NUMERICAL ALGORITHMS
and
SCIENTIFIC COOPERATION
Yaoundé, March 2009

In honor of Bernard Philippe

How to compute flow in 3D fracture networks

Collaboration between

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Work supported by ANR project MICAS

Surface water and groundwater

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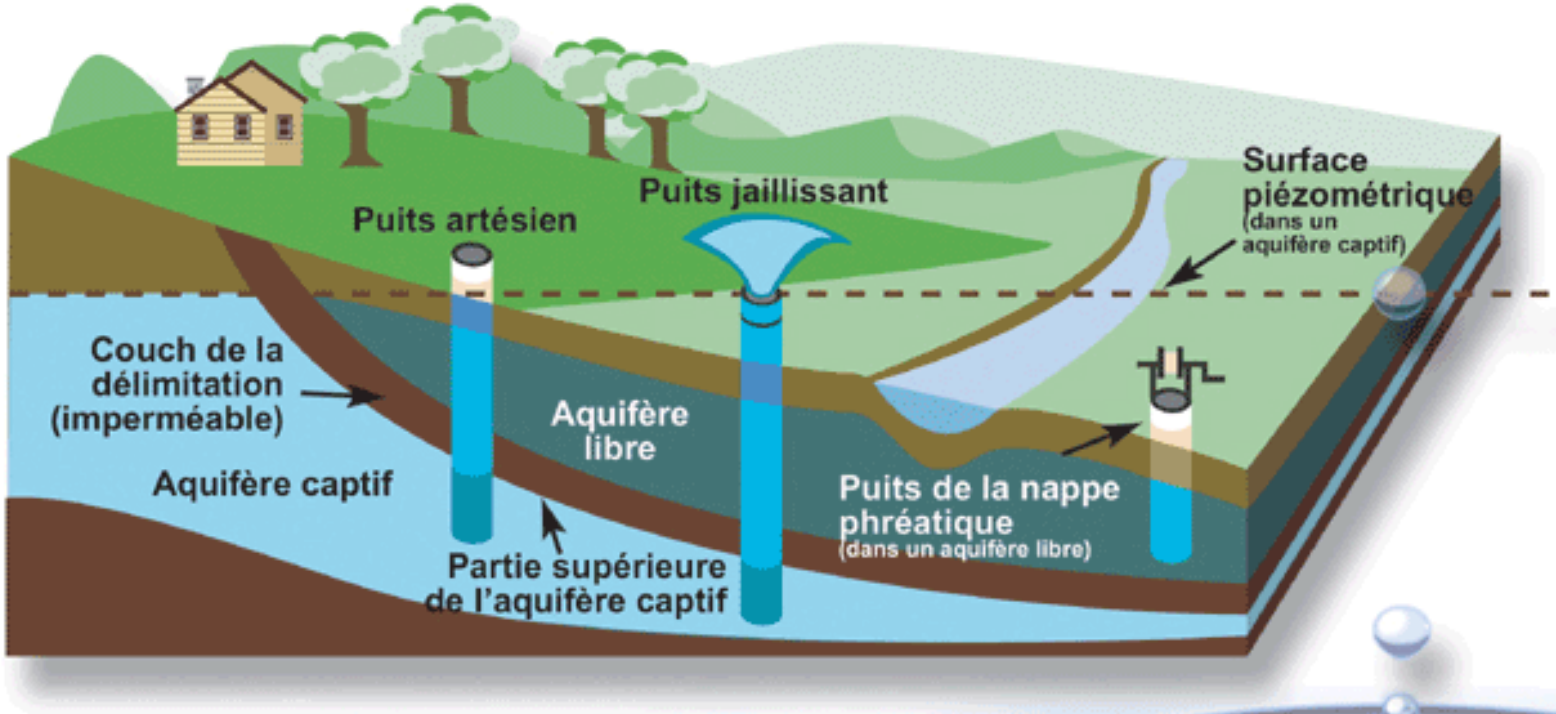
Aquifères et puits



