Rate-Based Transition Systems and Stochastic Process Algebras

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Outline. . .

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Motivations...

A number of stochastic process algebras have been proposed in the last two decades. These are based on:

1. Labeled Transition Systems (LTS)
   - for providing compositional semantics of languages
   - for describing *qualitative properties*
2. Continuous Time Markov Chains (CTMC)
   - for analysing *quantitative properties*

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Semantics of these calculi have been given by variants of the Structured Operational Semantics (SOS) approach but:

- there is no general framework for modelling the different formalisms
- it is rather difficult to appreciate differences and similarities of such semantics.

Stochastic Process Algebras - incomplete list

- TIPP (N. Glotz, U. Herzog, M. Rettelbach - 1993)
- Stochastic π-calculus (C. Priami - 1995, later with P. Quaglia)
- PEPA (J. Hillston - 1996)
- EMPA (M. Bernardo, R. Gorrieri - 1998)
- IMC (H. Hermanns - 2002)
- ...
- StoKlaim
- MarCaSPiS
- ...

More Calculi will come: Besides qualitative aspects of distributed systems it more and more important that performance and dependability be addressed to deal with issues related to quality of service.

Common ingredients of Stochastic PA

Randomized Actions

- It is assumed that action execution takes time
- Execution times is described by means of random variables
- Random Variables are assumed to be exponentially distributed
- Random Variables are fully characterised by their rates.

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Properties of Exponential Distributions

If X is *exponentially distributed* with *parameter* $\lambda \in \mathbf{R}_{>0}$:

- $\mathbb{P}\{X \leq d\} = 1 - e^{-\lambda \cdot d}$, for $d \geq 0$
- The average duration of X is $\frac{1}{\lambda}$; the variance of X is $\frac{1}{\lambda^2}$
- *Memory-less*: $\mathbb{P}\{X \leq t + d \mid X > t\} \; = \; \mathbb{P}\{X \leq d\}$

Continuous Time Markov Chains

Continuous Time Markov Chains are a successful mathematical framework for modeling and analysing performance and dependability of systems that rely on exponential distribution of states transitions.

CTMCs come with

- Well established Analysis Techniques
  - Steady State Analysis
  - Transient Analysis
- Efficient Software Tools:
  - Stochastic Timed/Temporal Logics
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A CTMC is a pair $(\mathcal{S}, \mathbf{R})$

- $\mathcal{S}$: a countable set of states
- $\mathbf{R} : \mathcal{S} \times \mathcal{S} \rightarrow \mathrm{R}_{\geq 0}$, the rate matrix

Stochastic process calculi

- A CTMC is associated to each process term;
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Process Calculi:

$$\alpha.P + \alpha.P = \alpha.P$$

$$\mathbf{rec}\ X\ .\ \alpha.X \mid \mathbf{rec}\ X\ .\ \alpha.X = \mathbf{rec}\ X\ .\ \alpha.X$$

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Stochastic Process Calculi:

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We introduce a variant of Rate Transition Systems (RTS), proposed by Klin and Sassone(FOSSACS 2008), and use them for defining stochastic behaviour of a few process algebras.

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Like most of the previous attempts we take a two step approach: For a given term, say T, we define an enriched LTS and then use it to determine the CTMC to be associated to T.

- Our variant of RTS associates terms and actions to functions from terms to rates
- The *apparent rate* approach, originally developed by Hillston for multi-party synchronisation (à la CSP), is generalized to deal "appropriately" also with binary synchronisation (à la CCS).

Semantics of stochastic process calculi

Stochastic semantics of process calculi is defined by means of a transition relation $\longrightarrow$ that associates to a pair (P, α) - consisting of process and an action - a total function ($\mathscr{P}$, $\mathscr{Q}$,...) that assigns a non-negative real number to each process of the calculus. Value 0 is assigned to unreachable processes.

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$P \xrightarrow{\alpha} \mathscr{P}$ means that, for a generic process Q:

- if $\mathscr{P}(Q) = x$ ($\neq 0$) then Q is reachable from P via the execution of α with rate/(weight) x
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We have that if $P \xrightarrow{\alpha} \mathscr{P}$ then

- $\oplus\mathscr{P} = \sum_Q \mathscr{P}(Q)$ represents the total rate/weight of α in P.

