# Filtrage particulaire et suivi multi-pistes

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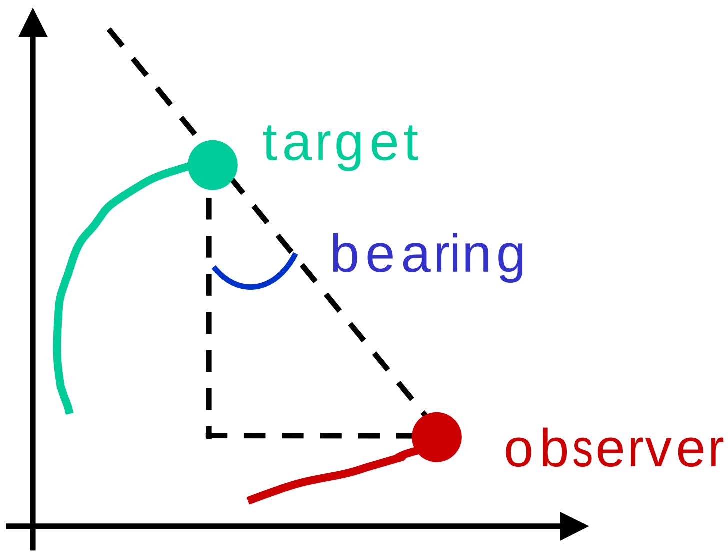
# Context

- **Applications:**
  - Signal processing: target tracking
    - bearings-only tracking
    - fusion of active and passive measurements
  - Image analysis: people tracking (in other work)



# Challenges

- **Generic tracking issues**
  - (re)initialization
  - Missing measurements
  - Clutter (false alarms)
  - Non-linearity
  - Non observability

- **Specific multiple-targets issues**
  - Complexity
  - Association and estimation simultaneoulsy
  - Varying number of targets

# Basic ingredients

- **State variables** $\boldsymbol{X}_t$ (position, velocity of the targets)

- **Data** $\boldsymbol{Y}_t$( bearings, ranges)

- **Estimation of posterior** $P(\boldsymbol{X}_t|\boldsymbol{Y}_0 \cdots \boldsymbol{Y}_t)$

  - Dynamics: $P(\boldsymbol{X}_t|\boldsymbol{X}_{t-1})$

  - Likelihood: $P(\boldsymbol{Y}_t|\boldsymbol{X}_t)$

# State variables and dynamics

- **Single-object state variable**: $\boldsymbol{X} = (Xx, Xy, v_x, v_y)$
- **Multiple-object state variable:** $\boldsymbol{X} = (\boldsymbol{X}^1 \cdots \boldsymbol{X}^n)$
- **Single-object dynamics prior**: Markovian process

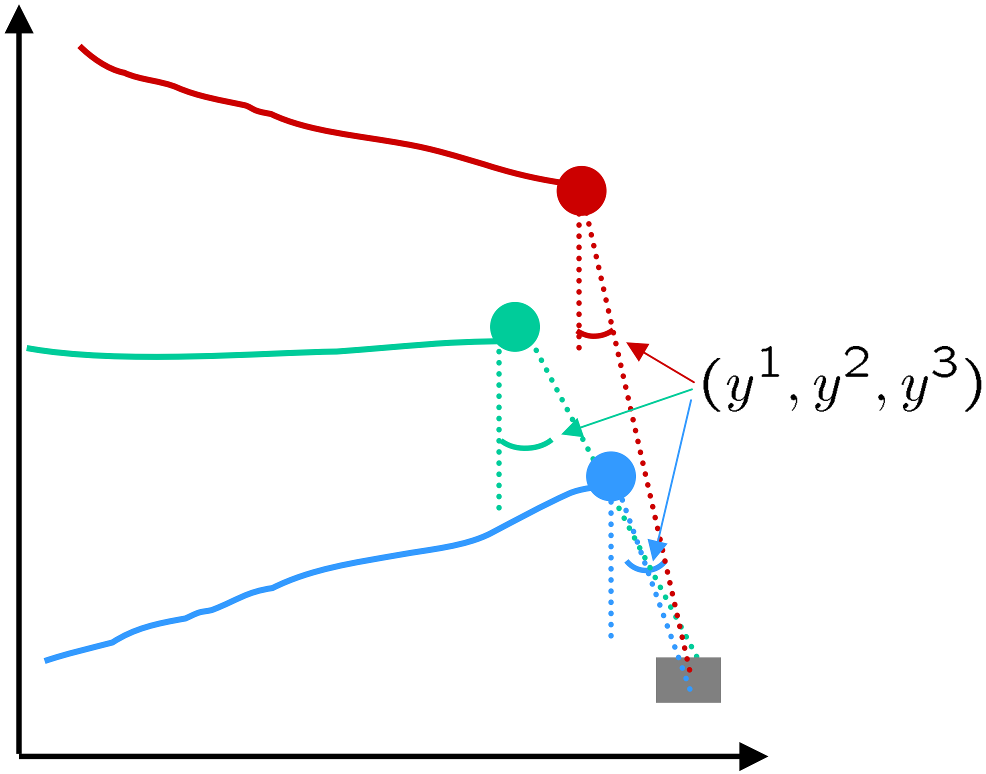
$$X^i_{t+\Delta t} = \begin{pmatrix} I_2 & \Delta t \cdot I_2 \\ 0 & I_2 \end{pmatrix} \cdot X^i_t + \begin{pmatrix} \dfrac{\Delta t^2}{2} I_2 \\ \Delta t \cdot I_2 \end{pmatrix} \cdot V_t$$

where $V_t \approx N\left(0, \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}\right)$

- **Multi-object dynamics:** independent single-object dynamics

# Data

- **Bearings-only tracking**: $\boldsymbol{Y}_t = (y^1 \cdots y^{m_t})$ highly non-linear wrt the state variable




➥How to assign the data to the states ?

# Data association

- **Notation**: association vector: $K^j = i$ if $y^j$ is issued from target $i$
- **Association assumptions**:

  (H1)one measurement can originate from one object or from the clutter (*false alarms*)

  (H2) one object can produce zero or one measurement

  (H3)one object can produce zero or several measurements at one time
- Algorithms based on **(extented) Kalman** filter:

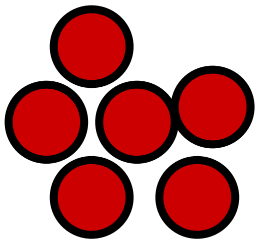
  (H1) + (H2) -> **jpdaf**

  (H1) + (H3) ->**pmht**
- **Our proposition**: particle filter using probabilistic association **(MOPF)**:

$$\pi^i = p(K^j = i)$$

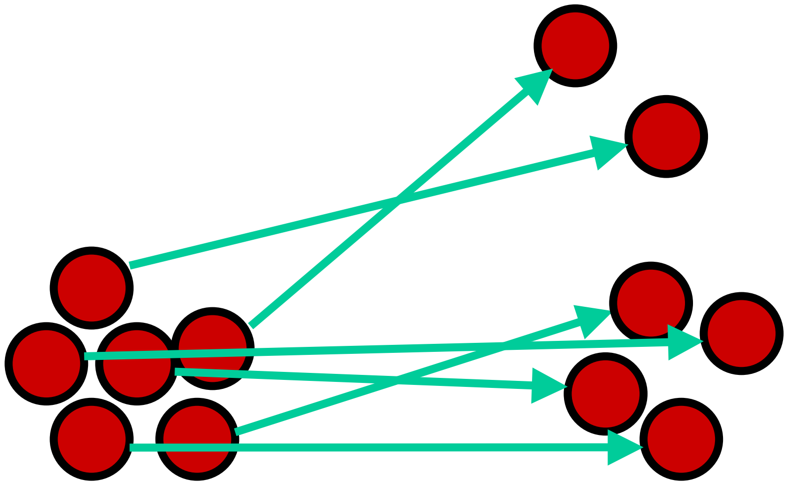
- Another proposed algorithm: particle filter based on jpda **(Sir-jpda)**