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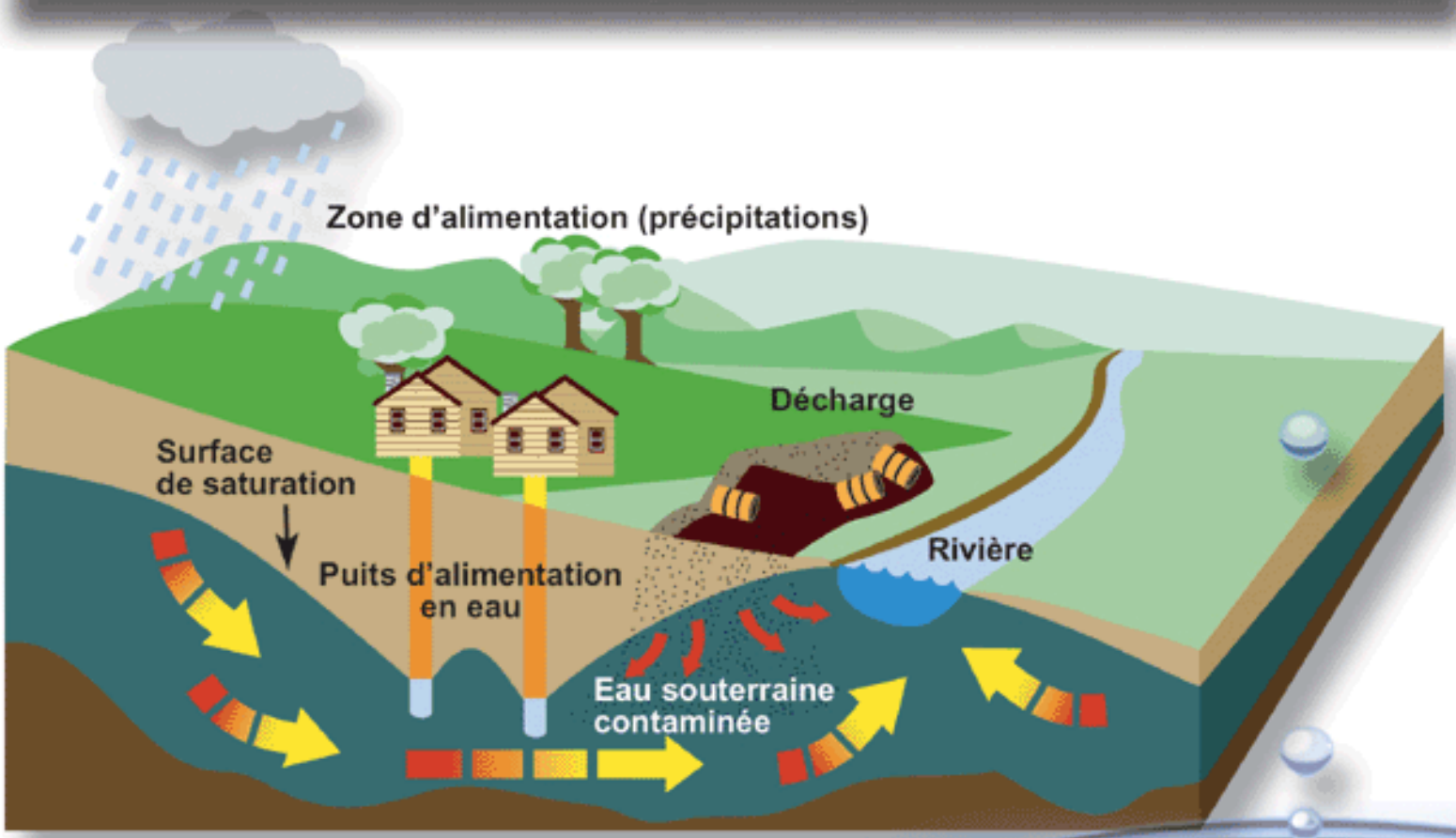
Contamination des eaux souterraines par un décharge



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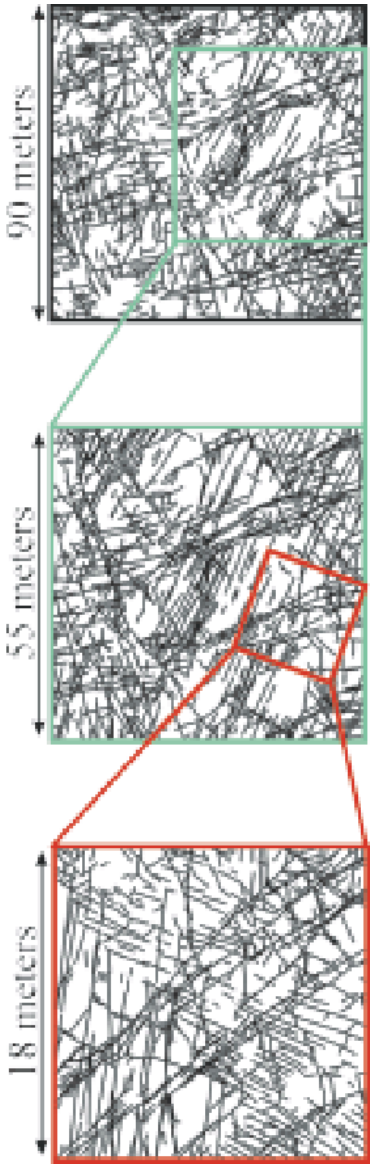
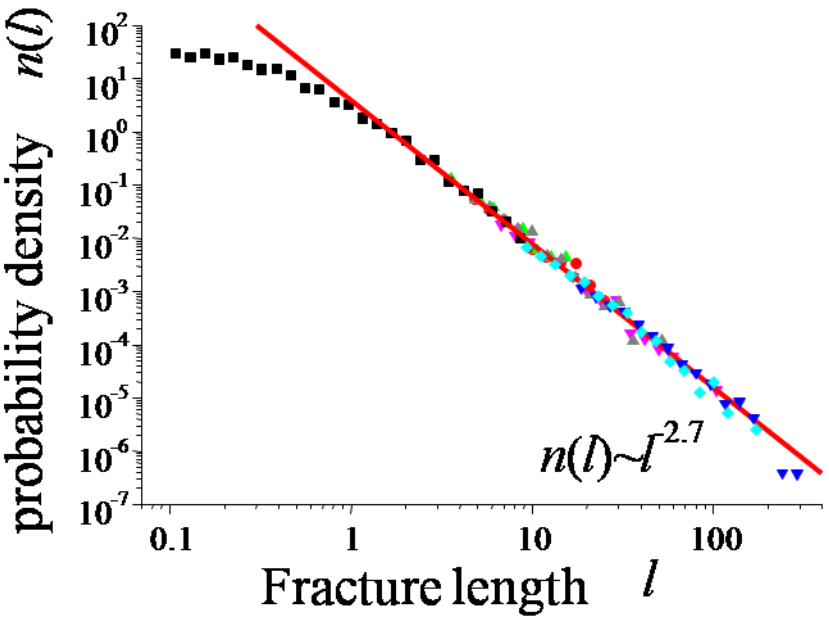
Groundwater numerical models