Rate transition systems

Definition

A rate transition system is a triple $(S, A, \longrightarrow)$ where:

- S is a set of states;
- A is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbf{R}_{\geq 0}]$

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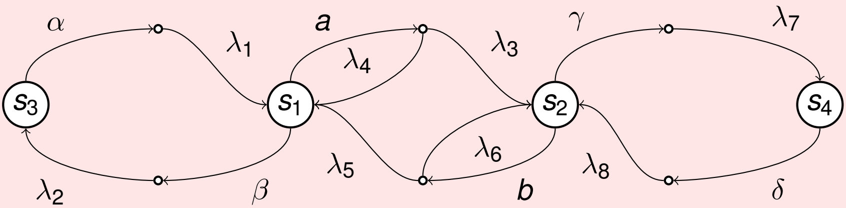
An example of RTS



Some Notation for Rate transition systems

- RTS will be denoted by $\mathcal{R}, \mathcal{R}_1, \mathcal{R}', \ldots,$
- Elements of $[S \to \mathbf{R}_{\geq 0}]$ are denoted by $\mathscr{P}, \mathscr{Q}, \mathscr{R}, \ldots$
- $[s_1 \mapsto v_1, \ldots, s_n \mapsto v_n]$ denotes the function associating v_i to s_i and 0 to all the other states.
- $\emptyset$ denotes the constant function 0.
- χ_s stands for $[s \mapsto 1]$.
- $\mathscr{P} + \mathscr{Q}$ denotes the function $\mathscr{R}$ such that: $\mathscr{R}(s) = \mathscr{P}(s) + \mathscr{Q}(s)$.
- $\mathscr{P} \cdot \frac{x}{y}$ denotes the function $\mathscr{R}$ such that: $\mathscr{R}(s) = \mathscr{P}(s) \cdot \frac{x}{y}$ if $y \neq 0$, and $\emptyset$ if $y = 0$.

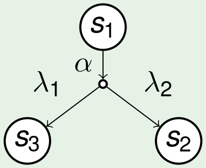
Rate transition systems

Definition

Let $\mathcal{R} = (S, A, \rightarrow)$ be an RTS, then:

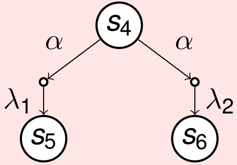
- $\mathcal{R}$ is *fully stochastic* if and only if for each $s \in S$, $\alpha \in A$, $\mathscr{P}$ and $\mathscr{Q}$ we have: $s \xrightarrow{\alpha} \mathscr{P}, s \xrightarrow{\alpha} \mathscr{Q} \Longrightarrow \mathscr{P} = \mathscr{Q}$
- $\mathcal{R}$ is *image finite* if and only if for each $s \in S$, $\alpha \in A$ and $\mathscr{P}$ such that $s \xrightarrow{\alpha} \mathscr{P}$ we have: $\{s' | \mathscr{P}(s') > 0\}$ is finite

A fully stochastic RTS...



...leads to a CTMC.

General RTS...



...leads to a CTM Decision Process.

From RTS to CTMC…

Reachable Sets of States

For sets $\mathcal{S}' \subseteq \mathcal{S}$ and $A' \subseteq A$, the set of derivatives of $\mathcal{S}'$ *through* A', denoted $Der(\mathcal{S}', A')$, is the smallest set such that:

- $\mathcal{S}' \subseteq Der(\mathcal{S}', A')$,
- if $s \in Der(\mathcal{S}', A')$ and there exists $\alpha \in A'$ and $\mathscr{Q} \in \Sigma_{\mathcal{S}}$ such that $s \xrightarrow{\alpha} \mathscr{Q}$ then $\{s' \mid \mathscr{Q}(s') > 0\} \subseteq Der(\mathcal{S}', A')$

Mapping $(\mathcal{S}, A, \rightarrow)$ into $(Der(\mathcal{S}', A'), \mathbf{R})$

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a *fully stochatics* RTS, for $\mathcal{S}' \subseteq \mathcal{S}$, the CTMC of $\mathcal{S}'$, when one considers only actions $A' \subseteq A$ is defined as $CTMC[\mathcal{S}', A'] \stackrel{def}{=} (Der(\mathcal{S}', A'), \mathbf{R})$ where for all $s_1, s_2 \in Der(\mathcal{S}', A')$:

$$\mathbf{R}[s_1, s_2] \stackrel{def}{=} \sum_{\alpha \in A'} \mathscr{P}^{\alpha}(s_2) \qquad \text{with } s_1 \xrightarrow{\alpha} \mathscr{P}^{\alpha}.$$

