

GEOFRAC : Large-scale computation of flow in complex 3D geological fractured porous media Proposal for ARC INRIA 2011

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Résumé

In the last twenty years, interest in geological fractured rocks has been renewed by a variety of energy-related applications, such as carbonate oil reservoir exploitation, geothermal energy production, geological storage of high level nuclear waste, and geological sequestration of CO₂. Fractures are highly permeable pathways within a less pervious but more porous medium generally called the rock matrix. Whatever the application or the details of the involved physical and chemical processes, the relative spatial organization of the fracture and matrix phases has a strong influence on exchanges between them. To date most models for flow in fractured media have relied on the separate homogenization of the fracture and matrix phases taken in a second step as two interacting continua. This so-called double-porosity approach has been motivated by the lack of extensive data on the fracture locations, by its conceptual simplicity and by the possibility of using existing developments on the more classical single porosity models. Important limitations of the double porosity approach have however recently come to light. An alternative approach is the discrete modeling of fractures. This approach encounters at least two challenging numerical issues. First, the fracture and matrix phases have very different hydraulic properties. Permeability is at least two orders of magnitude higher in the fractures than in the matrix. Specific numerical methods have to be used to cope efficiently with these highly localized discontinuities. Second, the complexity of the fracture structure creates intricate geometrical configurations which are difficult to mesh. We propose to address these issues by developing new numerical methods adapted to sharp discontinuities. The proposed methods should be highly robust and computationally efficient for handling upscaling problems in a stochastic modeling framework. They will be used in the last stage of this project for determining the upscaling laws in the most commonly encountered fracture formations. The numerical methods will be implemented in the H2OLab development platform used for modeling hydraulic processes in heterogeneous fractured reservoirs.

1 Participants

Our project is by nature multidisciplinary. For the sake of clarity, we identify four scientific domains : numerical analysis (more precisely numerical modeling for Partial Differential Equations), mesh

generation for solving PDEs, high performance computing (more precisely development of numerical parallel algorithms with implementation on distributed memory and grid architectures), geophysics (more precisely hydrogeology). Four partners contribute to the project : the team Estime (a new team, called Pomdapi, is submitted) of INRIA in Rocquencourt, the team Gamma3 of INRIA in Rocquencourt, the team Sage of INRIA in Rennes, the team hydrogeological transfers of the Geosciences department at the University of Rennes 1. The project is coordinated by Sage.

1.1 Participants and skills

Participants of ESTIME : J. Jaffré (DR), M. Kern (CR), J. Roberts (DR)

Skills of ESTIME : models of fractured media, Mixed Finite Element methods, domain decomposition methods.

Participants of GAMMA3 : P-L. George (DR), F. Alauzet (CR).

Skills of GAMMA3 : mesh generation, 3D mesh, surface mesh

Participants of SAGE : J. Erhel (DR), G. Pichot (CR), B. Poirriez (PhD), N. Soualem (engineer).

Skills of SAGE : Mixed Finite Element methods, large sparse linear solvers, high performance computing, software engineering.

Participants of Geosciences Rennes : P. Davy (DR CNRS), J-R. de Dreuzy (CR CNRS), T. Le Borgne (physicien CNAP). J-R. de Dreuzy is currently and for two years at UPC, Barcelona, Spain.

Skills of Geosciences Rennes : characterization of fracture networks and their influence on flow and transport properties, determination of upscaling laws.

1.2 Interactions between teams

The long-lasting collaboration between Sage and Geosciences Rennes existing since 1997 yields a common ground and way of working for this multidisciplinary project, ensuring good interactions between partners of different scientific fields. These two teams are currently involved in the project MICAS, funded by ANR and coordinated by Sage, and in the thematic network RISC-E, managed by OSUR (Observatoire des Sciences de l'Univers de Rennes), which is related to the national network RNSC (Réseau National des Systèmes Complexes). Moreover, Sage and Estime have been collaborating for several years. They are both involved in the MoMaS GNR and in projects of this group related to reactive transport and they frequently organize mini-symposia at the SIAM international conference on Geosciences. The three teams Geosciences, Sage and Estime were partners of the project HYDROGRID, coordinated by Estime. Recently, Sage, Geosciences Rennes and Estime contributed to public dissemination, with a paper in *La Recherche* (May 2009).

This collaborative research project is an opportunity to enhance the partnership between the three

teams Sage, Estime and Geosciences. Their skills are complementary and are required to complete successfully this project. Moreover, a major difficulty will be the generation of the mesh. Therefore, the expertise of the team gamma3 is also essential to this project.

2 Context and objectives

In the last twenty years, interest in geological fractured rocks has been rekindled by a variety of energy-related applications. Fractures are highly permeable pathways within a less pervious but more porous medium generally called the rock matrix. Differences in permeability and in porosity span one to several orders of magnitude [10]. Schematically, most of the fluids and contaminants are in the matrix but move primarily within the fracture network. For carbonate oil reservoirs, oil is located in the matrix but is drained to the production wells by the fractures [7]. For geothermal energy production, heat is diffused from the matrix to the fractures where it is advected by the flowing water to the well [35, 58]. For the geological storage of high level nuclear wastes in crystalline rocks, radionuclides are slowed down by trapping mechanisms in the matrix [44, 50]. For the geological sequestration of CO₂, long-term mineralization of CO₂ is conditioned by the extension of the interface between the fracture and matrix phases [54]. Whatever the application or the particular physical and chemical processes involved, the relative spatial organization of the fracture and matrix phases determines to a large extent the fluid exchanges between the two phases [33]. Intensity of exchanges critically depends not only on the dimension of the fracture matrix interface but also on the unfractured block size distribution. Typical trapping drainage times are determined by the "depth of the trap" taken here as the size of the block [38]. Drainage efficiency is conditioned by the structure of the drainage network (the fractures) within the area to be drained (the matrix).