# Particle filtering



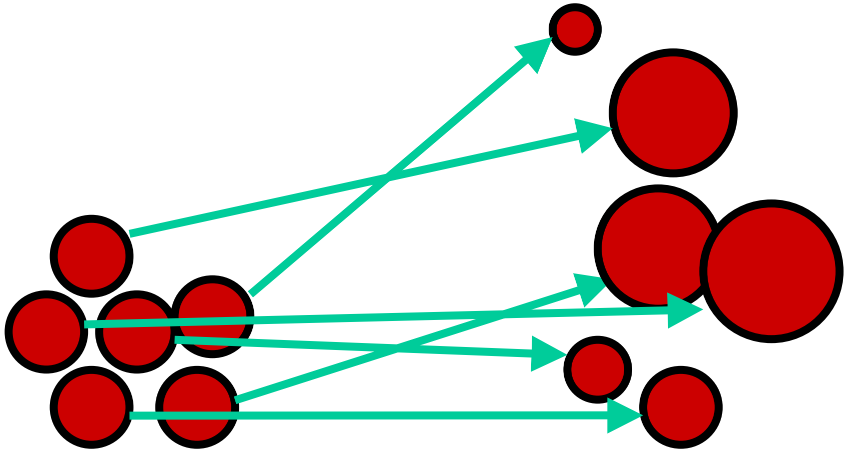
Current cloud at time t

# Particle filtering



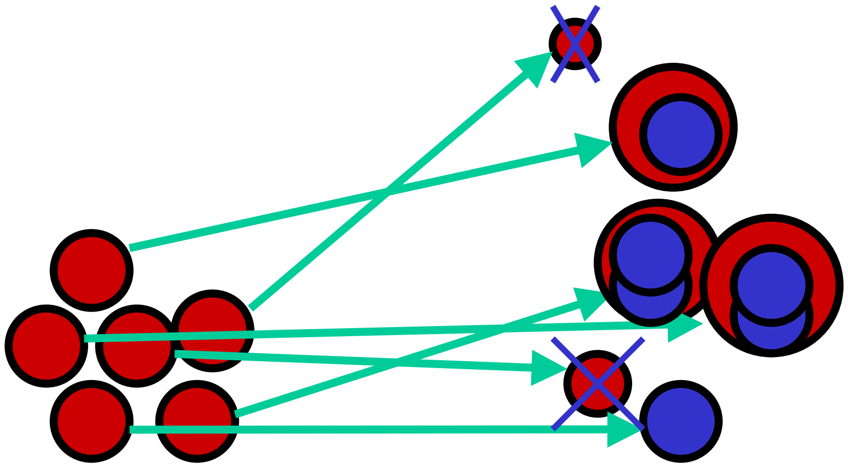
Propagation by sampling from the dynamic prior
or from an importance function based on the data

# Particle filtering



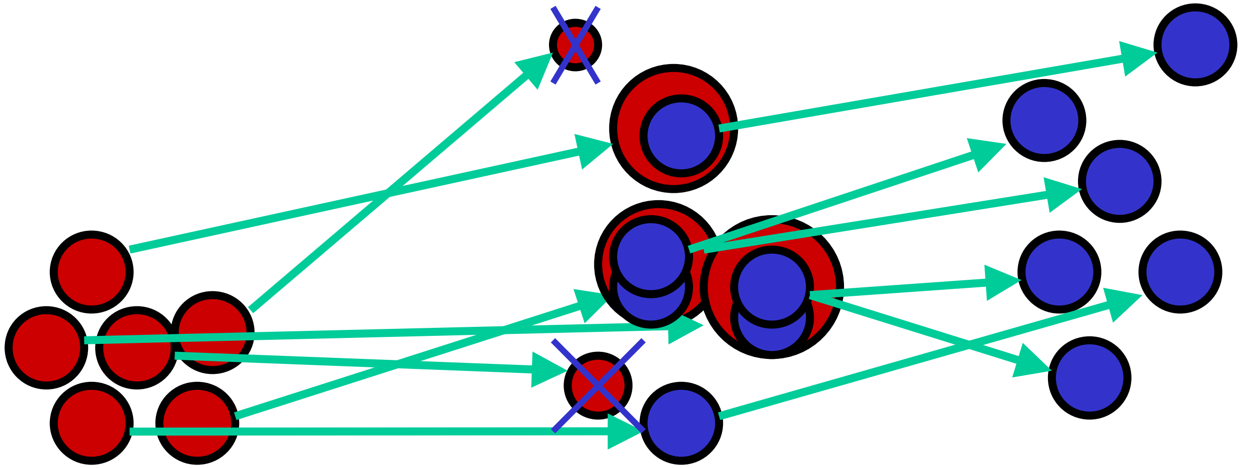
Weighting according to data likelihood

# Particle filtering



Resampling from the weighted particle set

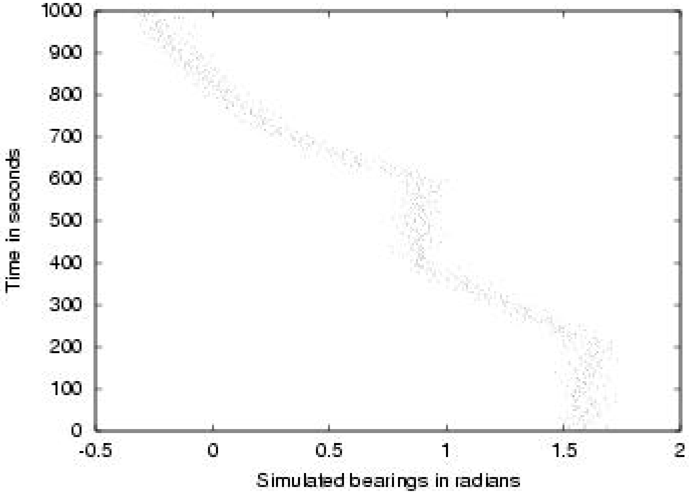
# Particle filtering



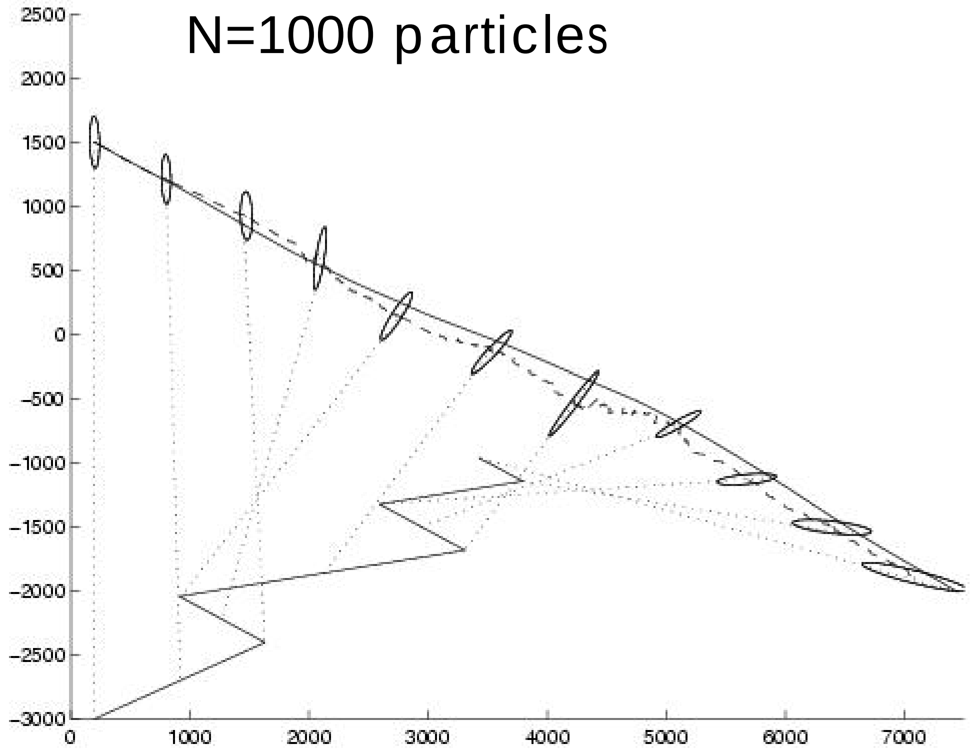
Prediction again ...