- **Understand physical phenomena**
- **Manage water resources**
- **Prevent risks of pollution**
- **Help in remediation**

Natural fractured media





Fractures exist at any scale with no correlation

Fracture length is a parameter of heterogeneity

- **Site of Hornelen, Norwegen**

Numerical models for fractured rocks

fractured geological media

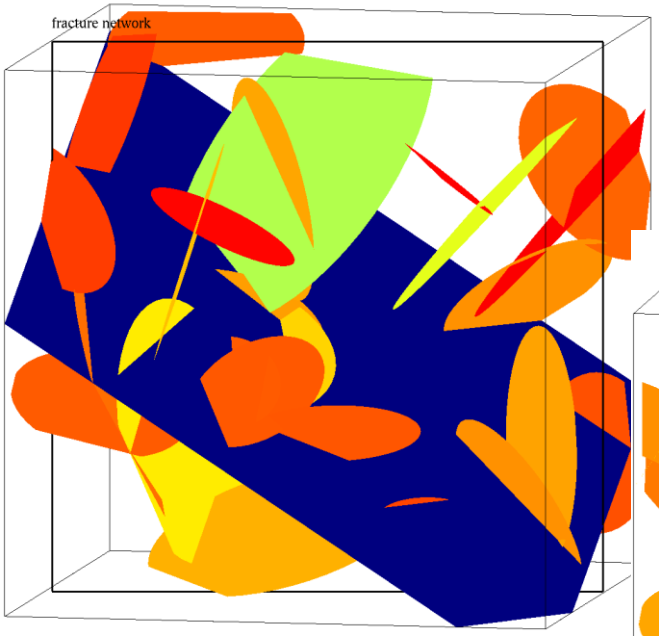
Spatial heterogeneity

Lack of observations

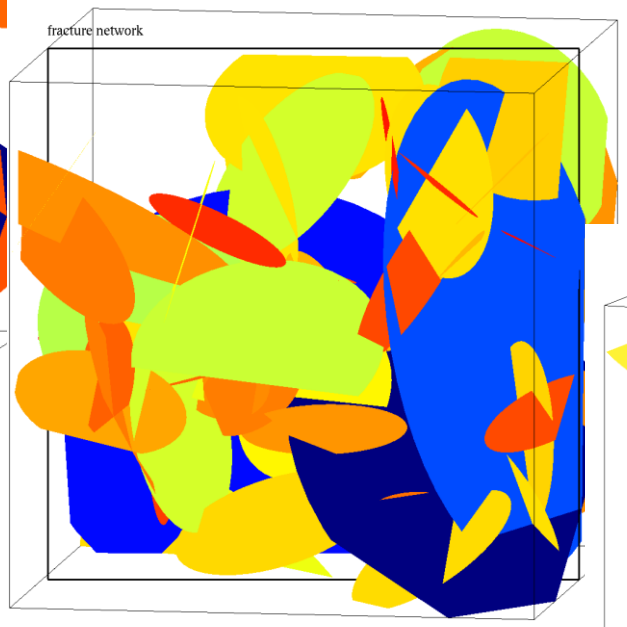
Stochastic models of flow and solute transport

-random velocity field
-random solute transfer time and dispersivity

Examples of fracture networks



a=2.5



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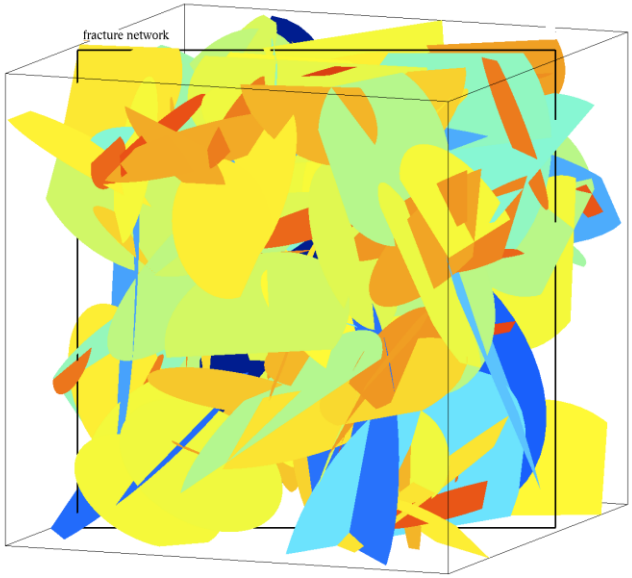