2.1 State of the art

Most models have relied so far on the separate homogenization of the fracture and matrix phases taken in a second step as two interacting continua [4, 5, 57]. This so-called double-porosity approach was motivated by the lack of extensive data on the fracture locations, by its conceptual simplicity and by the possibility of using the existing developments for the more classical single porosity models. However important limitations of the double porosity approach have recently been recognized.

First, fracture structures cannot be easily homogenized at the scale of interest, i.e. the reservoir scale or the confinement scale [15]. This is fundamentally due to the absence of a characteristic scale in the fracture length distribution and in the correlations between their positions [6]. Deterministic influences of long fractures and of long-range correlation patterns cannot be a priori statistically homogenized in an equivalent continuous phase [19, 20]. The direct consequence of the broad range of fracture structures and correlations is the segmentation of the matrix into a large variety of blocks of different sizes, shapes and connectivity with other blocks [11, 12]. The possible existence of an equivalent double-porosity model as well as its effective determination requires preliminary numerical simulations based on the relevant degree of geometric and hydraulic complexity mimicking those of the natural fractured media.

Second, new developments in geophysical and hydraulic field prospection methods give precise information about some of the main hydraulically active fractures [36, 55]. In that sense, it motivates the former development of approximate discrete double-porosity model which consists in keeping in a discrete fracture network approach all the main fractures while homogenizing all smaller fractures in an equivalent continuous heterogeneous porous approach (PhD of Delphine Roubinet at Geosciences Rennes, 2008-2010). The interest of this approach is to retain the main geometrical influence of fractures on flow and to yield a relatively inexpensive simulation method. The determination of the local-scale continuous permeabilities and of the exchange coefficients is performed on preliminary simulations with simplified boundary conditions [51]. This method is inspired from multi-scale finite element methods [22]. Modeling deterministically the fractures identified close to wells and man-made facilities is of great practical importance for enhancing the relevance of the "close-field" models. It however precludes the use of homogenization approaches and requires again the development of alternative discrete fracture models phenomenologically closer to the field settings.

Third, interest in fractured media has shifted from production applications to risk assessment studies especially for energy waste storage involving the determination of modeling uncertainties for highly complex phenomena [17, 26]. Homogenization approaches cannot be trusted in this context.

The alternative approach is the discrete modeling of fractures [39]. The teams Sage and Geosciences have developed a model where the matrix is considered impervious [24, 46, 47, 48]. The team Geosciences has analyzed the interactions between the matrix and the fractures in 2D complex geometries [51, 52] and has derived upscaling rules for lattice fractured networks [21]. On the other hand, the team Estime has developed a mathematical model of these interactions. The resulting model was studied and analyzed in [1] and an extension to the transport equation was given. The model described above was proposed and analyzed for the case of a single fracture in [40]. Numerical experiments were carried out in the 2D case. The case of intersecting fractures and a simple 3D experiment may be found in [2]. More complex 3D case will require the construction of three-dimensional meshes, as the ones developed by the team gamma3 [27]. The case of nonconforming grids can be found in [29] and an analysis and 2D numerical study is given in [28].

This approach comes with at least four numerical challenges. The fracture and matrix phases have very different hydraulic properties, with permeability which is at least two orders of magnitude larger in the fractures than in the matrix. Moreover, many fractures intersect each other, so that it is quite difficult to handle the exchange terms. Specific numerical models have to be used to cope efficiently with these highly localized discontinuities and highly complex topologies [45]. The complexity of the fracture structure generates intricate geometrical configurations which are really hard to mesh. Automatic mesh generation exists only for the fracture phase [24, 56]. Realistic computational domains composed of the matrix coupled with a network of fractures have never been automatically meshed [32]. In 3D, the linear system is large and sparse, with particular properties due to the model, so that some iterative linear solvers can fail.

2.2 Objectives

We propose to address these limitations by developing new numerical methods adapted for sharp discontinuities and stochastic fracture networks. We will focus on single-phase hydraulic processes.

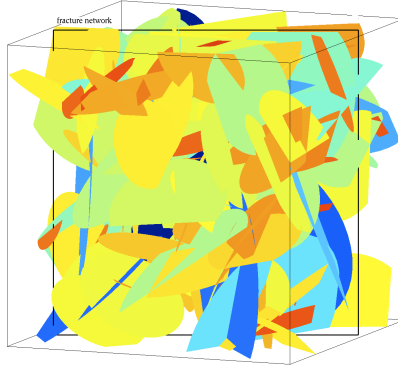


FIGURE 1 – An example of 3D fractured porous media.

