horizontal axis the pole frequency and as vertical axis the solution order. The pole is indicated by a symbol in this diagram (Figure 1). Physical poles are readily visible in the diagram. To show that the frequency (respectively damping and eigenvector) of a pole fall within certain bounds of the values obtained at a lower system order, this is indicated by a symbol (for example by an "f" for frequency stabilization, "d" for damping and frequency or "v" for eigenvector and frequency). As typical stability criteria, following values are used:

- 1% for frequency stability,
- 5% for damping stability,
- 2% for eigenvector stability.

These "defaults" reflect the accuracy of the estimates that can be expected in a wide range of industrial modal analysis applications.



Figure 1 Stabilization diagram: the principle. The symbols have following meaning: "o": new pole, "f": stable frequency, "d": stable frequency and damping, "v": stable frequency and eigenvector, "s" all criteria stable.

From such diagram, it is not only possible then to select the optimal system order, and for this order, the valid system poles, but it is even possible to select individual poles from different analyses. To this purpose, several criteria can be used such as the lowest order at which a pole becomes "stable", the frequency or damping trend when plotting a specific pole across the order etc. Of course, when selecting individual poles, a complete structural model is not available anymore. Recombining the poles into a new model usually solves this problem. In most modal parameter estimation methods (see also Section 3.2) a stabilization diagram is constructed based on pole λ_i and participation factor $\langle l_i^T \rangle$ information. After the interpretation of the stabilization diagram, the mode shapes $\{v_i\}$ and the lower and upper residuals are obtained as the linear least-squares solution of (3):

$$\left[H(\omega)\right] = \sum_{i=1}^{n} \frac{\left\{v_{i}\right\} < l_{i}^{T} >}{j\omega - \lambda_{i}} + \frac{\left\{v_{i}^{*}\right\} < l_{i}^{H} >}{j\omega - \lambda_{i}^{*}} - \frac{\left[LR\right]}{\omega^{2}} + \left[UR\right]$$
(3)

With respect to (1), the lower and upper residuals $[LR], [UR] \in \mathbb{R}^{l \times m}$, modelling the influence of the out-of-band modes in the considered frequency band, have been added. Solving (3), also solves the mode recombination problem that arises from selecting individual modes in the stabilization diagram.

As may be clear from this section, the process remains very interactive and requires a large user experience, especially for complex datasets. In some cases, it may even appear that the model order determination problem has been shifted to the problem of interpreting very unclear stabilization diagrams (see also Section 4).

3 AUTOMATIC MODAL ANALYSIS

3.1 Literature review

With the increasing use of modal analysis as a standard tool by many, also less experienced, users, the strong need is expressed to automate the process. Researched solutions include estimation methods that are much more robust with respect to the appearance of spurious poles such as PolyMAX (see Section 3.2). Nevertheless, the route to automation still requires discrimination methods to distinguish physical from mathematical poles.

Probably, the most "natural" way is trying to capture the decisions that an experienced modal analyst takes, based on a stabilization diagram, by rules that can be implemented as an autonomous procedure. Examples of such rules can be found in [6][7]. The rule-based approach outlined in [6] will be described in Section 3.3 and is applied to industrial examples in Section 4.

The amount of pole information that can be provided in a stabilization diagram is in fact bounded by the ability of the human mind to interpret them at a glance. Automated procedures do not suffer from this constraint and the pole classification can be based on much more information. Therefore research is performed on additional criteria that describe the stability of a pole. A first set of criteria is generic (i.e. independent of the system identification technique) and look for instance at the complexity of the modal vector [8][9]. As mathematical poles are often characterized by high complexity, this criterion is valid if real modes are expected (loosely, if the distribution of the damping mechanisms along the vibrating structure is similar to its mass and stiffness distribution). Another criterion to get confidence in a pole of an identified MIMO model is to verify whether the pole is still present if different SIMO or even SISO models are identified by selecting individual inputs or outputs from the data [6][8][10][11].

