A fully automatic parallel GMRES solver preconditionned by a Multiplicative Schwarz iteration

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Motivations and outlines

Outlines

1. Explicit Form of Multiplicative Schwarz.
2. Partitioning for explicit form of Multiplicative Schwarz.
3. Parallel GMRES preconditioned by Multiplicative Schwarz
4. Experimentation results.
5. Conclusion

Explicit Form of Multiplicative Schwarz(EFMS)

Problem

- Solve in parallel $M^{-1}Ax = M^{-1}b$, where M is defined by one iteration of multiplicative Schwarz .

Part I: Define M^{-1} explicitly

see [Atenekeng, Kamgnia, and Philippe: 2007]

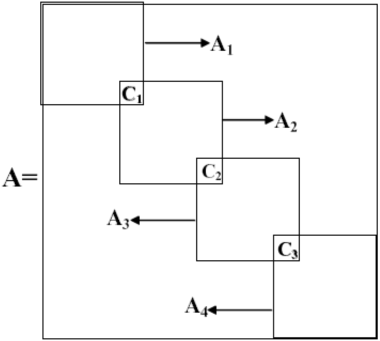
* It has been proven in previous work that

$$M^{-1} = \bar{A}_p^{-1}\bar{C}_{p-1}\bar{A}_{p-1}^{-1}...\bar{A}_2^{-1}\bar{C}_1\bar{A}_1^{-1}. \tag{1}$$

(Explicit Form of Multiplicative Schwarz(EFMS))

Explicit Form of Multiplicative Schwarz

$\bar{A}_i$, $\bar{C}_i$: A_i, C_i complete by identity to dimension of A.



- ⋆ With condition that: $\forall (k,l) \in I_i \times I_j [a_{kl} \neq 0 \Rightarrow |i-j| \leq 1]$. (1D partition of matrix)

Part II: 1D partitioning

Partition definition

Let $G = (V, E)$ be an adjancy graph obtained from A

- We need to partition V into $p > 1$, $(V_i)_{i=1,\dots,p}$.
  - Under the constraint that $(V_i)_{i=1,\dots,p}$ are almost balanced.
  - And that **distance between interacting partition=1**.
  - While minimizing vertex separators.
- Consequence: No possibility to use an existing partitioning software like METIS, HMETIS or PATOH, etc.

$\Longrightarrow$ Define new tools for 1D partition. This is done work [Atenekeng, Grigori, and Sosonkina 2008].

- Remark: This partition does not always exist. (Eg: When matrix have arrow profile).

GMRES Preconditioned By Explicit Formulation of MS(GPREMS)

Arnoldi Relation

1. A priori construction of Krylov subspace basis. See (Reichel,Erhel).
   Set $B = M^{-1}A$.

   $K_{m+1} = \{\sigma_0 v_0, \sigma_1 v_1, ..., \sigma_m v_m\}$, $v_{i+1} = \sigma_{i+1}(B - \lambda_i)v_i$.

   (λ_i): Ritz values of B ordered under Leja order.($i = 1...m$), $\sigma_i = \|v_{i-1}\|.(i = 1...m)$.

2. Orthogonalisation of Krylov basis. See (Sidje).

GPREMS(cont)

Parallel construction of Krylov basis

$$M^{-1} = \bar{A}_p^{-1}\bar{C}_{p-1}\bar{A}_{p-1}^{-1}\ldots\bar{A}_2^{-1}\bar{C}_1\bar{A}_1^{-1}.$$

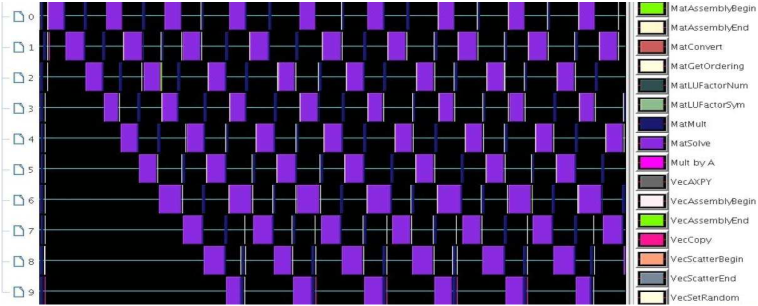


Figure: Krylov basis for $M^{-1}A$: [I Souopgui]

$\Rightarrow$ Asymptotic efficiency=1/3.