It is the basic process required for handling more advanced transport processes. It is also a necessary first step before solving multiphase flows. We will consider transient flow, since time plays a major role in exchanges between the fractures and the matrix. For steady-state flow, we could consider an impervious matrix. The proposed methods should be highly robust and computationally efficient for handling upscaling issues in a stochastic modeling framework. They will be used in the last phase of this project for determining the upscaling laws in the most commonly encountered fracture organizations [16]. The numerical methods will be implemented in the H2OLAB development platform used for modeling hydraulic processes in heterogeneous fractured reservoirs [23] (<http://h2olab.inria.fr/>).

The project will be organized in three phases : the design of numerical models handling sharp discontinuities, the development and the implementation of numerical methods using advanced mesh generation and high performance computing, and the application of the method to determine upscaling rules. The natural organization of tasks is sequential. The first two tasks should be successfully fulfilled to determine the upscaling rules in typical fractured media. Rather than following this sequential organization of tasks with non-negligible risks of failure due to stumbling blocks at different stages, we propose to partially separate the different tasks by using a series of fracture networks of evolving complexity. The series will be made up of simple 2D fracture networks, 3D fracture networks with fully crossing fractures, 3D fracture networks with broad distribution of fracture lengths and complex fracture orientations. An example of a 3D fractured porous media is given in Figure 1. This kind of network is the challenging target of the project. For example, the two first types of fracture networks present standard block configurations easy to mesh. They however contain a sufficient level of complexity for testing the numerical methods with sharp discontinuities on a large number of interfaces both in 2D and 3D.

3 Research program

For sake of clarity, the research program is described in three tasks but, again, these three tasks will be highly interconnected, with feedback from each other. The milestones are given in the post-doc program.

3.1 Numerical model of fractured porous media

The objective is to consider a porous fractured media where water can flow in both the matrix (the rock) and the fractures, with permeabilities spanning several orders of magnitude and with a stochastic network of fractures. The project will start from the results obtained by Estime. To model transient flow in a porous fractured medium with an interaction between the fractures and the matrix we consider the following model : we identify the porous medium with a domain Ω in R^3 . We suppose that the fractures may be identified with a set of planar surfaces $\gamma_{i,j}$ whose closures $\bar{\gamma}_{i,j}$ separate Ω into a finite collection of subdomains Ω_i where $\bar{\gamma}_{i,j} = \bar{\Omega}_i \cap \bar{\Omega}_j$. (Thus if $\bar{\Omega}_i \cap \bar{\Omega}_j$ is empty or is one dimensional then $\gamma_{i,j}$ is the empty set. We also define $\gamma_{i,i}$ to be the empty set.) We make the simplifying assumption that for each fracture $\gamma_{i,j}$, $\partial\gamma_{i,j} \subset \cup_{k,\ell \neq i,j} \partial\gamma_{k,\ell} \cup \partial\Omega$. Let $\gamma = \cup_i \partial\Omega_i \setminus \partial\Omega = \cup_{i,j} \bar{\gamma}_{i,j} \cap \Omega$. Let $\sigma = \cup_{i,j} \partial\gamma_{i,j} \setminus \partial\Omega$.

We suppose here that the fluid is a single phase, incompressible fluid, and for simplicity we ignore the effects of gravity. This type of model has been considered in a vertex centered finite volume scheme in [49]. Another possibility is to consider Forchheimer flow in the fractures. Such a model has been presented in [29] and analysed in [34].

Up to now, we dealt with a steady-state flow. A first task will be to generalize to the transient case the steady-state model which is described below. In each subdomain we suppose that Darcy's law as well as the law of mass conservation is satisfied :

$$\begin{aligned} \mathbf{u}_i &= -\mathbf{K}_i \nabla p_i, & \text{in } \Omega_i \\ \operatorname{div} \mathbf{u}_i &= f_i, & \text{in } \Omega_i, \end{aligned} \quad (1)$$

where p_i , \mathbf{u}_i , \mathbf{K}_i , and f_i are respectively the fluid pressure, the Darcy velocity, the permeability tensor, and the source term in Ω_i .

In each fracture we suppose that the flow is governed by Poiseuille's law and the law of mass conservation. Averaging the tangential components of Darcy's law and the law of mass conservation across a transverse cross section of the physical fracture represented by $\gamma_{i,j}$, one obtains

$$\begin{aligned} \mathbf{u}_{i,j} &= -\mathbf{K}_{i,j} \nabla_{\tau} p_{i,j}, & \text{in } \gamma_{i,j} \\ \operatorname{div}_{\tau} \mathbf{u}_{i,j} &= f_{i,j} + (\mathbf{u}_i \cdot \mathbf{n}_i + \mathbf{u}_j \cdot \mathbf{n}_j), & \text{in } \gamma_{i,j}, \end{aligned} \quad (2)$$

where as before $p_{i,j}$, $\mathbf{u}_{i,j}$, and $f_{i,j}$ are respectively the fluid pressure, the Darcy velocity, and the source term in $\gamma_{i,j}$. The effective transmissivity $\mathbf{K}_{i,j}$ in $\gamma_{i,j}$ is the tangential component of the fracture transmissivity, $\mathbf{K}_{\tau,i,j}$, multiplied by the fracture width function, $d_{i,j} : \mathbf{K}_{i,j} = d_{i,j} \mathbf{K}_{\tau,i,j}$. The operators $\operatorname{div}_{\tau}$ and ∇_{τ} are the tangential divergence and tangential gradient operators respectively. The vectors \mathbf{n}_i and \mathbf{n}_j are the unit exterior normal vectors on Ω_i and Ω_j , respectively so that $(\mathbf{u}_i \cdot \mathbf{n}_i + \mathbf{u}_j \cdot \mathbf{n}_j)$ represents a source term due to the difference in what flows into the fracture from one side and what flows out of the fracture into the other side.