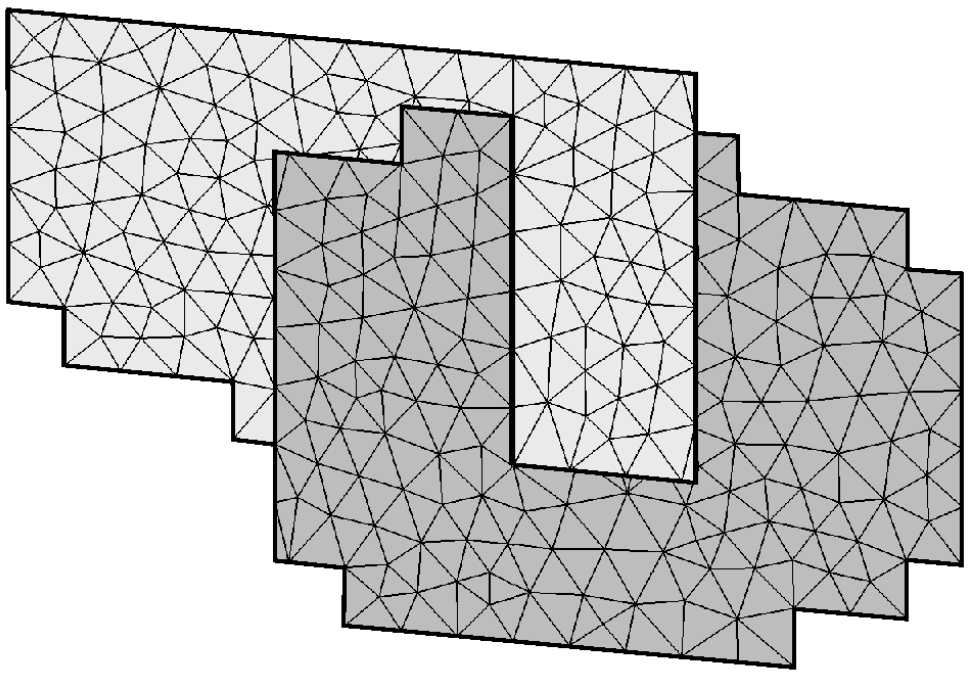
a=4.5

Numerical model

Physical equations

Darcy law and mass continuity in each fracture

$$\epsilon V = -K\nabla h, \nabla . V = 0$$



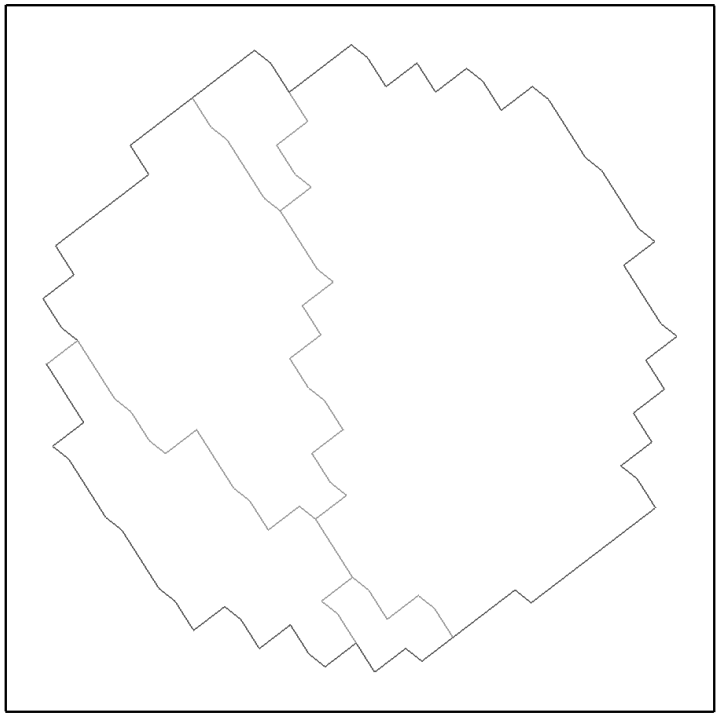
Continuity of hydraulic head h
and flux V.n at each intersection

Spatial discretization

- conforming mesh
- mixed hybrid finite element method
- easy to apply interface conditions