A translation from an RTS to a CTMC

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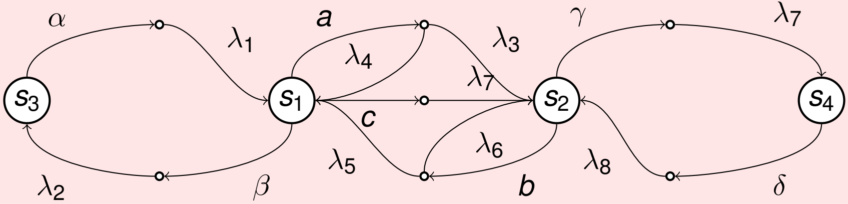
An RTS:



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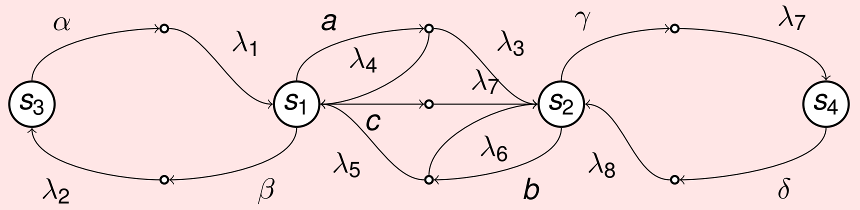
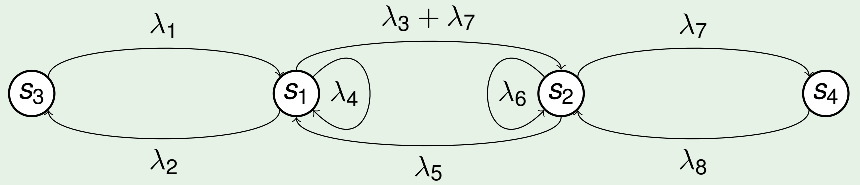


The corresponding CTMC:



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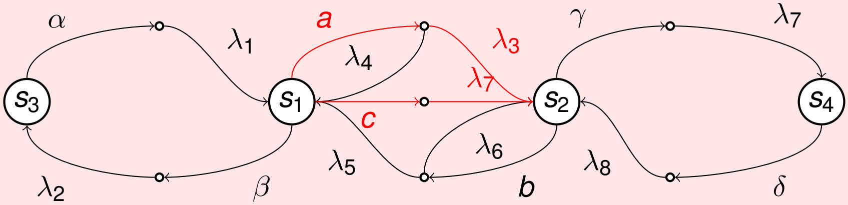

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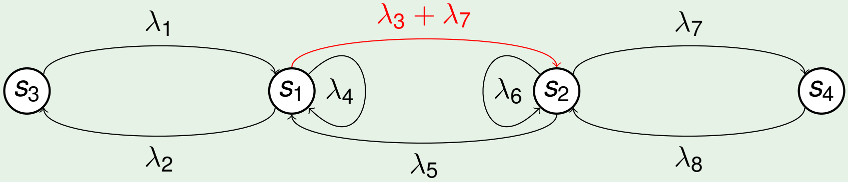
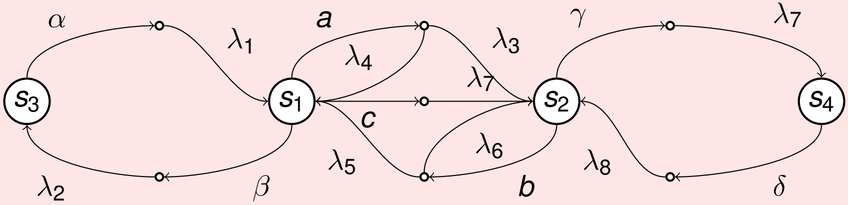
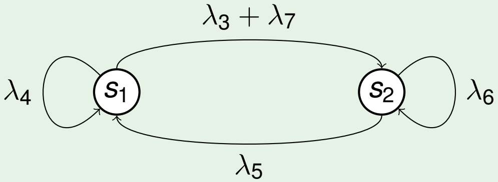