Boundary conditions for each subdomain Ω_i must be specified. On $\partial\Omega_i \cap \partial\Omega$ we suppose that appropriate boundary conditions have been specified. On $\gamma_{i,j}$ the condition, obtained by averaging the normal component of Darcy's law across the fracture, is

$$p_i = p_{i,j} + \kappa_{i,j}^{-1} (\xi \mathbf{u}_i + \bar{\xi} \mathbf{u}_j) \cdot \mathbf{n}_i, \quad (3)$$

where $\kappa_{i,j}$ is the normal component of the permeability, $K_{\nu,i,j}$, divided by half the width : $\kappa_{i,j} = K_{\nu,i,j}/(d_{i,j}/2)$, $\xi > 1/2$ is a quadrature parameter and $\bar{\xi} = 1 - \xi$.

The boundary conditions on $\partial\gamma_{i,j} \cap \partial\Omega$ are again determined by the given data for the problem, (by averaging in the case of a Dirichlet condition or by integrating in the case of a Neumann condition). Transmission conditions are given on $\sigma' = \partial\gamma_{i,j} \cap \sigma \cap_{k,\ell} \partial\gamma_{k,\ell}$ by

$$\begin{aligned} p_{i,j} &= p_{\sigma}, & \text{on } \sigma' \\ \mathbf{u}_{i,j} \cdot \mathbf{n}_{i,j} &= - \sum_{k,\ell} \mathbf{u}_{k,\ell} \cdot \mathbf{n}_{k,\ell}, & \text{on } \sigma'. \end{aligned} \quad (4)$$

We have supposed that $\partial\gamma_{i,j} \setminus \partial\Omega \subset \cup_{k,\ell \neq i,j} \partial\gamma_{k,\ell}$. This is not a major restriction as we can always extend fractures or add "fractures" that have the same characteristics as the surrounding medium. However other possibilities have been proposed in [3] and [41]. In the case of very thin fractures and large normal permeability (when $\kappa_{i,j}$ is very large) the condition given in (3) may be replaced by the continuous pressure condition

$$p_i = p_{i,j}. \quad (5)$$

In the model described above, at the intersection of two (or more) fractures, we have assumed the continuity of the pressure and the conservation of the flux across the intersection. Another possibility would be to suppose that the fluid may also flow along the intersection of the fractures in a 1D medium which is more permeable than the intersecting fractures.

Now, the research program is to use this model for complex 3D geometries, where many fractures intersect each other. Several difficulties have been handled with such fracture networks in the case of an impervious matrix. The challenge is here to simulate the exchanges between the matrix and the fractures. The model specifying these exchanges as well as the flow at the intersections of fractures will be adapted as the research will progress. We will progressively increase the complexity of the geometry. This modeling task will involve the three teams Estime, Sage and Geosciences.

3.2 Numerical methods and algorithms

Numerical discretization will be carried out with mixed finite element methods (MFE). In general, finite element methods allow one to deal with complex geometries and to use locally refined meshes. In a mixed finite element method, the unknowns are hydraulic head and velocity ; thus the velocity field is a good approximation, and useful for subsequent transport problems. Also this method guarantees both local and global mass conservation ; therefore, MFE methods and hybrid variants are very well-suited for solving flow problems [30], [31]. They have been successfully applied to complex fracture networks and to simple fractured media.

The first step is to define the mesh of the computational domain. In 2D, both teams Sage and Estime can provide the mesh. In 3D, we will use the mesh generator of fracture networks already available [24]. Then we will have to generate a mesh of the matrix and to combine both grids. This task will rely on the expertise of the team gamma3 in the construction of three-dimensional meshes for the numerical solution of partial differential equations [27]. Due to the complexity of the

geometry we plan to use tetrahedral meshes. The basic technique for constructing these meshes uses Delaunay triangulation algorithms which are modified in order to not mesh a set of points, but to mesh domains defined by a mesh of their boundary. Thus the data is a triangular mesh of a closed surface. It is also possible to have open surfaces inside the domain : in our problem they will be the fractures. The result depends both on the quality of the data (aspect ratio) and on the density of these data. The software which produces these meshes is GHS3D, developed by the gamma3 team. An alternative to this method of meshing would be to construct nonconforming meshes where meshes in the fractures do not match meshes on the two faces of the fractures. This is allowed in the proposed numerical model. In order to deal with these nonconforming grids, we will rely on previous work, using either the numerical method defined in [29] or the mortar-like method defined in [47, 48]. This first step will involve the three teams gamma3, Estime and Sage.