# Example: bearings-only tracking

Simulated bearings:

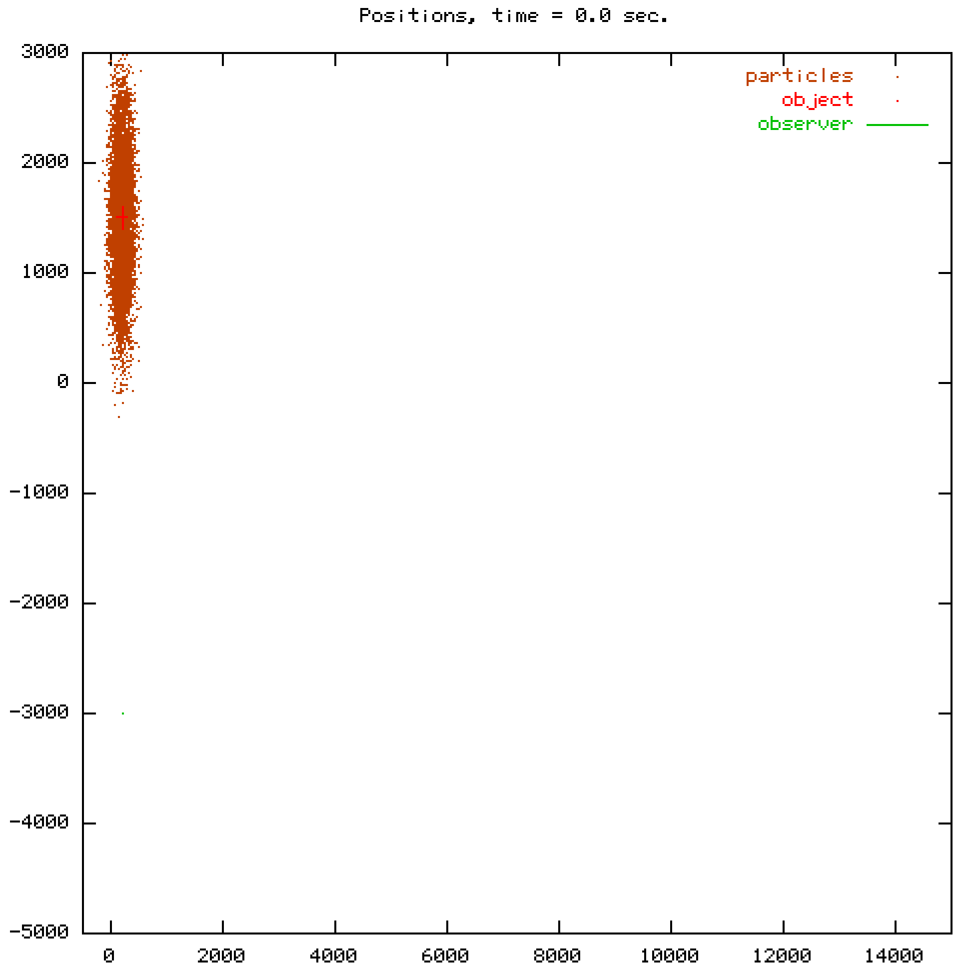



One particular run and confidence ellipses N=1000 particles



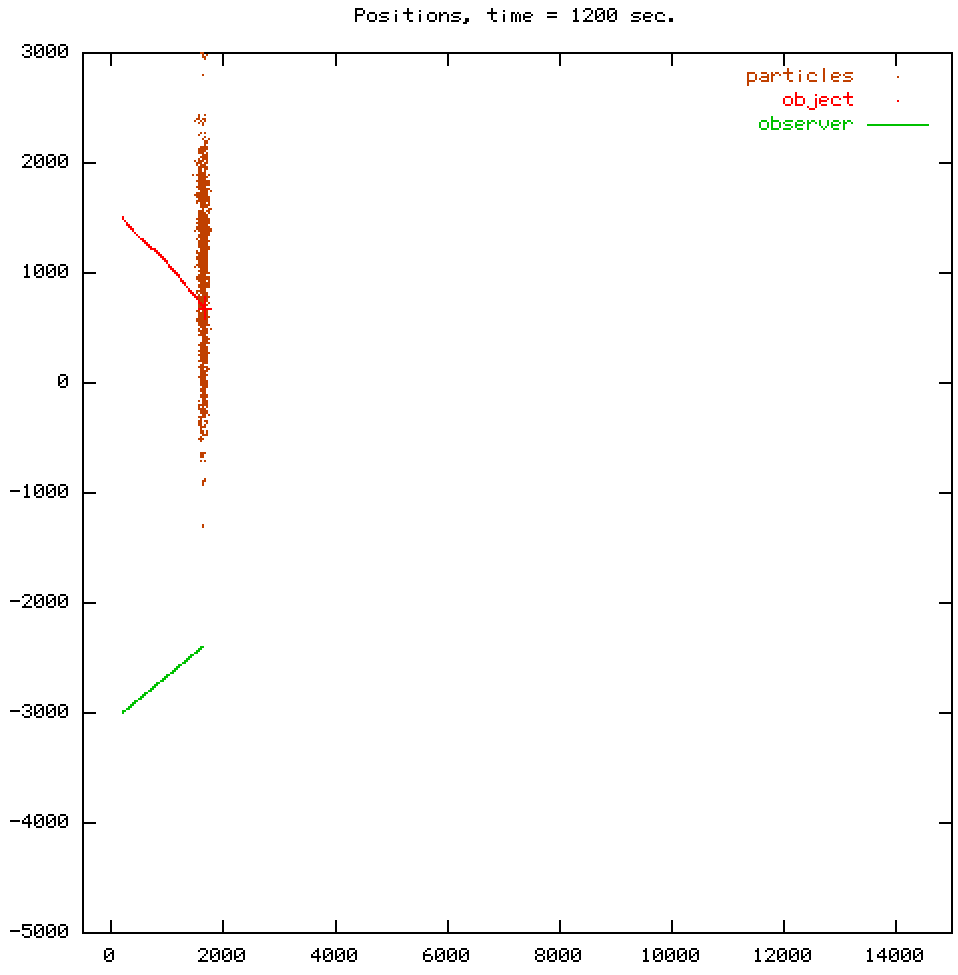
# Example: bearings-only tracking

Particle cloud for one object (N =10000 particles)