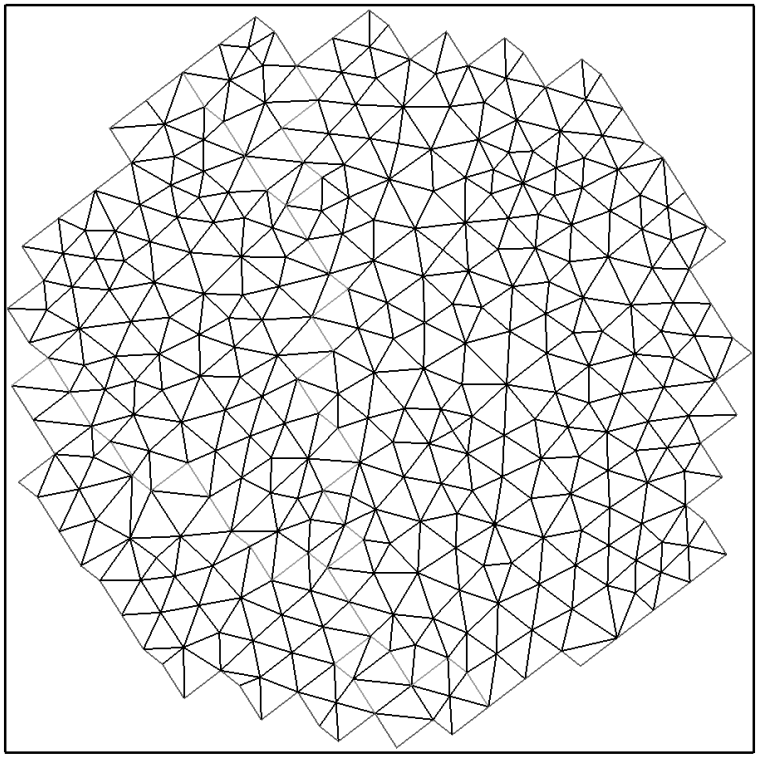
Mesh generation

discreted network



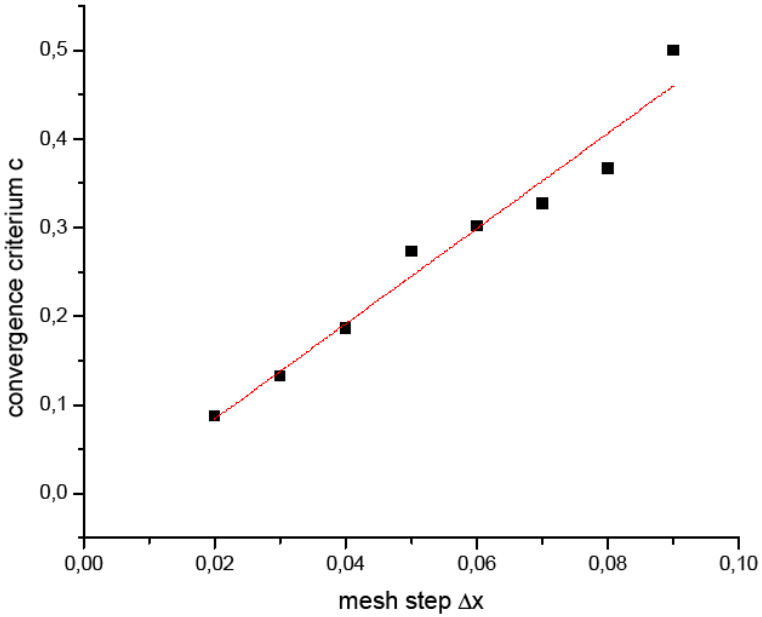
3D Discretization of intersections then projection onto the fracture

2D meshing



Conforming mesh at intersections

Convergence validation

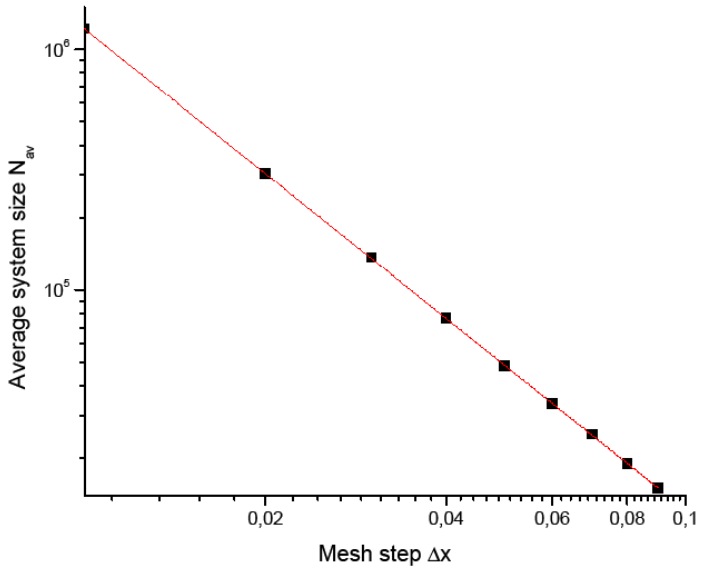


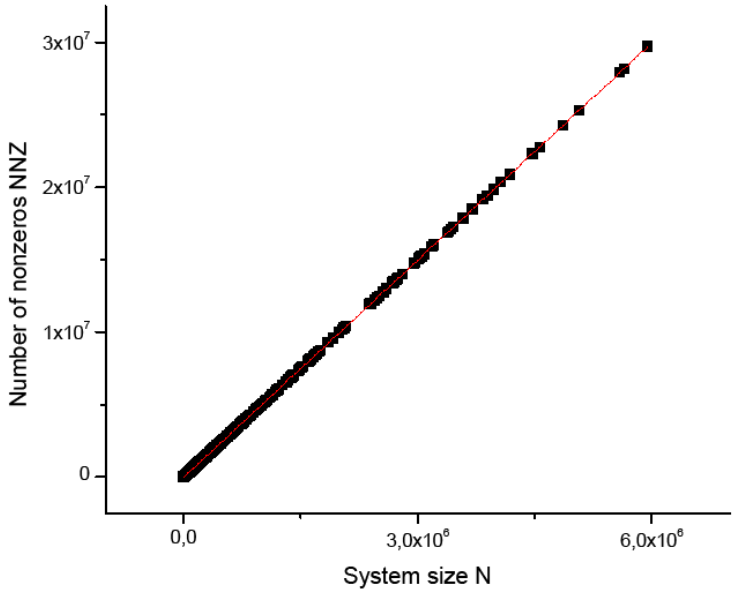
Flux through intersections

Classical Mixed Hybrid Finite Element method is of order 1

Here, interface conditions and
3D discretization and
projection with local corrections