Since the problem is linear, the main computational task is to solve a large sparse linear system, or a series of linear systems in the case of transient flow. This is the topic of the second step. Direct sparse solvers are very efficient but require too much memory for the large 3D cases considered. Therefore, we will have to deal with either iterative or semi-iterative methods. Clearly, a multilevel method is required here, in order to increase the size of the system without inordinately increasing the number of iterations. This multilevel approach can be used in domain decomposition methods such as Schur combined with Neumann-Neumann for example or as a preconditioner in PCG (Preconditioned Conjugate Gradient). We will first investigate a multigrid preconditioner then a Schwarz preconditioner [9]. For both Schur and Schwarz approaches, global preconditioners such as coarse grid correction or deflation are required [9, 42, 43, 53, 25]. In particular, we will investigate how domain decomposition and deflation methods can be adapted to these specific geometries. Indeed, the geometry is naturally decomposed into blocks, yielding many ways of substructuring the computational domain. However, it is not clear how to assemble these blocks in order to get a relatively small number of subdomains. Another challenge is to deal with the exchange terms, which are not at all classical and yield a linear system with specific properties. High performance computing is mandatory to deal with large computational domains. A challenge will be to design a parallel version of domain decomposition methods. Indeed, the geometry is very complex and we aim at using nonconforming grids. This second step will involve the three teams Sage, Estime and Geosciences.

3.3 Upscaling flow in typical fracture configuration cases

The objective is to determine the influence of small scale fracture organization on larger scale exchanges between fracture and matrix. Results will be expressed in the form of the exchange coefficients also called shape factor in the double-porosity framework [37]. Although the final results can only be obtained at the completion of the previous tasks, we plan to estimate the upscaling rules on increasingly complex fracture structures all along the project. At the beginning of the project, we will use approximated discrete double-porosity model presented in section 2.1 to determine the large-scale exchange coefficients on 2D fracture networks. The method heavily relies on this smaller scale fracture network simulations basically performed with the superposition of a regular grid. We will first use the existing simulation methodology to estimate the upscaling rules in 2D fracture structures. We will especially focus on highly differentiated structures where block distributions are fractal like in fragmentation and Sierpinski lattice models [8, 18]. These existing methods would greatly benefit from the methods developed in the first tasks of this project. The smaller scale

methods will be performed with the newly developed more accurate numerical methods. A broader range of 2D fracture structures will be tested including the Universal Fracture Model recently proposed by Davy et al. [16]. This more realistic model relies on simple excluded volume rules for the generation of the fracture network. It has been positively evaluated on the well-documented Swedish sites studied for nuclear waste disposals [13, 14]. Depending on the state of completion of the previous tasks, full 3D network structures will be studied. We will focus first on strongly correlated large scale fracture structures easier to mesh. As in 2D, we will study the outcome of upscaling for a 3D Sierpinski lattice. The objective is to find and understand the differences between 2D and 3D upscaling rules. This research task will involve the three teams Geosciences, Estime and Sage.

4 Results and perspectives

We plan to contribute to the improvement of the simulation capacities for fracture/matrix geological systems and to the determination and understanding of upscaling laws and potential scale dependence of fracture/matrix exchanges [59]. We list here three projected results of this project :

1. A methodology for simulating flow in complex fracture/matrix systems. This result will take the form of a series of scientific publications in applied mathematics and computational journals like SIAM or JCP.
2. A software prototype for fracture/matrix flow simulations. This software will be developed in the hydrogeological simulation platform H2OLab. It will be built on the competences and developments assimilated through this project. It will be made up of the 3D matrix mesh generator, spatial and temporal discretization schemes, adapted system solvers and visualization tools. It will be fully designed for parallel computations. Depending on its final state of completion, this software may be licensed under LGPL.
3. Upscaling rules for 2D and simple 3D network structures. Results will be published in geosciences journals like *Advances in Water Resources* or *SPE (Society of Petroleum Engineers)*.

In a longer term, we plan first to develop the fracture/matrix interacting models for simulating more complex multiphase flow models. Multiphase flow is more complex both in terms of numerical methods and geological applications. The side objective is to get closer to oil and CO₂ sequestration applications. The second objective is to use the developed methods as a basis for handling the inverse problem (identification of geological structures from remote hydraulic data). The simulation model will be used as the direct model within the inverse approach. The teams Geosciences Rennes and Sage are increasingly involved in inverse problem studies both from the field perspective (O. Bour, T. Le Borgne, PhD of Maria Klepikova 2010-2012) and from the development of efficient parametrization and inversion methods (PhD of Romain Le Goc 2005-2009, PhD of Sinda Khalfallah 2010-2012 in collaboration with A. Ben Abda (LAMISIN, Tunis)).

This project is a natural continuation of the current ANR project MICAS 2008-2011 and a preliminary step towards the submission of a new ANR project (young researcher, led by G. Pichot). We will also submit a european project (COST program), involving mainly the teams from UPC, Barcelona (Spain) where J-R. de Dreuzy is working now, the team from UFZ, Leipzig (Germany) where G. Pichot spent three months in 2010, Switzerland and Great Britain.

TABLE 1 – Projected budget 2011-2012 of GEOFRAC project (in €).

Type of cost	2011	2012
travel expenses in France	6000	6000
travel expenses from Spain	3000	3000
Post-doc stays in Paris	2000	2000
Post-doc stays in Barcelona	3000	3000
staff cost (post-doc)	24 000	48 000
computer	2000	
total	40 000	62 000

5 Budget

Costs of the project include travel expenses, both in France and in Spain. We ask for a post-doctoral position during 18 months in order to design the numerical method and to develop the software prototype described above (the post-doctoral subject is detailed below). We ask also for a laptop. The post-doctoral researcher will be a member of team Sage and will work in close relation with Geosciences Rennes. We plan several stays at INRIA Rocquencourt in order to collaborate with Estime and gamma3 teams. We plan also several stays at UPC, Barcelona, in order to collaborate with J-R. de Dreuzy and the team there.