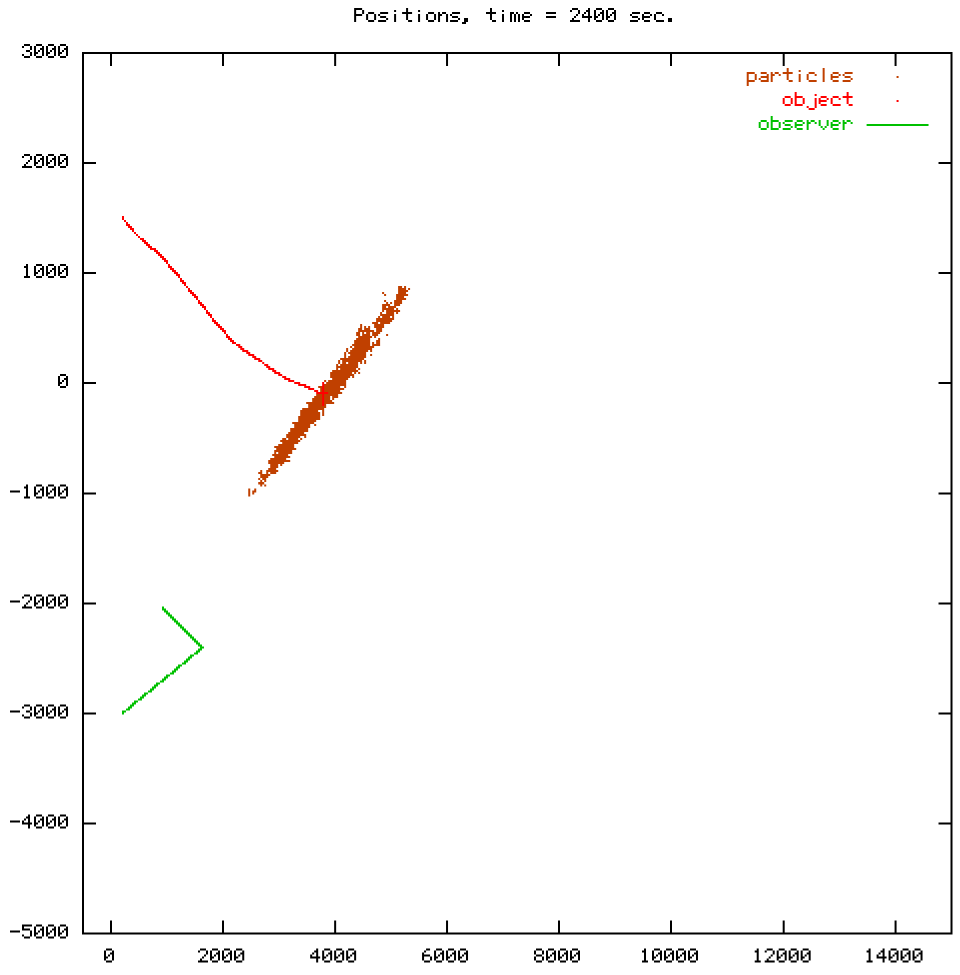
# Example: bearings-only tracking

Particle cloud for one object



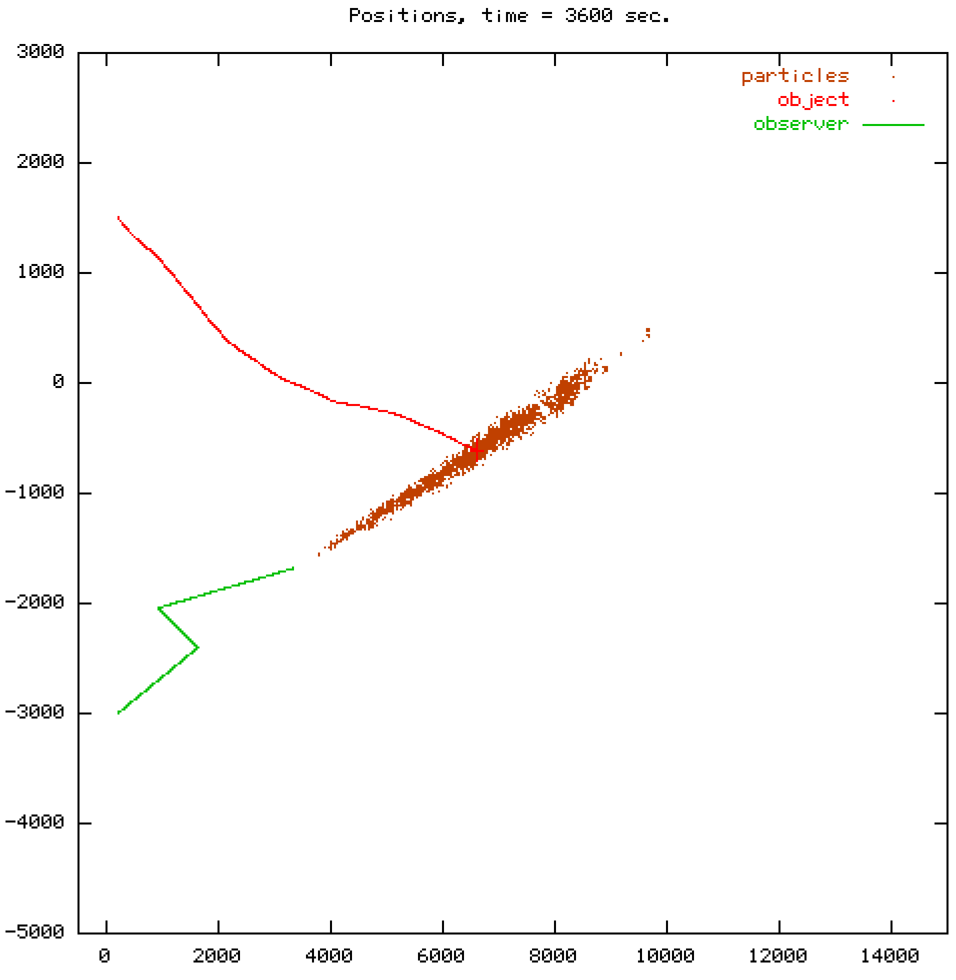
# Example: bearings-only tracking

Particle cloud for one object



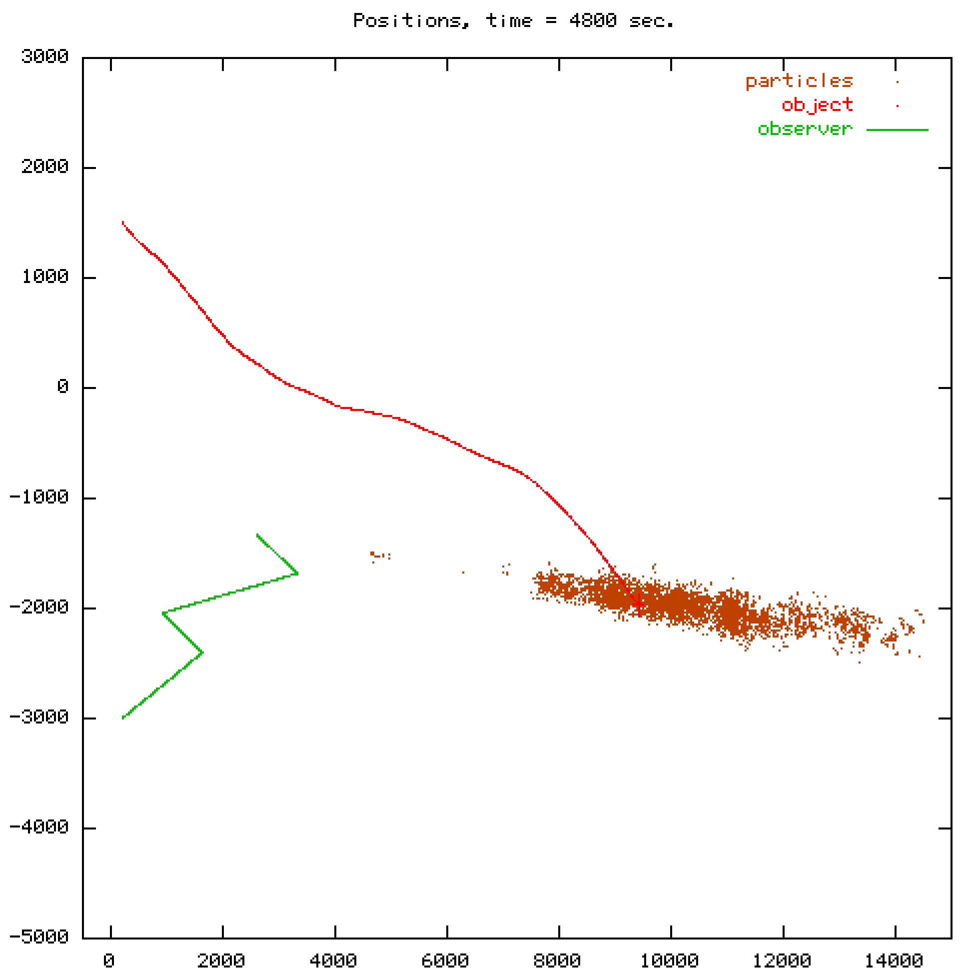
# Example: bearings-only tracking

Particle cloud for one object



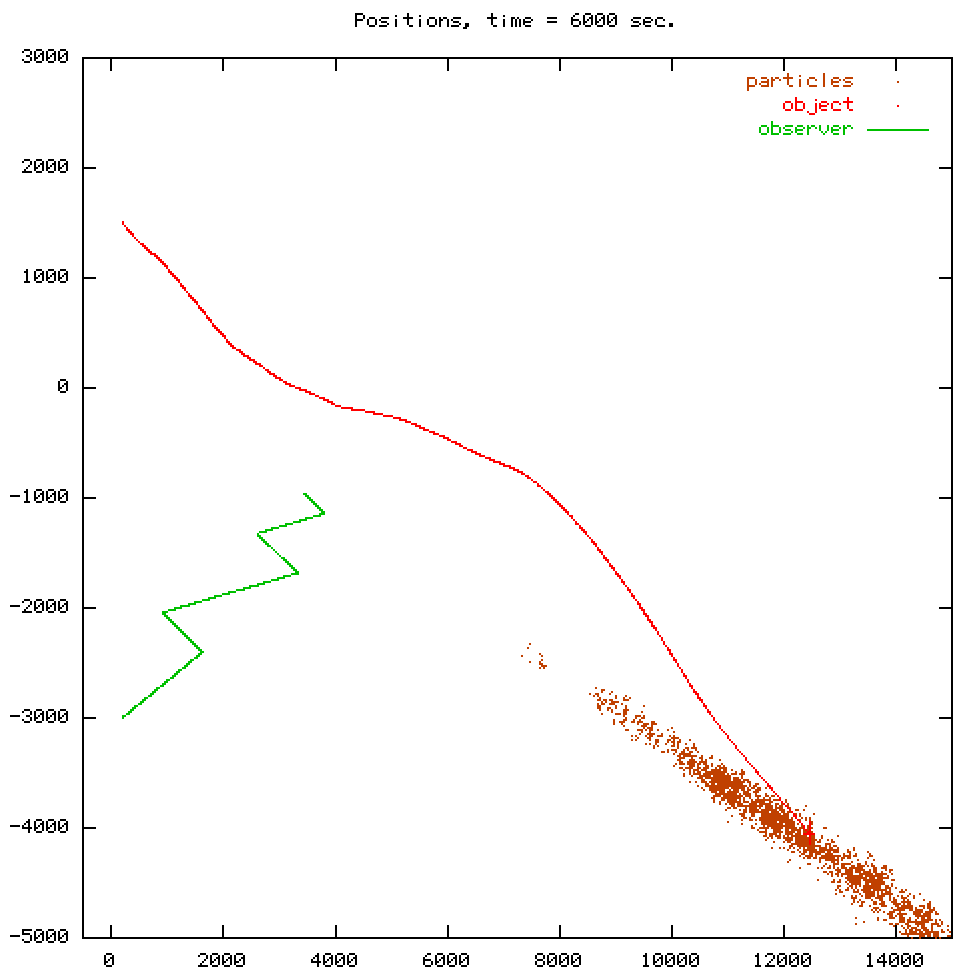
# Example: bearings-only tracking

Particle cloud for one object



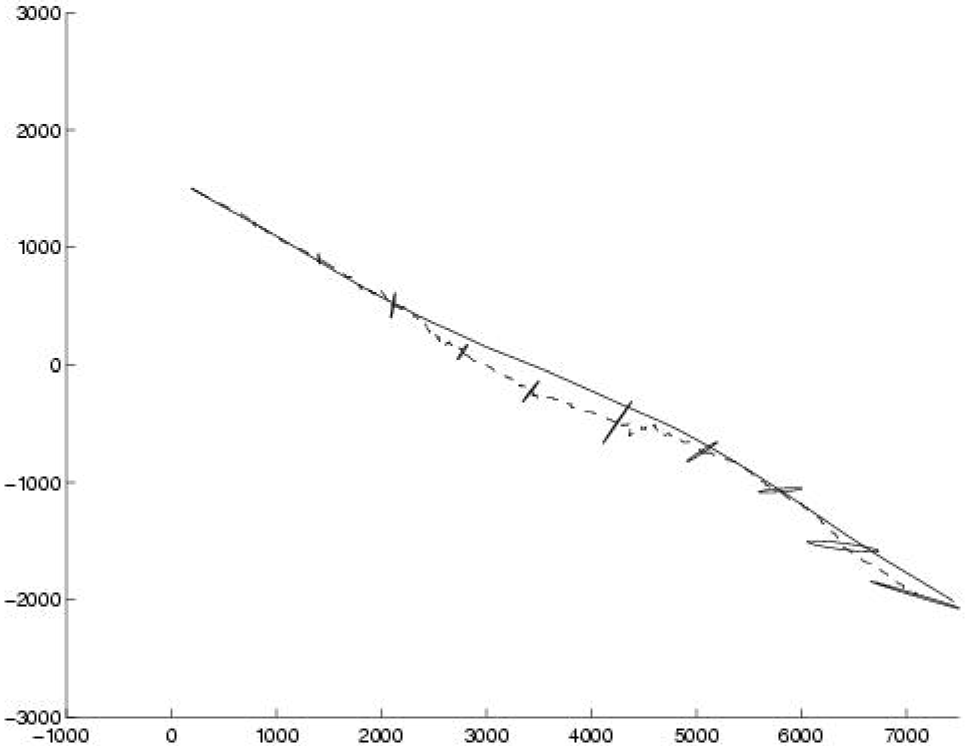
# Example: bearings-only tracking

Particle cloud for one object



# Example: bearings-only tracking

Averaged estimate over 100 runs and 2 $\sigma$ confidence ellipses
N=1000 particles



# Multi-object data likelihood

$$p(Y_t = (y_t^1, \ldots, y_t^{m_t}) \mid X_t = (x_t^1, \ldots, x_t^M))$$

$$\underline{\underline{H3}} \prod_{j=1}^{m_t} p(y_t^j \mid X_t = (x_t^1, \ldots, x_t^M))$$

$$= \prod_{j=1}^{m_t} \frac{\pi_t^0}{V} + \sum_{i=1}^{M} \pi_t^i \, p(y_t^j \mid X_t^i = x_t^i)$$

$\pi_t^0, \cdots, \pi_t^M$ à estimer