Other pole validation criteria emerged in combination with specific system identification techniques. For instance in [12], the autonomous procedure heavily relies upon the "Consistent-Mode Indicator" which was presented as a by-product of the Eigensystem Realization Algorithm (ERA). Based on the realization theory, ERA identifies a state-space model from impulse response functions. Very interesting are the stochastic pole validation criteria that were formulated in combination with frequency-domain maximum likelihood estimation [8][9][13][14]. These stochastic criteria propagate the data uncertainty (variances on the FRFs) to features such as:

- Uncertainty on pole estimate: large uncertainty may indicate a mathematical pole;
- Pole-zero pairs: a large amount of zeros within a certain confidence circle around pole is an indication of a mathematical pole (it will be cancelled by the zeros);
- Pole-zero correlation coefficient: a high correlation again indicates a non-physical origin of pole.

A final set of criteria were developed to operate in combination with subspace identification [6][11][15]:

- Modal model reduction: the effect of removing a mode from the state-space model is studied. A small effect
 may indicate a mathematical pole.
- Relation between balanced and modal form: the energy interpretation of the states of a balanced model is heuristically transferred to the poles of the system. "Low-energy" poles are considered as non-physical.
- Forced pole-zero cancellations: if the effect on the model of shifting a pole towards its closest zeros is limited, this was probably a mathematical pole.

Evidently none of the methods works in all cases: many of them may give unclear separation in case of low signal-to-noise ratios; some of them may fail to label a local or weakly-excited pole as a physical one; the mode complexity indicators fail if the structure exhibits complex physical modes. However, combining multiple criteria by clustering techniques drastically improves the pole-discrimination capacity. For instance, in [8][14] Fuzzy C-means clustering is used for this purpose: the result is a membership function value for each pole that indicates whether it belongs to the class of physical or mathematical poles.



Figure 2. Stabilization diagram in frequency – damping format. Final set of poles obtained by applying a genetic algorithm and Fuzzy C-Means clustering [16].

Clustering techniques are also used to automatically interpret a classical stabilization diagram. In such an approach, the final cluster centers that pass certain thresholds are considered as the set of physical poles [6][9]. Figure 2 illustrates the method outlined in [16]: a Genetic Algorithm is used to find the initial values for the cluster centers. The final values are obtained by applying iteratively Fuzzy C-Means clustering.

3.2 Methods with clear stabilization diagrams

A major research topic has been the development of system identification algorithms with improved stabilization behaviour. Impressive results are for example obtained with the "Least Squares Complex Frequency Domain (LSCF)" method [17][18]. This discrete frequency-domain method uses a least squares or total least squares approach to fit a rational fraction polynomial model to a MIMO FRF matrix. Originally a common-denominator model structure was used in LSCF. Recently a right matrix-fraction variant of the method was introduced. In this so-called "PolyMAX" method following model is identified from the FRF data:

$$\left[H(\omega)\right] = \sum_{r=0}^{p} z^{r} \left[\beta_{r}\right] \cdot \left(\sum_{r=0}^{p} z^{r} \left[\alpha_{r}\right]\right)^{-1}$$
(4)

where $[\beta_r] \in \mathbb{C}^{l \times m}$ are the numerator matrix polynomial coefficients; $[\alpha_r] \in \mathbb{C}^{m \times m}$ are the denominator matrix polynomial coefficients and p is the model order. Please note that a so-called z-domain model (i.e. a frequency-domain model that is derived from a discrete-time model) is used in (4), with:

$$z = e^{j\omega\Delta t}$$
(5)

where Δt is the sampling time. Equation (4) can be written down for all values ω of the frequency axis of the FRF data. Basically, the unknown model coefficients $[\alpha_r], [\beta_r]$ are then found as the least-squares solution of these equations (after linearization). More details about this procedure can be found in [19][20].