Size complexity

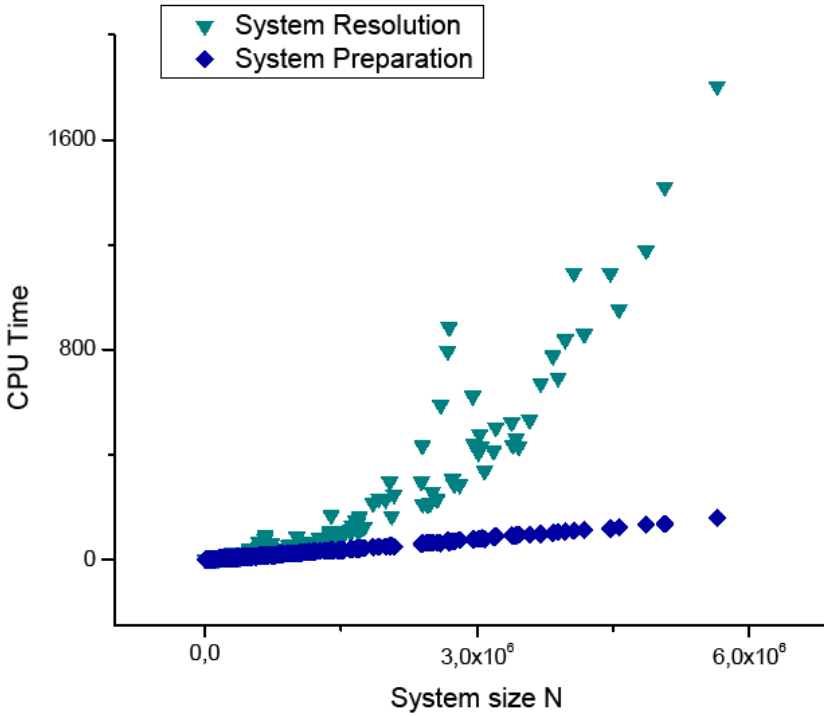




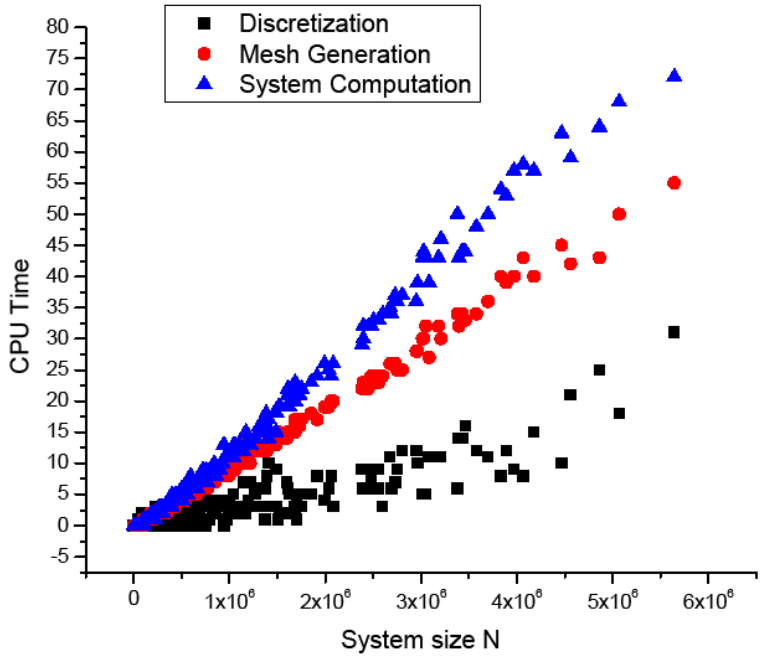
System size N is linear with mesh step

Number of nonzeros NNZ is linear With system size N

Time complexity



CPU time for preparing and solving



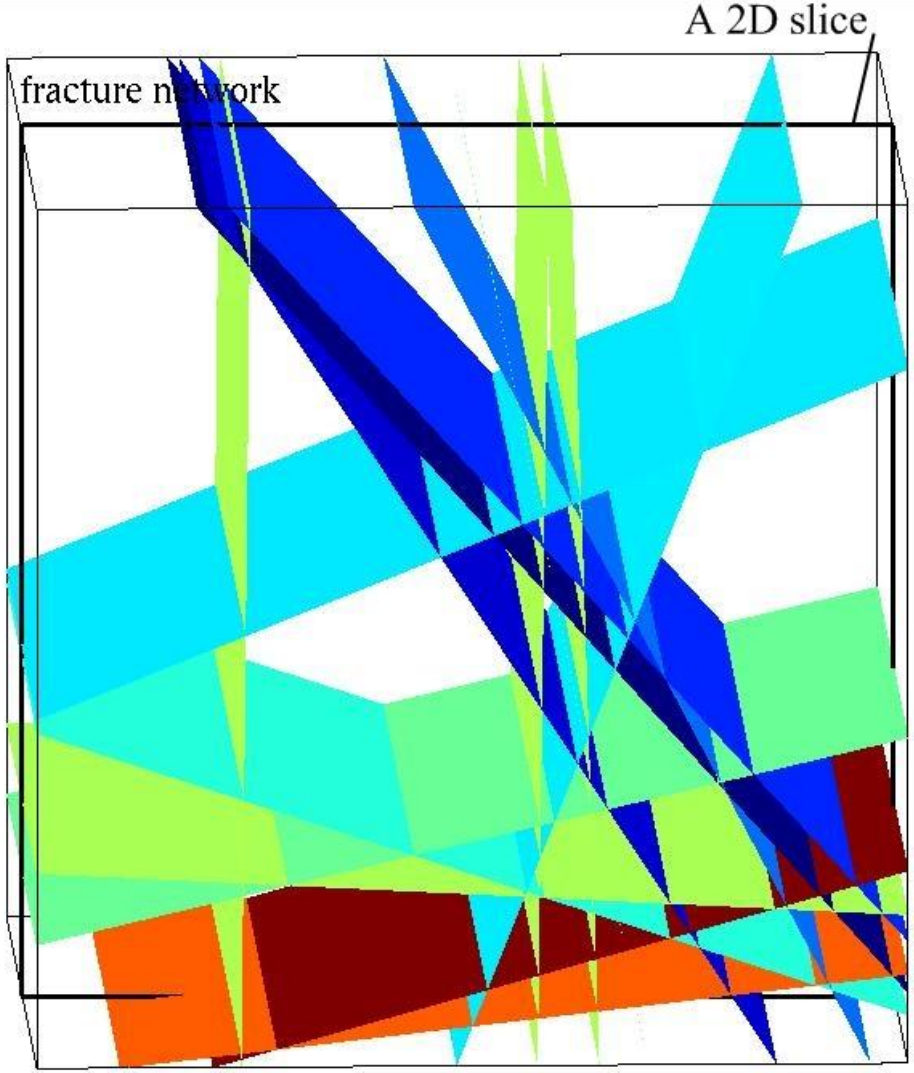
CPU time for preparing is linear with system size N