**MOPF**

$$\underline{\underline{H2}} \sum_{all\ associations} p(Y_t = (y_t^1, \ldots, y_t^{m_t}) \mid X_t, K_t^u) \, p(K_t^u)$$

$$= \sum_{all\ associations} \prod_{j=1}^{m_t} p(y_t^j \mid X_t, K_t^u) \, p(K_t^u)$$

**SIR-JPDA**

# MOPF: use of a Gibbs sampler

- Estimation of the association probabilities vector with a Gibbs sampler:

For $\tau = 1, \ldots \tau_{end}$ :

- Sample $K_{t,\tau+1}$ from $p(K_t^j \mid X_{t,\tau}, \Pi_{t,\tau}, Y_{0:t})$
- Sample $\Pi_{t,\tau+1}$ from $p(\Pi_t^i \mid X_{t,\tau}, K_{t,\tau+1}, Y_{0:t})$
- Sample $X_{t,\tau+1}$ from $p(X_t^i \mid K_{t,\tau+1}, \Pi_{t,\tau+1}, Y_{0:t})$

Compute $\hat{\pi}_t = \dfrac{1}{\tau_{end} - \tau_{beg}} \displaystyle\sum_{\tau=\tau_{beg}}^{\tau=\tau_{end}} \pi_{t,\tau}$

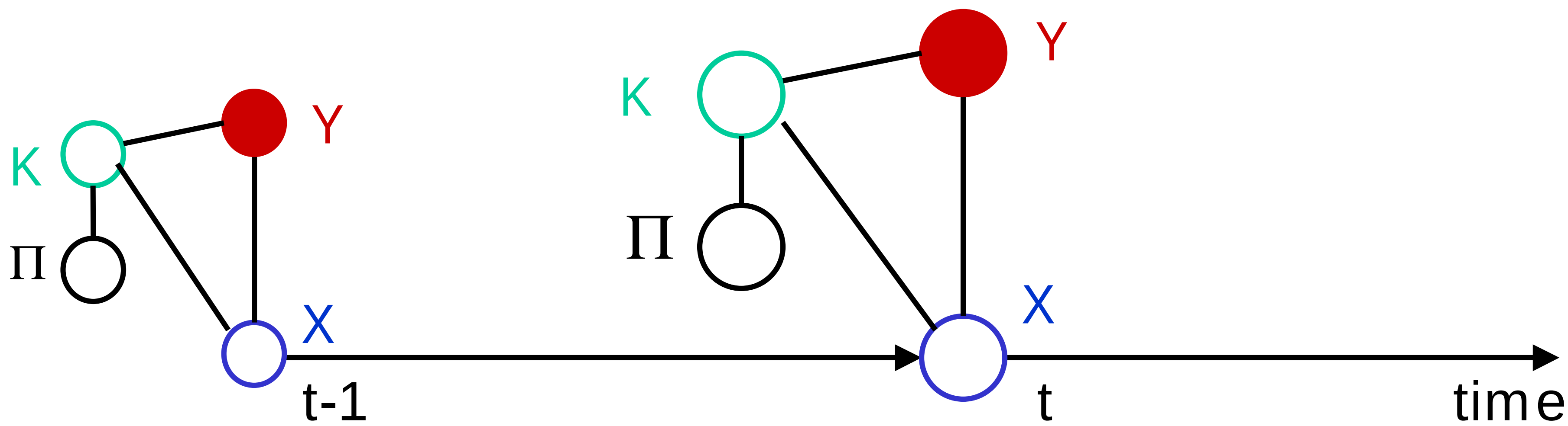
# One iteration of the Gibbs sampler

- Sample $K^j_{t,\tau+1}$ from $p(K^j_{t,\tau+1}=i) \propto \begin{cases} \pi^i_{t,\tau} p(y^j_t \mid x^i_{t,\tau}) & \text{if } i=1,\ldots,M \\ \pi^0_t / V & \text{if i} = 0 \end{cases}$