Very high system orders (over 50) are clearly identified in a single-step procedure, leading to extremely clear stabilization diagrams, and hence drastically improving the quality and the interpretability of the result. It is a feature of the PolyMAX identification method to estimate the mathematical poles with negative damping ratio. Hence these poles are readily excluded before constructing the stabilization diagram. The fact that inherently unstable models are identified is not a problem as a new model is recomposed (3) after selection of the stable poles from the stabilization diagram. Figure 3 shows the comparison between stabilization diagrams obtained from an analysis on a vehicle body by a best-of-class industrial approach (Poly-reference LSCE [1]) and the PolyMAX method. The difference in stabilization quality is obvious.



Figure 3. Comparison between stabilization diagrams obtained from an analysis on a full vehicle by a standard method, i.e. the poly-reference LSCE method (Left), and the PolyMAX method (Right).

3.3 AMPS: Automatic Modal Parameter Selection

The selection of poles in a stabilization diagram has classically been done by an expert engineer visually inspecting the symbols, which are based on similarity in frequency, damping ratio and/or mode vector between poles belonging to subsequent model orders (see Section 2). Very typical questions that are raised in this context by many modal analysis users are:

- How to select poles in a stabilization diagram?
- How to speed up the iterative process of poles selection on stabilization diagram?
- How to ensure that consistent analyses are obtained from different people from the same database?

An automatic selection procedure is the answer to these questions. The so-called Automatic Modal Parameter Selection (AMPS) procedure implemented in [21] and relying on [6] is an intelligent rule-based approach in which the knowledge of experienced analysts is captured. Especially the combination of PolyMAX and AMPS has the advantages that it is much faster than the time-consuming iterative process of manual selecting poles, that less-experienced analysts have access to expert knowledge and that user-independent results are generated.

4 INDUSTRIAL VALIDATION

4.1 **Proof of concept**

A benchmark analysis was performed to evaluate the results derived from the new Automatic Modal Parameter Selection tool. The goal of the benchmark test was to provide a qualitative assessment of the AMPS tool, and to place it within the spectrum ranging from novice to experienced modal analysts. Eight people were selected for the test, including 4 novices and 4 experts, all of whom had an engineering background. The novices received a short description of the task they had to carry out.

In the test a MIMO data set from a fully trimmed car body was used. It has 2 inputs and 264 measurement points distributed over the entire car body, leading to 528 FRFs. The parameter estimation was done with both a time-domain method (Poly-reference LSCE) and a frequency-domain method (PolyMAX) for the frequency band 35-75Hz. Both stabilization diagrams were created to a model size of 64.

An initial examination of the two stabilization diagrams (Figure 4) show that the LSCE diagram is rather complicated and clouded by spurious, mathematical poles, especially at higher model orders, whereas the PolyMAX diagram clearly shows the stable poles throughout the whole frequency band. This resulted in large differences in the number of poles selected by the different participants. The time taken by each participant to make the assessment was measured, and in general, the LSCE task took about twice as long to complete as the PolyMAX task. In addition, it was found that the experts spent about twice as much time in the assessment as the novices, who were so overwhelmed by the complexity of the LSCE diagram that they quickly gave up.



Figure 4. Car body stabilization diagrams. (Left) Poly-reference LSCE; (Right) PolyMAX.

Figure 5 shows a frequency spectrum of the pole selection of all the test participants on the two stabilization diagrams. Users 1-4 are the novices, and users 5-8 are the experts. The vertical dotted lines show the selection made by the AMPS tool. Relying on the LSCE diagram (Figure 4 - Left), it was clear that the novices encountered difficulties in the 45-60Hz band: not only did they miss poles; there was also a wide variation in those that were selected. Even the experts did not find it easy in this frequency range, as the poles they selected did not line up well, indicating differences in the frequencies (Figure 5 - Left). Above 65 Hz, the experts agreed quite well, but the novices missed some of the poles completely. Relying on the PolyMAX diagram (Figure 4 - Right), the majority selected all poles. All users (experts and novices) agree much better, as indicated by the nicely aligned dots (Figure 5 - Right).