- Every processor have set of consecutive rows of Krylov subspace.

GPREMS(cont)

Recovering Arnoldi relation

1. QR factorization $K_{m+1} = V_{m+1} R_{m+1}$.[No global communication (see Sidje)].

2. $BK_m = V_{m+1} \bar{H}_m$,
   where $\bar{H}_m = R_{m+1} \bar{T}_m R_m^{-1}$.

$$\bar{T}_m = \begin{pmatrix} \lambda_1 & & 0 \\ \sigma_1 & \ddots & 0 \\ & \ddots & \lambda_m \\ 0 & 0 & \sigma_m \end{pmatrix}.$$

GPREMS(cont)

Condition number of the Krylov basis

- Problem with this approach is that K_{m+1} become intraitable when m increases.

  Because $cond(K_{m+1}) = cond(R_{m+1})$, become bad.

$\Rightarrow$ We then need to find $l \leq m$ such that $cond(R_l)$ is acceptable in order to correct the roundoff error on the correction to add to the current solution.

GPREMS(cont)

Acceptable condition number of R_{m+1}?

- z: roundoff error which occurs when computing y(Solution of least square problem).

  We can estimate z by the following relation.

  $$\|z\| = O(\epsilon\|y\|cond(R_m)max(\|B\|, \|\bar{T}_m\|, 1)). \tag{2}$$

  ϵ: machine precision.
  (Incremental Conditioning Estimator)
- Condition number of R_l is acceptable if $\|z\| < \|r\|$. Where r is residual vector

Experimental results

Software

- Petsc
- MUMPS and Superlu

Test Matrices

Matrix	Size	nnz	Cond
bjtcai	27,628	442,898	6.46e19
C07	233,786	27,333,984	-

Parameters for GMRES

- b=Ae
- restart: 64
- tol: $1e-12$
- Systems scaled by 1-norm

Observations

- Parallelism: Speedup and efficiency
- Convergence: Number of iteration

Machine: Cluster of Grid5000

CPU	Intel Xeon	Bi$-$processor	2.33Ghz
	Dual Core	Memory 4Go	Gigabit Ethernet

Experimental results

Circuit simulation: bjtcai

ND \ NP		4	8	16	32	Speedup	Efficiency
4	Times	12,10					
	Iterations	134					
8	Times	31,53	22,11			1,42	0,32
	Iterations	341	341				
16	Times	38,79	29,81	23,26		1,66	0,13
	Iterations	484	484	484			
32	Times	168,82	97,73	62,98	46,60	3,66	0,13
	Iterations	1633	1633	1633	1633		

- Iterations increases while number of subdomains augmented.
- Good efficiency when ND=NP=8.

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CFD: C07

NP / ND		4	8	16	32	Speedup	Efficiency
4	Times	**93,34**					
	Iterations	114					
8	Times	120,53	**80,40**			1,49	0,37
	Iterations	243	243				
16	Times	171,5	123,78	**97,39**		1,76	0,14
	Iterations	576	576	576			
32	Times	270,4	220	157,98	**118,44**	2,28	0,08
	Iterations	1152	1152	1152	1152		

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- We provide an automatic parallel GMRES solver preconditioned by multiplicative Schwarz.
- Our implementation need a 1D partition and we see that 1D partition cannot be found when adjancy graph of matrices have small diameter.
- Exprimentation result show that efficiency is asymptotic 1/3.
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Thanks you