- Sample $\Pi^{1:M}_{t,\tau+1}$ from $Multinomial\,(\pi;(n^i(K_{t,\tau+1}))_{i=1,\ldots,M})$

  where $n^i(K) \stackrel{\Delta}{=} Card\{j : K^j = i\}$

- Sample $X^i_{t,\tau+1}$ from a law based on the predicted cloud



# Sir-jpda

- Énumération exhaustive des associations

$$p(Y_t = (y_t^1, \dots, y_t^{m_t}) \mid X_t = (x_t^1, \dots, x_t^M))$$

$$= \sum_{all\ associations} \prod_{j=1}^{m_t} p(y_t^j \mid X_t, K_t^u) p(K_t^u)$$

avec $$p(K_t^u) = \frac{\Phi^u !}{m_t !} p_F(\Phi^u) \prod_{i=1}^{M} P_d^{D^u(i)} \prod_{i=1}^{M} (1 - P_d)^{1-D^u(i)}$$

$\Phi^u$ = nombre de fausses alarmes dans $K_t^u$

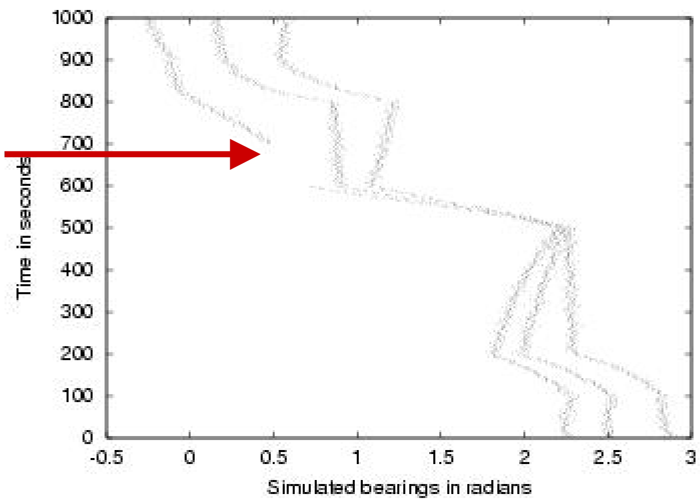
# Example: bearings-only tracking

- Measurement equation

$$Y_t^j = \text{atan}\frac{Xx_t^i - Xx_t^{obs}}{Xy_t^i - Xy_t^{obs}} + W_t \text{ if } K_t^j = i \text{ where } W_t \sim N(0, \sigma^2)$$

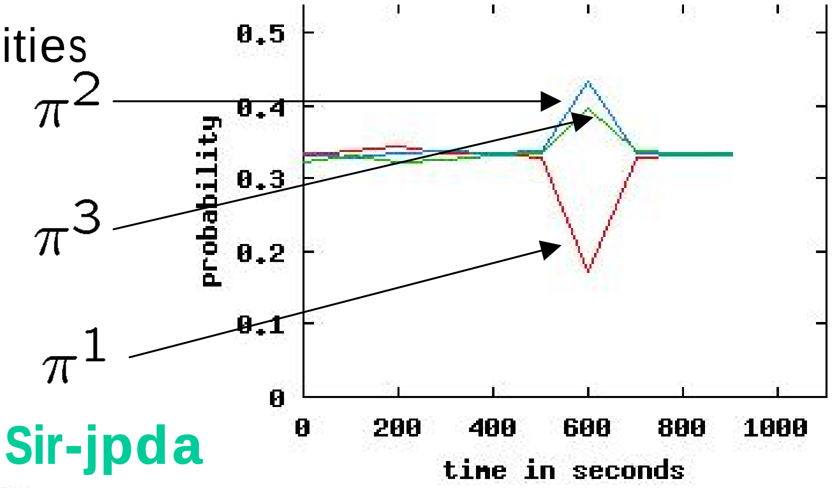
- Simulated bearings for the three objects

Occlusion period for target 1



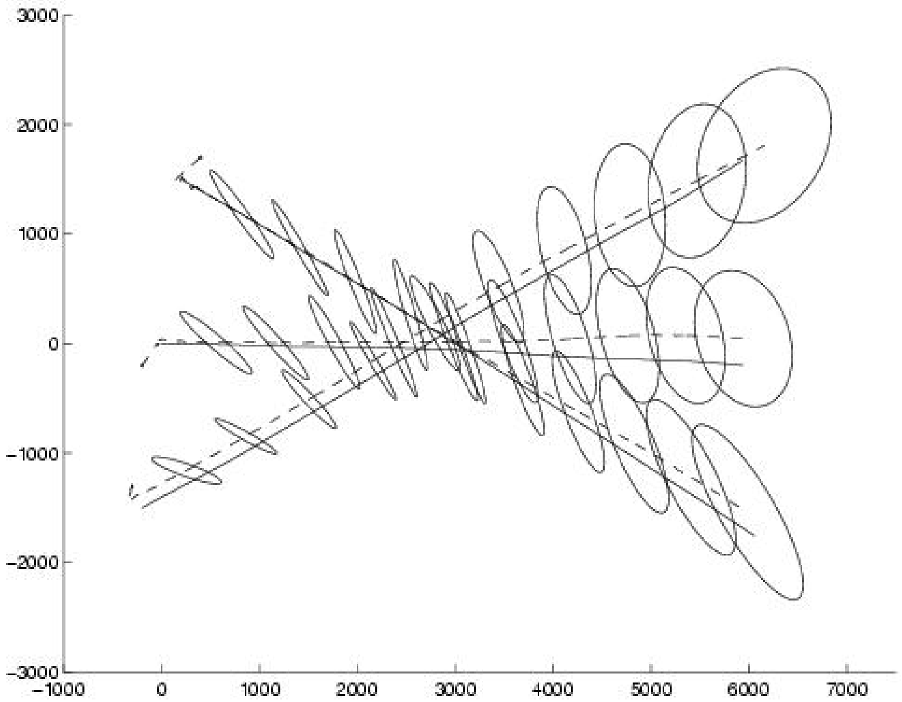
# Example: bearings-only tracking

- MOPF- estimated association probabilities

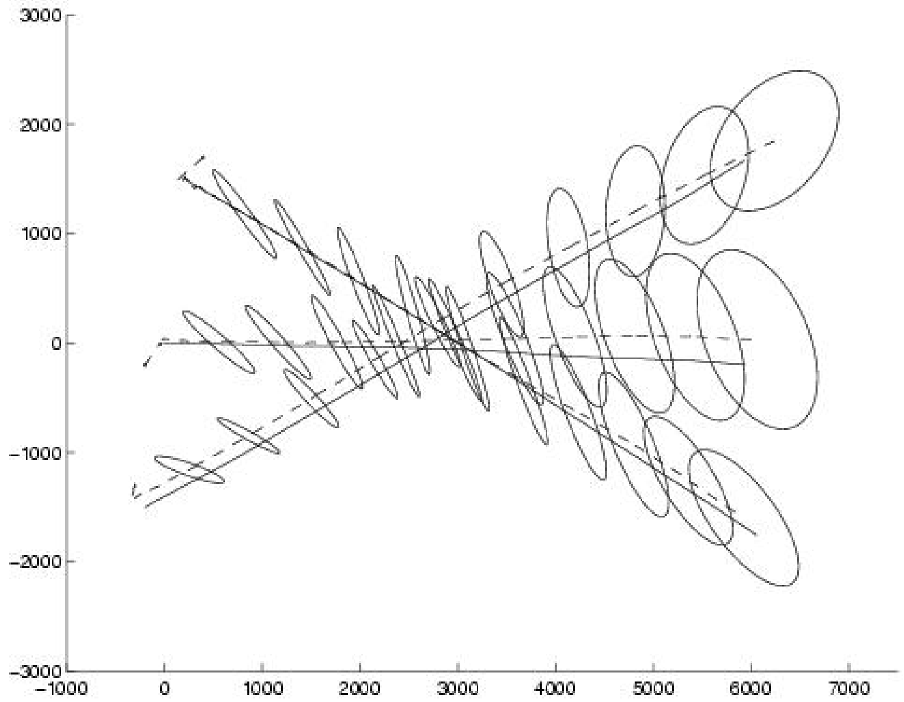



- Estimated trajectories:

**MOPF**




**Sir-jpda**



# Example: tracking with high clutter

- detection probability for each target $P_d = 0.9$
- clutter ~ Poisson law of parameter $\lambda \cdot V = 0, 1, 2, 3$

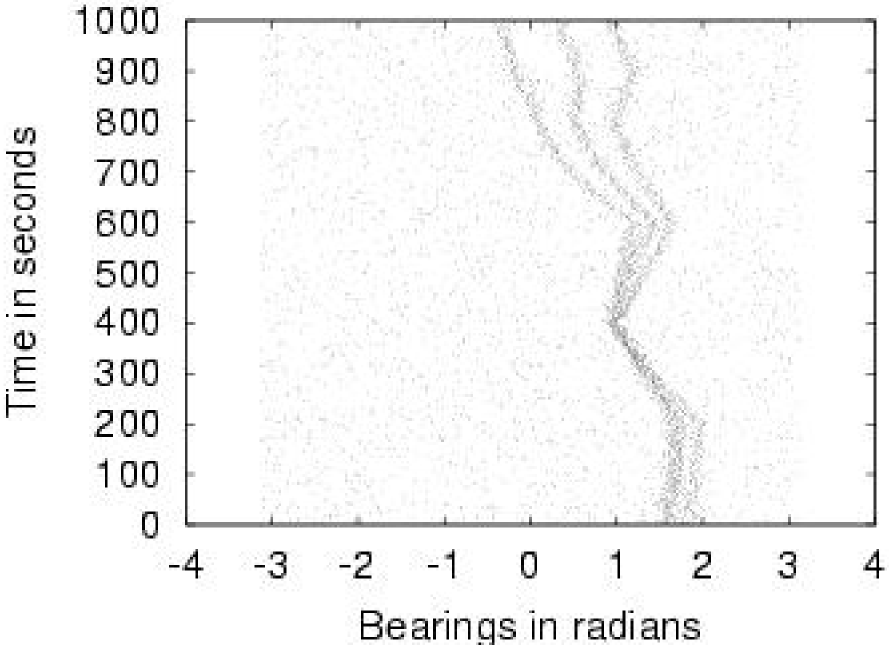


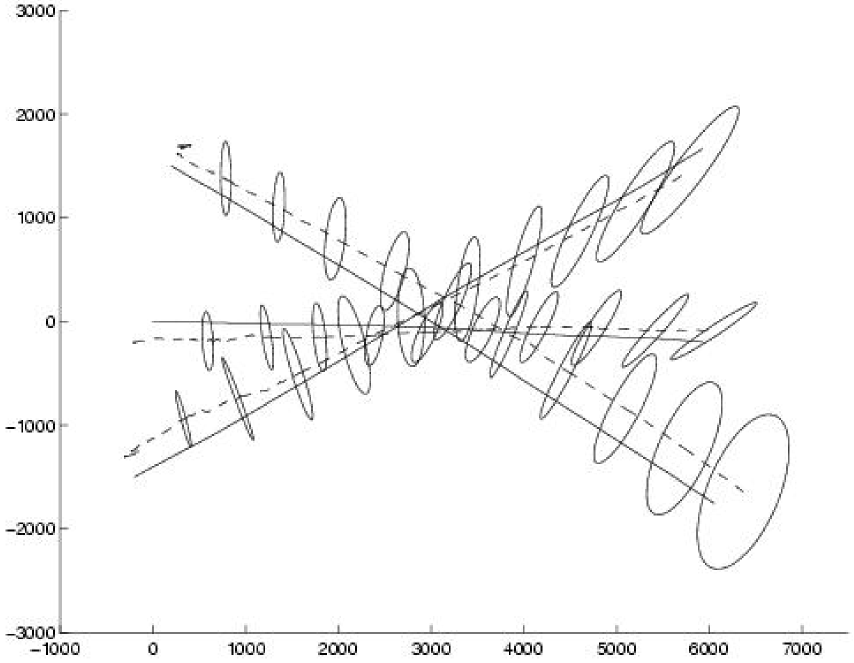

TABLE I

*Computational cost for one iteration on a 863 Mhz Pentium III.*

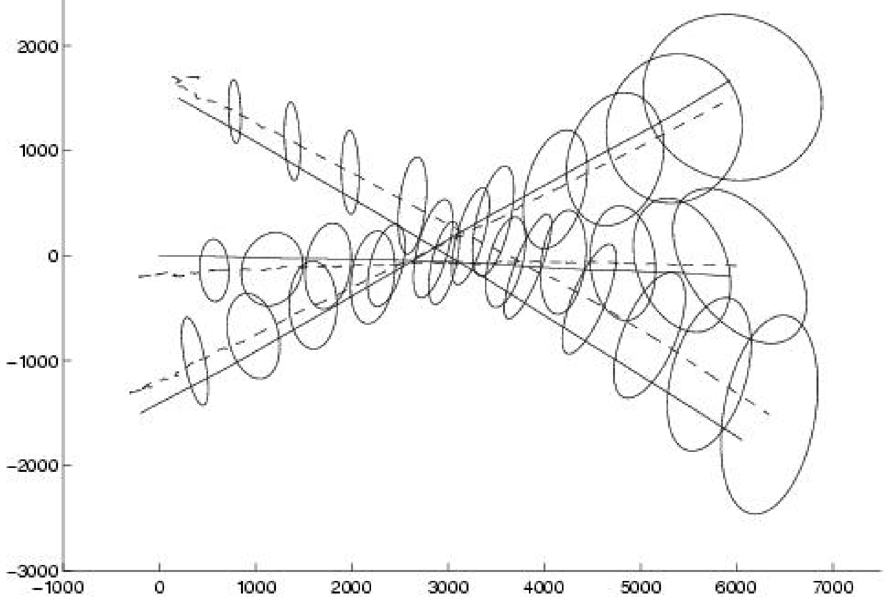
| clutter density | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| SIR-JPDA | 75ms | 155ms | 290ms | 540ms |
| MOPF | 385ms | 400ms | 415ms | 430 ms |