Figure 5. Car body pole selection results per user (novices:1-4, experts 5-8). (Left) LSCE; (Right) PolyMAX.

Figure 6 shows a comparison of the damping ratios of the selected poles. The crosses (\times) represent the AMPS selection. A cluster of dots (\bullet) represents consensus over the selection by the different test participants, whereas a scatter over the damping would indicate, in case of novices, lack of experience and feeling for physical damping values. In general the participants demonstrated more consensus with respect to damping in the PolyMAX diagram (Figure 6 - Right) than in the LSCE diagram (Figure 6 - Left). AMPS largely agrees with this consensus. The sparse number of selections on the 45-60Hz band explains the differences. There is a remarkable improvement for both novices and experts in that band in the PolyMAX plot.



Figure 6. Car body pole selection: frequency versus damping for all users (•) and AMPS (×). (Left) LSCE; (Right) PolyMAX.

It is clear that the association of the PolyMAX method and AMPS generate user-independent result and can be an educational tool for both novices and experts. More benchmark results and details can be found in [22].

4.2 Increased productivity

Using AMPS, the iterative process of pole selection from a stabilization diagram is much less time-consuming. More poles are selected in just a few seconds and the pole selection procedure can be made faster by using a larger band and a higher maximum model order. This is illustrated using data from a body-in-white of a midsize car. The data consists of 2 inputs and as much as 2005 response DOFs. A complete modal analysis was performed by an expert modal analyst over 2 weeks which resulted in 233 poles being found. The iterative procedure, used in this type of modal analysis cannot be applied to the whole band of interest; different frequency bands are analyzed and the poles estimated from each frequency band are then merged into one analysis. With

AMPS, the whole frequency range (with 1437 spectral lines) of interest was treated at the same time with a model size of 256. After 40 seconds, 112 poles were highlighted and automatically selected in the stabilization diagram. Figure 7 shows the results from a more detailed study of the range between 36 Hz and 79 Hz. Not only is the PolyMAX diagram much clearer, also AMPS is able to find much more poles. Figure 8 shows the excellent quality of the synthesized FRFs of the modal model generated by the PolyMAX - AMPS selection.

Most of the AMPS modes agree well with a (manual) expert analysis. The remaining modes need to be estimated in an iterative way. For a simple structure, all modes are identified by AMPS, and the time gain achieves a significant 95%. On a more complex structure, where interactive modal analysis is performed on several smaller bands, on average 80% of the modes are identified with high confidence by AMPS. Taking the remaining interactive analysis time into account, the overall productivity gain still is an impressive 50%.



Figure 7. Car body-in-white stabilization diagrams with AMPS results indicated by vertical lines. (Left) LSCE; Right (PolyMAX).



Figure 8. Measured (Red) and synthesized (green) FRFs.

5 OUTLOOK AND CONCLUSIONS

Simplified or even automated identification of the parameters of complex systems will offer the key to a whole series of model-based engineering applications. In many problems, structural identification is closely related to detecting changes in the system dynamics. An example is the flight qualification of aircraft, requiring the repeated in-flight modal analysis at different airspeeds. At each air speed, resonance frequencies and damping ratios of critical modes are checked to verify the absence of aero-elastic instability (flutter). As during the change from one flight condition to the next one, the dynamics may change due to imminent flutter, the damping must be monitored continuously. Examples of automated in-flight data analysis can be found in [6][11][23].

Another example is the use of changes in the modal system model to detect structural damage [24] or in the assessment of the integrity of a structure after the forced loading during a qualification test. An example of such a structural health monitoring (SHM) approach can be found in [10][12]: as part of scheduled major maintenance, each orbiter in the NASA Space Shuttle fleet undergoes modal testing on a regular basis. It was demonstrated that automatic procedures could reduce the data analysis efforts from one month to one hour. In vibration-based SHM of civil engineering structures, ad-hoc automatic modal analysis [25] as well as statistical-test methods are pursued [26]. The same methodology can be applied to problems such as variant analysis (product scatter) or end-of-line product testing.