Travel expenses in France and from Spain are computed on the basis of 3 meetings each year in Rennes, or Paris, with 4 persons moving in France and one person moving from Spain, thus 12 national journeys per year at 500 € each and 3 international journeys per year at 1000 € each.

Travel expenses also include stays for the post-doctoral fellowship. More precisely, they include two stays at Rocquencourt of 5 days each year, at 1000 € each; and one stay at UPC of 12 days each year, at 3000 € each.

Staff cost (post-doctoral position) is evaluated at 24 000 € for six months.

5.1 Post-doctoral subject

The objective of the 18 months-postdoctoral position is to study in detail the numerical models and methods for simulating flow in fractured porous media. The post-doctoral researcher will work with all the teams, mainly on the two first phases described above. Several numerical tools need to be developed : mesh generation ; spatial discretization with conforming or nonconforming grids ; temporal discretization for stiff systems ; domain decomposition methods adapted for fractured media ; sparse linear solvers ; scalable parallel algorithms.

As we said in section 2.2, rather than performing the task sequentially, the post-doctoral fellow will study networks of increasing complexity. During the first 6 months of the project, before the arrival of the post-doctoral fellow, we will adapt the mathematical model to the transient case and

to fracture networks with many fractures intersecting each other. We will first develop a Matlab prototype with some simple 2D configurations and the steady-state case. We will also continue to work on sparse linear solvers for the steady-state case, considering networks of fractures and an impervious matrix. This preliminary step will provide sufficient material to start the post-doctoral program. Three work packages have been defined and scheduled :

- The first 6 months period (2011) is devoted to the 2D porous fractured case for which both the teams Sage and Estime are able to provide a mesh with intersecting fractures. The main objective of this first step is to generalize the model developed by Estime (section 3.1) for a stochastic generation of fractures and to study numerical convergence of temporal and spatial discretizations. This model will be implemented in a Matlab prototype, in order to understand the numerical behaviour as well as some physical properties by varying the contrast of permeability between the matrix and the fractures.
- During the next 6 months (2012), the second step is to work on 3D fractured porous media. At this stage, the expertise of the team gamma3 will be essential for the challenging mesh generation. At first, a limited number of fractures will be considered with simple crossing configurations. The objective is to rewrite the model and its mathematical formulation in 3D. Once the linear system (or the series of linear systems in case of transient flow) is set up, the next challenge is to solve the problem using domain decomposition methods. The main outcome should be a software prototype, integrated in the platform H2OLab.
- The last stage (2012) will consist in increasing the number of fractures and the complexity of their geometry and orientation. The challenge is the mesh generation as well as the writing of the exchange terms between the fractures and the matrix when fractures are highly intricate. With the increase of the size of the linear system, the development of parallel domain decomposition methods is mandatory. The main difficulty is to deal with a large number of subdomains and interfaces. Depending on the issues, we may think of using a non conforming mesh generation.

Références

- [1] C. Alboin, J. Jaffré, J. Roberts, and C. Serres. Modeling fractures as interfaces for flow and transport in porous media. In Zhangxin Chen and Richard E. Ewing, editors, *Fluid flow and transport in porous media : mathematical and numerical treatment (South Hadley, MA, 2001)*, number 295 in Contemp. Math., pages 13–24, Providence, RI USA, 2002. Amer. Math. Soc.
- [2] L. Amir, M. Kern, V. Martin, and J. E. Roberts. Décomposition de domaine pour un milieu poreux fracturé : un modèle en 3d avec fractures qui s’intersectent. *ARIMA*, 5 :11–25, 2006.
- [3] Ph. Anjot, F. Boyer, and F. Hubert. Asymptotic and numerical modelling of flows in fractured porous media. *M2AN*, 43(2) :239–275, 2009.
- [4] T. Arbogast, J. Douglas, and U. Hornung. Derivation of the double porosity model of single-phase flow via homogenization theory. *SIAM J. Math. Anal.*, 21 :823–836, 1990.
- [5] G. I. Barenblatt, I. P. Zheltov, and I. N. Kochina. Basic concept in the theory of seepage of homogeneous liquids in fissured rocks. *J. Appl. Math.*, 24 :1286–1303, 1960.
- [6] E. Bonnet, O. Bour, N. Odling, P. Davy, I. Main, P. Cowie, and B. Berkowitz. Scaling of fracture systems in geological media. *Reviews of Geophysics*, 39(3) :347–383, 2001.
- [7] O. Bour and P. Davy. On the connectivity of three dimensional fault networks. *Water Resources Research*, 34(10) :2611–2622, 1998.