# Example: tracking with high clutter

$$\lambda \cdot V = 1 \, ; \, N = 1000 \, particles$$



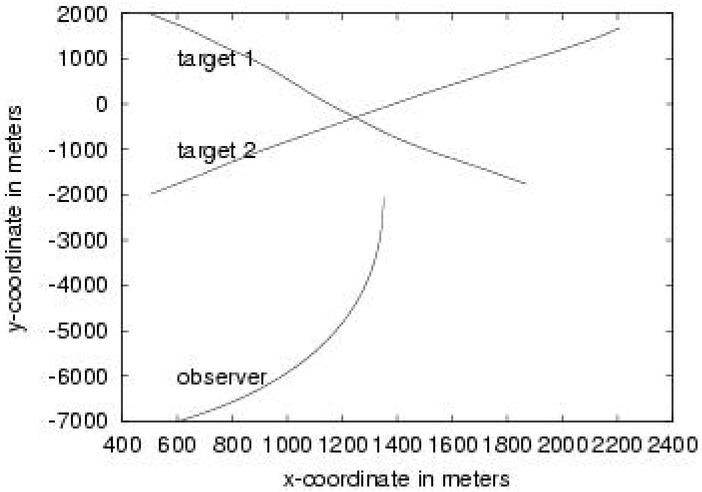

MOPF

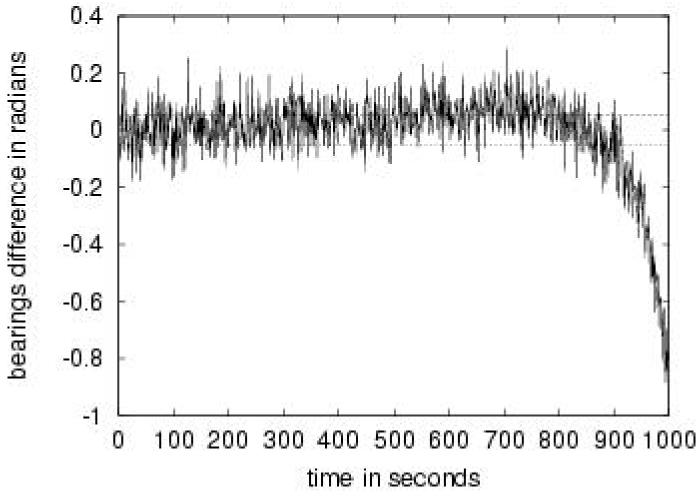



Sir-jpda

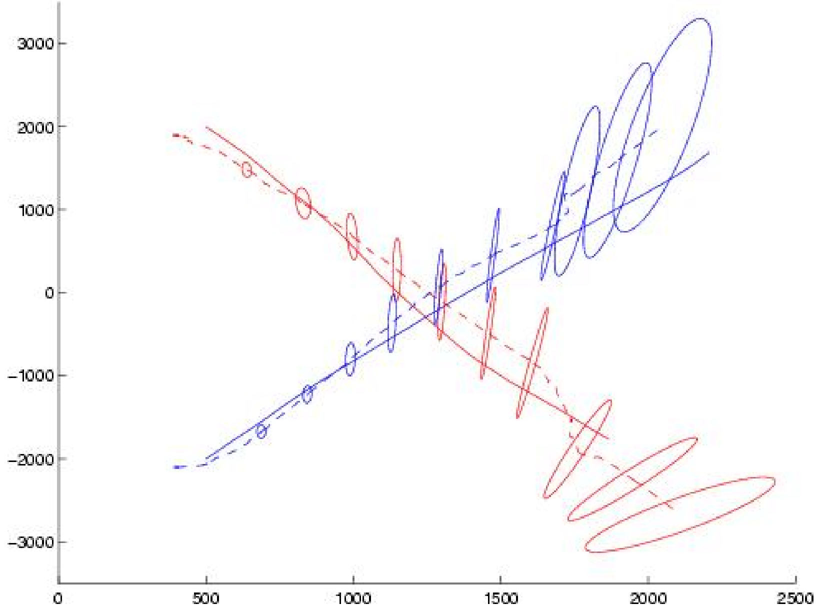
# Active and passive measurements:

- Too difficult scenario:only very close bearings




-> bad estimation



# MOPF using multiple receivers

- measurement $y^j$ is received by receiver $r^j$ known (bearing or range)

$$p(Y_t = (y^1_{t,r^1},\dots,y^{m_t}_{t,r^{m_t}}) \mid X_t = (x^1_t,\dots,x^n_t)) = \prod_{j=1}^{m_t} p(y^j_{t,r^j} \mid X_t = (x^1_t,\dots,x^n_t))$$

$$= \prod_{j=1}^{m_t}\left[\frac{\pi^0_t}{V} + \sum_{i=1}^{n} \pi^i_t \cdot p(y^j_{t,r^j} \mid X^i_t = x^i_t)\right]$$

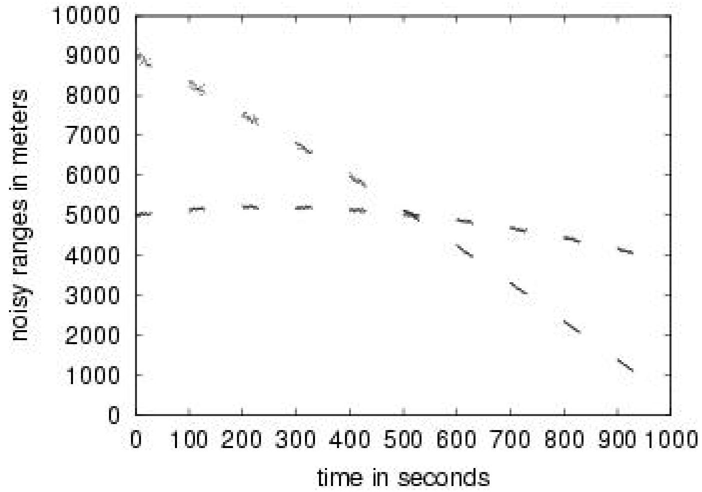
bearings-likelihood or range likelihood

# Example:range and bearings

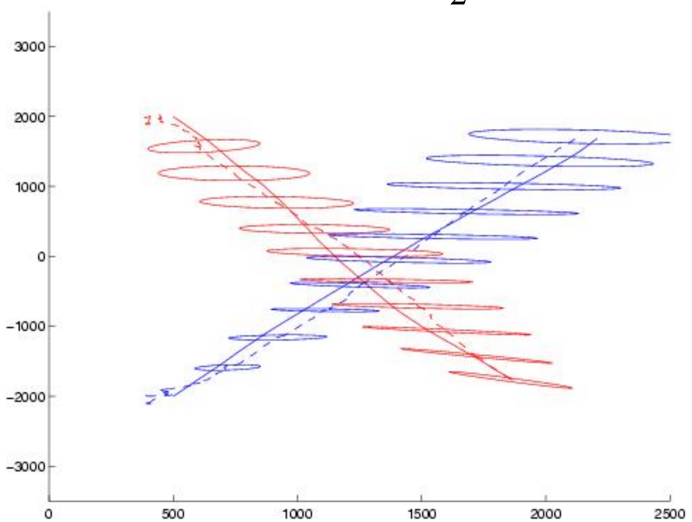
- Range measurements added at certain times:

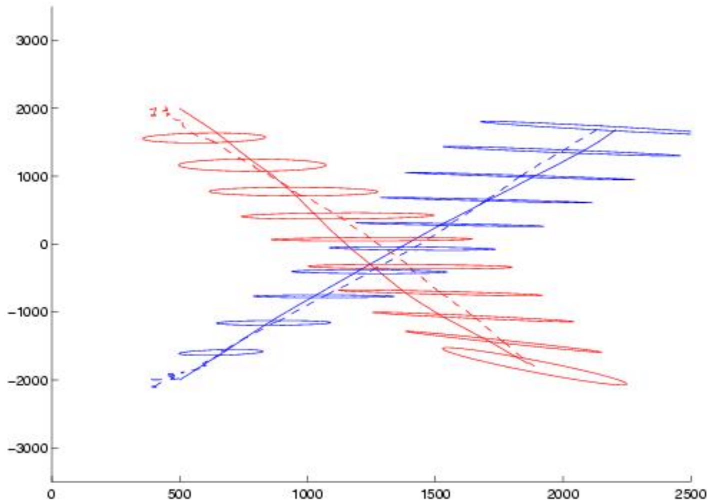
$$\rho_t^j = \sqrt{\left(x_t^i - x_t^{obs}\right)^2 + \left(y_t^i - y_t^{obs}\right)^2}$$

$$R_t^j = \rho_t^j + Z_t; \quad Z_t \approx N\left(0, \sigma_z \rho_t^j\right)$$




$$\sigma_z = 10^{-6}$$




20 percent of active measurements; 50 percent

# Conclusion and further work

- **MOPF: Generic multi-object tracker based on a mix of particle filtering and Gibbs sampling**
  - ➥ suitable for various applications
- **Sir-jpdaf:** particle filtering and jpda
- **Drawbacks**
  - Fixed number of objects
  - Costly
- **Perspectives**
  - Test with varying number of objects
  - Compare the results with *a posteriori* Cramer-Rao bounds

The End

Questions?