Significant progress has been made in addressing the key problem of discriminating physical system poles from mathematical poles in the identification of the dynamics of complex structures. The stabilization diagram offers a heuristic approach hereto. Novel model estimation algorithms with clear stabilization behaviour ease the process dramatically, opening the way to a fully automated process.

The combination of the PolyMAX modal parameter estimation method with the Automatic Modal Parameter Selection (AMPS) procedure means that the myth of automating modal analysis is close to becoming a reality. In this paper, the proof of concept, the increased productivity and the industrial applicability have been demonstrated.

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REFERENCES

- [1] W. HEYLEN, S. LAMMENS, AND P. SAS, *Modal Analysis Theory and Testing*. Dept. of Mech. Eng., K.U.Leuven, Belgium, 1995.
- [2] N. MAIA AND J SILVA, *Theoretical and Experimental Modal Analysis.* Baldock, UK: Research Studies Press Ltd., 1997.
- [3] H. VAN DER AUWERAER., System identification for structural dynamics and vibroacoustics design engineering. In *Proc. SYSID-2003, the 13th IFAC Symposium on System Identification*, Rotterdam, The Netherlands, 27–29 Aug. 2003.
- [4] H. VAN DER AUWERAER, Requirements and opportunities for structural testing in view of hybrid and virtual modelling. In *Proceedings of ISMA 2002, the International Conference on Noise and Vibration Engineering*, Leuven, Belgium, September 2002.
- [5] L. LJUNG, System Identification Theory for the User, 2nd Ed. Upper Saddle River, NJ, USA: Prentice Hall PTR, 1999.
- [6] M. SCIONTI, J. LANSLOTS, I. GOETHALS, A. VECCHIO, H. VAN DER AUWERAER, B. PEETERS, AND B. DE MOOR, Tools to improve detection of structural changes from in-flight flutter data. In *Proc.* 8th Int. Conf. on Recent Advances in Structural Dynamics, Southampton, UK, 14–16 July 2003.
- [7] B. PEETERS, System Identification and Damage Detection in Civil Engineering. PhD thesis, Dept. of Civil Eng., K.U.Leuven, Belgium, Dec. 2000.
- [8] P. VERBOVEN, E. PARLOO, P. GUILLAUME, AND M. VAN OVERMEIRE, Autonomous structural health monitoring part I: modal parameter estimation and tracking. *Mechanical Systems and Signal Processing*, Vol. 16, No. 4, pp. 637–657, 2002.