Another translation

$(\{s_1, s_2, s_3, s_4\}, \{\alpha, \beta, \gamma, \delta, a, b, c\}, \rightarrow)$



$CTMC[\{s_1, s_2\}, \{a, b, c\}]$



Strong Markovian Bisimilarity

Definition (Bisimulation)

Given a generic CTMC $(S, \mathbf{R})$

- An equivalence relation $\mathcal{E}$ on S is a Markovian bisimulation on S if and only if for all $(s_1, s_2) \in \mathcal{E}$ and for all equivalence classes $C \in S_{/\mathcal{E}}$ the following condition holds: $\mathbf{R}[s_1, C] \; = \; \mathbf{R}[s_2, C]$.

Definition (Bisimilarity)

Given a generic CTMC $(S, \mathbf{R})$

- Two states $s_1, s_2 \in S$ are strongly Markovian bisimilar, written $s_1 \sim_M s_2$, if and only if there exists a Markovian bisimulation $\mathcal{E}$ on S with $(s_1, s_2) \in \mathcal{E}$.

Rate aware bisimulation

Definition (Rate Aware Bisimilarity)

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a RTS:

- An equivalence relation $\mathcal{E} \subseteq \mathcal{S} \times \mathcal{S}$ is a *rate aware* bisimulation if and only if, for all $(s_1, s_2) \in \mathcal{E}$, and $\underline{S} \in \mathcal{S}_{/\mathcal{E}}$, and for all α and $\mathscr{P}$:

$$s_1 \xrightarrow{\alpha} \mathscr{P} \Longrightarrow \exists \mathscr{Q} : \ s_2 \xrightarrow{\alpha} \mathscr{Q} \wedge \mathscr{P}(\underline{S}) = \mathscr{Q}(\underline{S})$$

- Two states $s_1, s_2 \in S$ are *rate aware bisimilar* ($s_1 \sim s_2$) if there exists a rate aware bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Theorem

Let $\mathcal{R} = (\mathcal{S}, A, \longrightarrow)$*, for each* $A' \subseteq A$ *and for each* $s_1, s_2 \in \mathcal{S}$ *and* $(S, \mathbf{R}) = CTMC[\{s_1, s_2\}, A']$*:* $s_1 \sim s_2 \Longrightarrow s_1 \sim_M s_2$

Notice that *rate aware bisimilarity* and *strong bisimilarity* coincide when one does not take into account actions.

Outline. . .

PEPA: Performance Process Algebra

Systems

PEPA systems are the result of *components* interaction via *activities*:

- Components reflect the behaviour of relevant parts of the system,
- activities capture the actions that the components perform.

Activities

Each PEPA activity consists of a pair (α, λ) where:

- α symbolically denotes the performed action;
- $\lambda > 0$ is the rate of the (negative) *exponential* distribution.

Syntax

If $\mathcal{A}$ is a set of *actions*, ranged over by $\alpha, \alpha', \alpha_1, \ldots$, then $\mathcal{P}_{PEPA}$ is the set of process terms $P, P', P_1, \ldots$ defined by:

$$P ::= (\alpha, \lambda).P \mid P + P \mid P \parallel_L P \mid P/L \mid A$$

PEPA Stochastic semantics…

$$\frac{}{(\alpha,\lambda).P \xrightarrow{\alpha} [P \mapsto \lambda]}\ (\textsc{Act}) \qquad \frac{\alpha \neq \beta}{(\alpha,\lambda).P \xrightarrow{\beta} \emptyset}\ (\emptyset\text{-}\textsc{Act})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q}}{P + Q \xrightarrow{\alpha} \mathscr{P} + \mathscr{Q}}\ (\textsc{Sum}) \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \parallel_L Q \xrightarrow{\alpha} \mathscr{P} \parallel_L \chi_Q + \chi_P \parallel_L \mathscr{Q}}\ (\textsc{Int})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \parallel_L Q \xrightarrow{\alpha} \mathscr{P} \parallel_L \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}}\ (\textsc{Coop})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad \alpha \notin L}{P/L \xrightarrow{\alpha} \mathscr{P}/L}\ (\text{P-}\textsc{Hide}) \qquad \frac{\alpha \in L}{P/L \xrightarrow{\alpha} \emptyset}\ (\emptyset\text{-}\textsc{Hide})$$

$$\frac{P \xrightarrow{\tau} \mathscr{P}_\tau \quad \forall \alpha \in L. P \xrightarrow{\alpha} \mathscr{P}_\alpha}{P/L \xrightarrow{\tau} \mathscr{P}_\tau/L + \sum_{\alpha \in L} \mathscr{P}_\alpha/L}\ (\textsc{Hide}) \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad A \triangleq P}{A \xrightarrow{\alpha} \mathscr{P}}\ (\textsc{Call})$$