- [8] O. Bour and P. Davy. Clustering and size distributions of fault patterns : theory and measurements. *Geophysical Research Letters*, 26(13) :2001–2004, 1999.
- [9] L. Carvalho, L. Giraud, and P. Le Tallec. Algebraic two-level preconditioners for the Schur complement method. *SIAM J. Sci. Comput.*, 22 :1987–2005, 2001.
- [10] National Research Council. *Rock Fractures and Fluid Flow*. National Academy Press, Washington, D.C., 1996.
- [11] C. Darcel, O. Bour, and P. Davy. Cross-correlation between length and position in real fracture networks. *Geophysical Research Letters*, 30(12), 2003.
- [12] C. Darcel, O. Bour, P. Davy, and al. Connectivity properties of two-dimensional fracture networks with stochastic fractal correlation. *Water Resources Research*, 39, 2003.
- [13] C. Darcel, P. Davy, O. Bour, and al. Alternative dfn model based on initial site investigations at simpevarp. Skp report, Geosciences Rennes, 2004.
- [14] C. Darcel, P. Davy, O. Bour, and al. Discrete fracture network for the Forsmark site. Skb report, Geosciences Rennes, 2006.
- [15] P. Davy, O. Bour, J.-R. De Dreuzy, and C. Darcel. Flow in multiscale fractal fracture networks. In 261, editor, *Geological society, London, special publications*, pages 31–45, 2006.
- [16] P. Davy, R. Le Goc, C. Darcel, O. Bour, J.-R. de Dreuzy, and R. Munier. A universal model of fracture scaling and its consequence for crustal hydro-mechanics. *Journal of Geophysical Research - solid earth*, in press.
- [17] F. P. J. de Barros and Y. Rubin. A risk-driven approach for subsurface site characterization. *Water Resources Research*, 44, 2008.
- [18] J.-R. de Dreuzy and P. Davy. Relation between fractional flow and fractal or long-range permeability field in 2d. *Water Resources Research*, 43, 2007.
- [19] J.-R. de Dreuzy, P. Davy, and O. Bour. Hydraulic properties of two-dimensional random fracture networks following a power law length distribution : 1-effective connectivity. *Water Resources Research*, 37 :2065–2078, 2001.
- [20] J.-R. de Dreuzy, P. Davy, and O. Bour. Hydraulic properties of two-dimensional random fracture networks following a power law length distribution : 2-permeability of networks based on log-normal distribution of apertures. *Water Resources Research*, 37 :2079–2095, 2001.
- [21] J.-R. de Dreuzy, P. de Boiry, G. Pichot, and P. Davy. Use of power-averaging for quantifying the influence of structure organization on permeability upscaling in on-lattice networks under mean parallel flow. *Water Resources Research*, 46(W08519) :11 pages, 2010.
- [22] Y. Efendiev and T. Hou. Multiscale finite element methods for porous media flows and their applications. *Applied Numerical Mathematics*, 57 :577–596, 2007.
- [23] J. Erhel, J.-R. de Dreuzy, A. Beaudoin, E. Bresciani, and D. Tromeur-Dervout. A parallel scientific software for heterogeneous hydrogeology. In Ismail H. Tuncer, Ulgen Gulcat, David R. Emerson, and Kenichi Matsuno, editors, *Parallel Computational Fluid Dynamics 2007*, volume 67 of *Lecture Notes in Computational Science and Engineering*, pages 39–48. Springer, 2009. invited plenary talk.
- [24] J. Erhel, J.-R. de Dreuzy, and B. Poirriez. flow simulations in three-dimensional discrete fracture networks. *SIAM Journal on Scientific Computing*, 31(4) :2688–2705, 2009.
- [25] J. Erhel and F. Guyomarc’h. An augmented conjugate gradient method for solving consecutive symmetric positive definite systems. *SIAM Journal on Matrix Analysis and Applications*, 21(4) :1279–1299, 2000.

- [26] R.A. Freeze, J. Massmann, L. Smith, and al. Hydrogeological decision-analysis .1. a framework. *Ground Water*, 28 :738–766, 1990.
- [27] P.J. Frey and P.L. George. *Mesh Generation*. ISTE and Wiley, 2008.
- [28] N. Frih, V. Martin, J. E. Roberts, and A. Saada. Modeling fractures as interfaces with non-matching grids. *in preparation*, 2010.
- [29] N. Frih, J. E. Roberts, and A. Saada. Modeling fractures as interfaces : a model for Forchheimer fractures. *Comput. Geosci.*, 12 :91–104, 2008.
- [30] H. Hoteit, J. Erhel, R. Mosé, B. Philippe, and P. Ackerer. Numerical reliability for mixed methods applied to flow problems in porous media. *Computational Geosciences*, 6 :161–194, 2002.
- [31] H. Hoteit, R. Mosé, B. Philippe, P. Ackerer, and J. Erhel. The maximum principle violations of the mixed-hybrid finite-element method applied to diffusion equations. *International Journal for Numerical Methods in Engineering*, 55(12) :1373–1390, 2002.
- [32] T. Kalbacher, R. Mettier, C. McDermott, W. Wang, G. Kosakowski, T. Taniguchi, and O. Kolditz. Geometric modelling and object-oriented software concepts applied to a heterogeneous fractured network from the Grimsel rock laboratory. *Computational Geosciences*, 11(1) :9–26, Mar 2007.
- [33] M. Kfoury, R. Ababou, B. Noetinger, and al. Upscaling fractured heterogeneous media : Permeability and mass exchange coefficient. *Journal of Applied Mechanics-Transactions of the Asme*, 73 :41–46, 2006.
- [34] P. Knabner and J. E. Roberts. Forchheimer flow in porous media with fractures. *in preparation*, 2010.
- [35] O. Kolditz and C. Clauser. Numerical simulation of flow and heat transfer in fractured crystalline rocks : Application to the hot dry rock site in rosemanowes (u.k.). *Geothermics*, 27 :1–23, 1998.
- [36] F. L. Paillet O. Bour al. Le Borgne, T. Cross-borehole flowmeter tests for transient heads in heterogeneous aquifers. *Ground Water*, 44 :444–452, 2006.
- [37] K.T. Lim and K. Aziz. Matrix-fracture transfer shape factors for dual-porosity simulators. *J. Pet. Sci. Eng.*, 13 :169–178, 1995.
- [38] H.H. Liu, Y. Q. Zhang, and F. J. Molz. Scale dependence of the effective matrix diffusion coefficient : Some analytical results. *Vadose Zone Journal*, 6 :679–683, 2007.
- [39] J. C. S. Long, J. S. Remer, C. R. Wilson, and P. A. Witherspoon. Porous media equivalents for networks of discontinuous fractures. *Water Resources Research*, 18(3) :645–658, 1982.
- [40] V. Martin, J. Jaffré, and J. Roberts. Modeling fractures and barriers as interfaces for flow in porous media. *SIAM J. Sci. Comput.*, 26(5) :1667–1691, 2005.
- [41] F. Moreles and R.E. Showalter. The narrow fracture approximation by channeled flow. *JMAA*, 36(5) :320–331, 2010.
- [42] R. Nabben and C. Vuik. A comparison of Deflation and Coarse Grid Correction applied to porous media flow. *SIAM J. Numer. Anal.*, 42 :1631–1647, 2004.
- [43] R. Nabben and C. Vuik. A comparison of deflation and the balancing preconditioner. *SIAM J. Sci. Comput.*, 27 :1742–1759, 2006.
- [44] I. Neretnieks. Diffusion in the rock matrix : An important factor in radionuclide retardation ? *Journal of Geophysical Research*, 85 :4379–4397, 1980.