- [9] P. VERBOVEN, P. GUILLAUME, B. CAUBERGHE, E. PARLOO, AND S. VANLANDUIT, Stabilization charts and uncertainty bounds for frequency-domain linear least squares estimators, In *Proc. of IMAC 21, the Int. Modal Analysis Conf.*, Kissimmee, FL, USA, Feb. 2003.
- [10] K.S. CHHIPWADIA, D.C. ZIMMERMAN, AND G.H. JAMES III, Evolving autonomous modal parameter estimation, In Proc. of IMAC 17, the Int. Modal Analysis Conf., pp. 819–825, Kissimmee, FL, USA, Feb. 1999.
- [11] I. GOETHALS AND B. DE MOOR, Subspace identification combined with new mode selection techniques for modal analysis of an airplane. In *Proc. SYSID-2003, the 13th IFAC Symposium on System Identification*, Rotterdam, The Netherlands, 27–29 Aug. 2003.
- [12] R.S. PAPPA, G.H. JAMES III, AND D.C. ZIMMERMAN, Autonomous model identification of the Space Shuttle tail rudder. *Journal of Spacecraft and Rockets*, Vol. 35, No. 2, pp. 163–169, March-April, 1998.
- [13] P. VERBOVEN, E. PARLOO, P. GUILLAUME, AND M. VAN OVERMEIRE, Autonomous modal parameter identification based on a statistical frequency-domain maximum likelihood approach. In *Proc. of IMAC 19, the Int. Modal Analysis Conf.*, pp. 1511–1517, Kissimmee, FL, USA, Feb. 2001.
- [14] S. VANLANDUIT, P. VERBOVEN, J. SCHOUKENS, AND P. GUILLAUME, An automatic frequency domain modal parameter estimation algorithm. In *Proc. Int. Conf. on Structural System Identification*, pp. 637–646, Kassel, Germany, Sep. 2001.
- [15] I. GOETHALS AND B. DE MOOR, Model reduction and energy analysis as a tool to detect spurious modes. In *Proc. ISMA 2002, the Int. Conf. on Noise and Vibration Eng.*, pp. 1307–1314, Leuven, Belgium.
- [16] J. LANSLOTS, M. SCIONTI, AND A. VECCHIO, Fuzzy clustering techniques to automatically assess stabilization diagrams. In Proc. 7th Int. Conf. on the Application of Artificial Intelligence to Civil and Structural Eng., Egmond-aan-Zee, The Netherlands, 2003.
- [17] P. GUILLAUME, P. VERBOVEN, AND S. VANLANDUIT, Frequency-domain maximum likelihood identification of modal parameters with confidence intervals. In *Proc. ISMA 23, the Int. Conf. on Noise and Vibration Eng.*, Leuven, Belgium, 16–18 Sep. 1998.
- [18] H. VAN DER AUWERAER, P. GUILLAUME, P. VERBOVEN, AND S. VANLANDUIT, Application of a fast-stabilizing frequency domain parameter estimation method. ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 123, No. 4, pp. 651–658, Dec. 2001.
- [19] P. GUILLAUME, P. VERBOVEN, S. VANLANDUIT, H. VAN DER AUWERAER, AND B. PEETERS, A poly-reference implementation of the least-squares complex frequency-domain estimator. In *Proc. of IMAC 21, the Int. Modal Analysis Conf.*, Kissimmee, FL, USA, Feb. 2003.
- [20] B. PEETERS, H. VAN DER AUWERAER, P. GUILLAUME, AND J. LEURIDAN, The PolyMAX frequency-domain method: a new standard for modal parameter estimation? *Shock and Vibration*, Special Issue dedicated to Professor Bruno Piombo, **11**, 395-409, 2004.
- [21] LMS INTERNATIONAL. LMS Test.Lab Modal Analysis Rev 7A, Leuven, Belgium, www.lmsintl.com, 2006.
- [22] J. LANSLOTS, B. RODIERS, AND B. PEETERS. Automated pole-selection: proof-of-concept & validation. In Proceedings of the ISMA 2004 International Conference on Noise and Vibration Engineering, Leuven, Belgium, 20–22 September 2004.
- [23] P. VERBOVEN, B. CAUBERGHE, AND P. GUILLAUME, A structural health monitoring approach for flutter testing. In *Proc. of ISMA 2002 Int. Conf on Noise and Vibration Eng.*, pp. 1631-1642, Leuven, Belgium, Sep. 2002.
- [24] S.W. DOEBLING, C.R. FARRAR, M.B. PRIME, AND D.W. SHEVITZ, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review. Los Alamos National Laboratory Report, LA-13070-MS, 1996.
- [25] B. PEETERS AND G. DE ROECK, One-year monitoring of the Z24-Bridge: environmental effects versus damage events. *Earthquake Engineering and Structural Dynamics*, Vol. 30, No. 2, pp. 149–171, 2001.
- [26] M. BASSEVILLE, M. ABDELGHANI, AND A. BENVENISTE, Subspace-based fault detection algorithms for vibration monitoring. *Automatica*, Vol. 36, No. 1, pp. 101–109, 2000.