Particle Filter Track Before Detect Algorithms
Theory and Applications

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Outline

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- Filtering
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TBD integrates the information over time.

Detection is based on power/energy that has been integrated over time (multiple scans).

Classical tracking: single scan based detection.

* TBD provides higher probability of detection ($P_d$) at the same level of probability of false alarm ($P_{fa}$)
* TBD circumvents the data association problem.
Twofold problem

The TBD problem is twofold:

1. Filtering

2. Detection
Filtering

The System

\[ s_{k+1} = f(t_k, s_k, d_k, w_k), \quad k \in \mathbb{N} \]

\[ \text{Prob}\{d_{k+1} = j \mid d_k = i\} = [\prod(t_k)]_{ij} \]

\[ z_k = h(t_k, s_k, d_k, v_k), \quad k \in \mathbb{N} \]

Filtering Problem: Determine \( p(s_k, d_k|Z_k) \)
Basic idea of the particle filter

"Describe the a posteriori pdf \( p(s_k, d_k | Z_k) \) by a cloud of \( N \) particles that propagates in time such that the cloud approximately equals an \( N \)-sample drawn from \( p(s_k, d_k | Z_k) \) "

NOTE:
This is more than just a (point) estimate !!!!
Filtering

Kalman vs. PF representation

\[ \text{Graphs showing the comparison between Kalman and PF representations.} \]
Filtering

Using a (proper) particle filter on the system:

The following holds

\[ \sum_{i=1}^{N} \frac{1}{N} \delta(s - \tilde{s}_i^k) \xrightarrow{a.s.} p(s_k | Z_k) \]

i.e. almost sure convergence...

Popular (point) estimators obtained from particle cloud:

\[ \hat{s}_k^{MV} = \int_{\mathbb{R}^n} s_k p(s_k | Z_k) ds_k \approx \sum_{i=1}^{N} \frac{1}{N} \tilde{s}_i^k \]

\[ \hat{s}_k^{MAP} = \arg \max_{s_k \in \mathbb{R}^n} p(s_k | Z_k) \approx \hat{s}_i^* \]

where \( \hat{s}_i^* = \arg \max_i q_k^i \)
Detection

Deciding upon presence of target:

Hypothesis testing:

Given two hypotheses

• $\mathcal{H}_0$ : no signal present

\[ z(j) = v(j), \quad j = 0, \ldots, k \]

• $\mathcal{H}_1$ : signal present

\[ z(j) = h(s(j), v(j)), \quad j = 0, \ldots, k \]

where $s(k)$ evolves according to dynamical system
Detection

Using particle filter output for detection

Every optimal detector can be expressed in terms of a Likelihood Ratio Test:

\[ L(Z(k, l)) \leq \tau \]

**THEOREM:**

\[
L(Z(k, l)) = \frac{p(z(k - l + 1), \ldots, z(k) \mid \mathcal{H}_1)}{p(z(k - l + 1), \ldots, z(k) \mid \mathcal{H}_0)} \approx \frac{\prod_{j=k-l+1}^{k}(\sum_{i=1}^{N} \tilde{q}^i(j))}{N^l \prod_{j=k-l+1}^{k} p_v(z(j))}
\]
Detection

Using particle filter output for detection

Elements of the Proof:

\[ p(z(l), \ldots, z(m) \mid \mathcal{H}_0) = \prod_{j=l}^{m} p_v(z_j) \]

\[ p(z(l), \ldots, z(m) \mid \mathcal{H}_1) = \prod_{j=l}^{m} p(z(j) \mid Z(j - 1)) \]

where

\[ p(z(j) \mid Z(j - 1)) = \int_S p(z(j), s \mid Z(j - 1)) ds \]

\[ = \int_S p(z(j) \mid s, Z(j - 1)) p(s \mid Z(j - 1)) ds \]

\[ = E_{p(s \mid Z(j - 1))} p(z(j) \mid s, Z(j - 1)) \]

\[ \approx \frac{1}{N} \sum_{i=1}^{N} p(z(j) \mid s^i(j)) = \frac{1}{N} \sum_{i=1}^{N} \tilde{q}^i(j) \]
Example - Detection

Linear Gaussian scalar system:

\[ s(k + 1) = s(k) + w(k) \]

\[ z(k) = s(k) + v(k) \]

\( w(k) \sim N(0,1) \), \( v(k) \sim N(0,1) \) and \( s(0) \sim N(0,10) \)

Data has been generated according to the above model.

Particle filter solution (200 particles) and the \textbf{exact (Kalman) solution} have been calculated.
Example - Detection

True states and estimates

Ratio exact and p.f. likelihood
The Fighter-Missile Example:

Multi target track before detect application for *small to very small closely spaced targets*. Early detection is crucial.

Modelling details in:
Example - MTT TBD

System:

\[ s_{k+1} = f(t_k, s_k, d_k) + g(t_k, s_k, d_k) w_k \]

where

\[
 f(t_k, s_k, d_k) = \begin{pmatrix}
 1 & 0 & T & 0 \\
 0 & 1 & 0 & T \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{pmatrix} s_k
\]

The process noise input model is given by

\[
 g(t_k, s_k, d_k) = \begin{pmatrix}
 \frac{1}{2}(\frac{1}{3} a_{x,\text{max}})T^2 & 0 \\
 0 & \frac{1}{2}(\frac{1}{3} a_{y,\text{max}})T^2 \\
 \frac{1}{3} a_{x,\text{max}} T & 0 \\
 0 & \frac{1}{3} a_{y,\text{max}} T
\end{pmatrix}
\]
System:

The discrete mode $d_k$ represents one of three hypotheses (each have a different measurement equation!!)

- $d_k = 0$: There is no target present.
- $d_k = 1$: The prime target is present.
- $d_k = 2$: There are two targets present.

Markov process:

$$\Pi(t_k) = \begin{pmatrix} 0.90 & 0.10 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.00 & 0.10 & 0.90 \end{pmatrix}$$
Simulations

Initially, there is no target present. The first target (fighter: SNR=13dB) appears after 5 seconds ($T = 1s$) and moves at a constant velocity of $200ms^{-1}$ towards the sensor.

After 20 seconds, a second target (missile: SNR 3dB) spawns from the first and accelerates to a velocity of $300ms^{-1}$ in 3 scans.

1000 particles have been used in a ’plain vanilla particle filter implementation’
Simulations

Matlab movies
Estimation of the mode

True mode

- o no target present
- * 1 target present
- * 2 targets present
Overview

Related work (co)authored by presenter:


Some Other Related work:


Overview

General Particle Filter literature:

As an excellent general book on Particle Filtering with a lot of theory, applications and references:

Conclusions

Specific Conclusions

Every optimal detector can be expressed in terms of PF weights.....Very important result both from a theoretical and practical point of view.

A multi target particle filter for closely spaced targets has been presented for a TBD application. The algorithm can be applied in real time.

Questions/Remarks/Discussions