- [45] B. Noetinger, T. Estebenet, and P. Landereau. A direct determination of the transient exchange term of fractured media using a continuous time random walk method. *Transport in Porous Media*, 44, 2001.
- [46] G. Pichot, J.-R. de Dreuzy, J. Erhel, and P. Davy. Flow in multi-scale fracture networks : numerical optimization by use of a mortar-like method. In B. Amaziane, D. Barrera, M. Fortes, M. Ibanez, M. Odunlami, A. Palomares, M. Pasadas, M. Rodriguez, and D. Sbibih, editors, *Proceedings of the third international conference on approximation methods and numerical modelling in environment and natural resources, MAMERN'09*, volume 2, pages 761–766. EUG, 2009.
- [47] G. Pichot, J. Erhel, and J.R. de Dreuzy. A mixed hybrid mortar method for solving flow in discrete fracture networks. *Applicable Analysis*, 89(10) :1629–1643, 2010.
- [48] G. Pichot, J. Erhel, and J.R. de Dreuzy. A generalized mixed hybrid mortar method for solving flow in stochastic discrete fracture networks. *SIAM Journal on scientific computations*, submitted.
- [49] V. Reichenberger, H. Jakobs, P. Bastian, and R. Helmig. A mixed-dimensional finite volume method for multiphase flow in fractured porous media. *Advances in Water Resources*, 29(7) :1020–1036, 2006.
- [50] B.E. Rotter, D. A. Barry, J. I. Gerhard, and al. Modeling u(vi) biomineralization in single- and dual-porosity porous media. *Water Resources Research*, 44, 2008.
- [51] D. Roubinet, J.-R. de Dreuzy, and P. Davy. Connectivity-consistent mapping method for 2d discrete fracture networks. *Water Resources Research*, 46(W07532), 2010.
- [52] D. Roubinet, H.-H. Liu, and J.-R. de Dreuzy. A new particle-tracking approach to simulating transport in heterogeneous fractured porous media. *Water Resources Research*, in press.
- [53] Y. Saad, M. Yeung, J. Erhel, and F. Guyomarc'h. A deflated version of the conjugate gradient algorithm. *SIAM Journal on Scientific Computing*, 21(5) :1909–1926, 2000.
- [54] C.F. Tsang, J. Birkholzer, and J. Rutqvist. A comparative review of hydrologic issues involved in geologic storage of CO₂ and injection disposal of liquid waste. *Environmental Geology*, 54 :1723–1737, 2008.
- [55] G.P. Tsoffias, J. P. Van Gestel, P. L. Stoffa, and al. Vertical fracture detection by exploiting the polarization properties of ground-penetrating radar signals. *Geophysics*, 69 :803–810, 2004.
- [56] M. Vohralík, J. Maryska, and O. Severýn. Mixed and nonconforming finite element methods on a system of polygons. *Applied Numerical Mathematics*, 57 :176–193, 2007.
- [57] J.E. Warren, P. J. Root, and M. Aime. The behavior of naturally fractured reservoirs. *Society of Petroleum Engineers Journal*, 1963.
- [58] T.F. Xu, E. Sonnenthal, N. Spycher, and al. Toughreact - a simulation program for non-isothermal multiphase reactive geochemical transport in variably saturated geologic media : Applications to geothermal injectivity and co₂ geological sequestration. *Computers & Geosciences*, 32 :145–165, 2006.
- [59] Q.L. Zhou, H. H. Liu, F. J. Molz, and al. Field-scale effective matrix diffusion coefficient for fractured rock : Results from literature survey. *Journal of Contaminant Hydrology*, 93 :161–187, 2007.