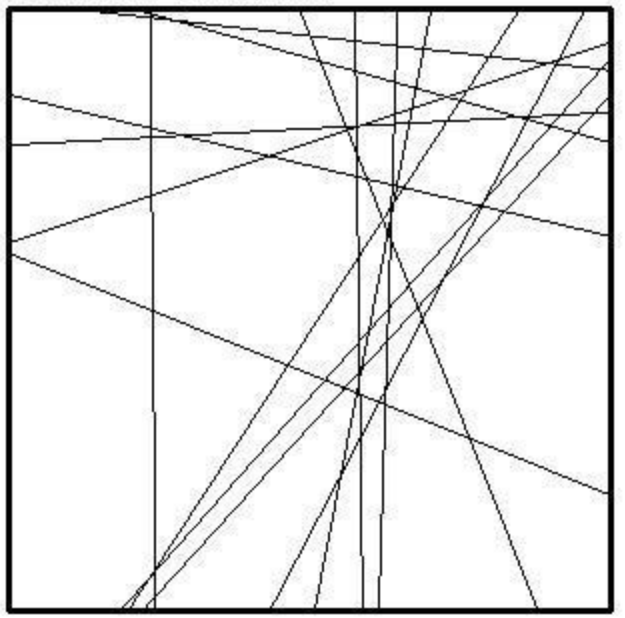
Numerical method with Mortar

Interface conditions written using mortar spaces
Geometrically conforming intersections: slave side and master side
Hydraulic head on slave side is L2 projection of hydraulic head on master side
Mass continuity through all edges of intersections

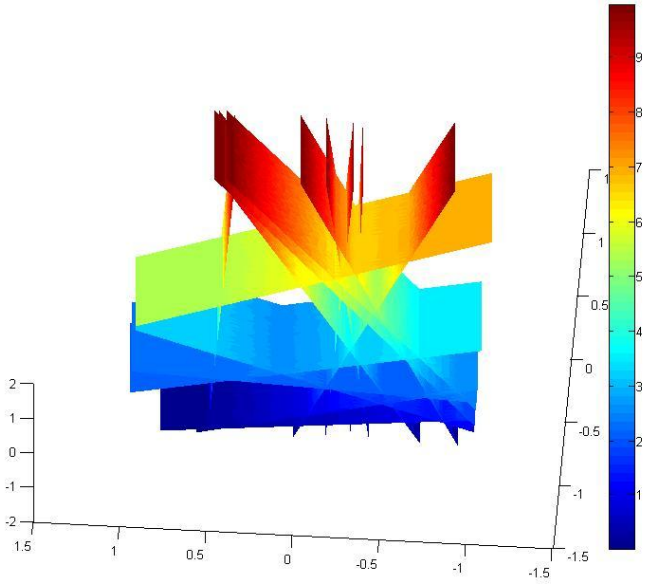
Geometrically non conforming intersections: duplication of some edges
Similar to a second level of mortar method

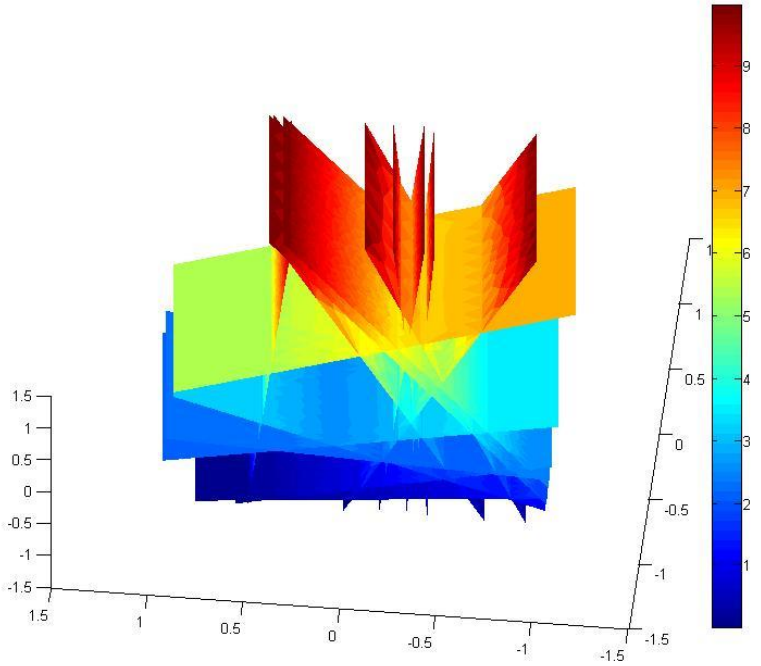
First numerical experiments



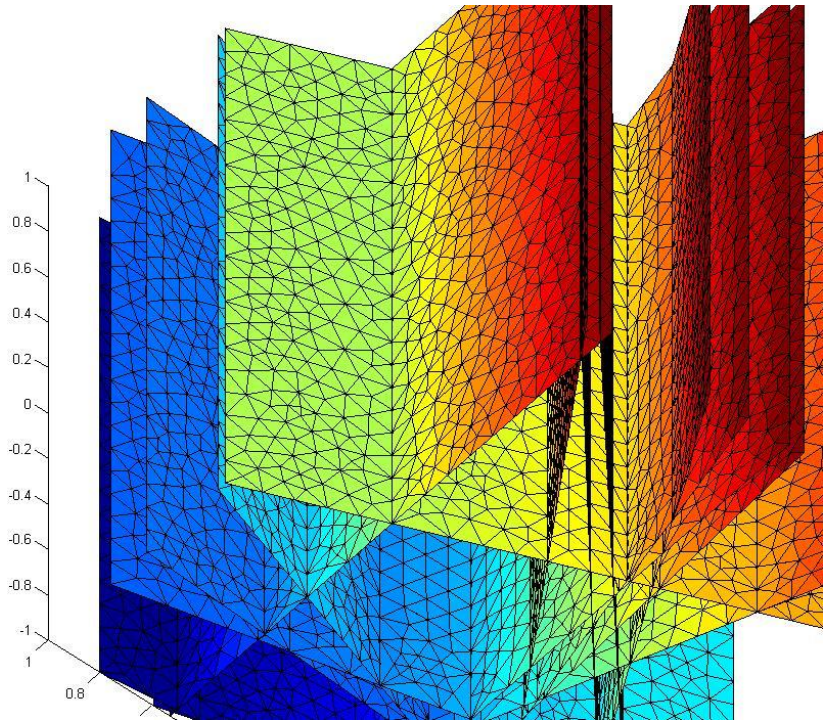


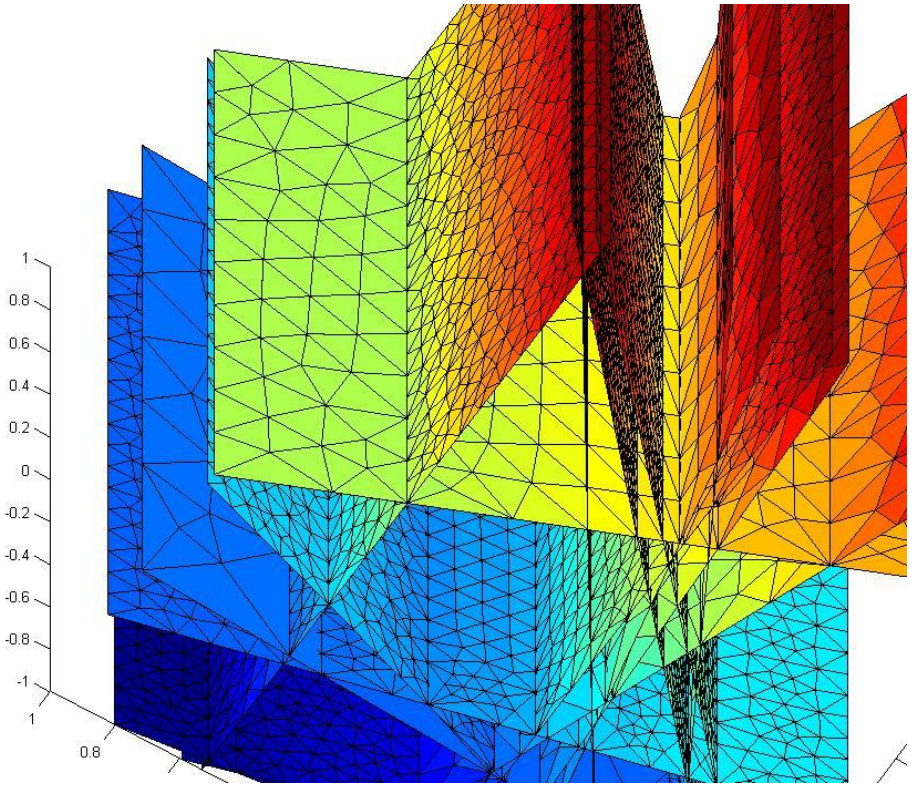
Results: hydraulic head





Results: hydraulic head (zoom)





conclusion and future work

- a priori convergence estimation
- large fracture networks
- parallel sparse linear solvers
- uncertainty quantification methods

- upscaling rules and classification of networks
- transient flow in fracture networks
- solute transport by advection and dispersion
- exchange of flow and solute with porous matrix