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Prefixes and Sums

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An example derivation

$$\frac{}{((\alpha,\lambda_1).P_1 + (\beta,\lambda_2).P_2) + (\alpha,\lambda_3).P_3 \xrightarrow{\alpha}}$$

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PEPA Stochastic semantics

Interleaving and Multiparty Synchronization

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \,||_L\, Q \xrightarrow{\alpha} \mathscr{P} \,||_L\, \chi_Q + \chi_P \,||_L\, \mathscr{Q}} \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \,||_L\, Q \xrightarrow{\alpha} \mathscr{P} \,||_L\, \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}}$$

- remember that χ_P is:

$$\chi_P(R) = \begin{cases} 1 & \text{if } R = P \\ 0 & \text{otherwise} \end{cases}$$

- $\mathscr{P} \,||_L\, \mathscr{Q}$ denotes the function $\mathscr{R}$ such that:

$$\mathscr{R}(R) = \begin{cases} \mathscr{P}(P) \cdot \mathscr{Q}(Q) & \text{if } R = P \,||_L\, Q \\ 0 & \text{otherwise} \end{cases}$$

A couple results for our PEPA semantics

Theorem

$\mathcal{R}_{PEPA}$ is fully stochastic and image finite.

Theorem

For all $P, Q \in \mathcal{P}_{PEPA}$ and $\alpha \in \mathcal{A}$ the following holds:

$$P \xrightarrow{\alpha} \mathscr{P} \wedge \mathscr{P}(Q) = \lambda > 0 \Leftrightarrow P \xrightarrow{\alpha,\lambda}_P Q$$

where $\longrightarrow_P$ stands for the transition relation defined by Hillstone in [Hil96].

Outline...

StoCCS: Stochastic CCS

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StoCCS is a Markovian extension of CCS where:

- *output activities* are enriched with *rates* characterizing random variables with exponential distributions, modeling their duration;
- *input activities* are equipped with *weights* characterizing the relative selection probability

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Like for PEPA , and for most of the other calculi, the CTMC for StoCCS specifications are obtained by only considering internal actions and channel interactions.

StoCCS: Transitions rates

STOCCS: Transitions rates

- The rate of a binary complementary synchronization mainly depends on the one of the triggering *activity*

STOCCS: Transitions rates

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- The synchronization rate of $\overline{a}$ and a depends on the rate of $\overline{a}$, on the weight of the *selected a* and on the *total weight* of a (i.e. on the *sum* of the weights of *all a*-transitions).

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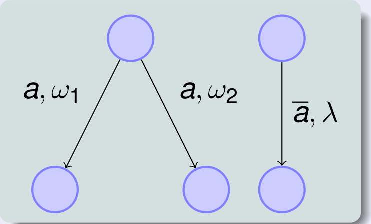

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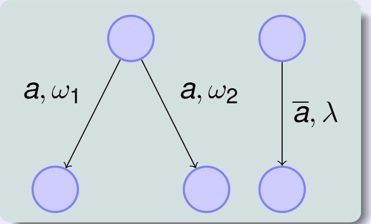
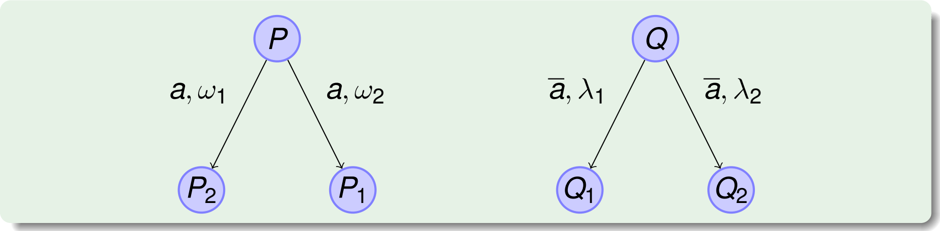


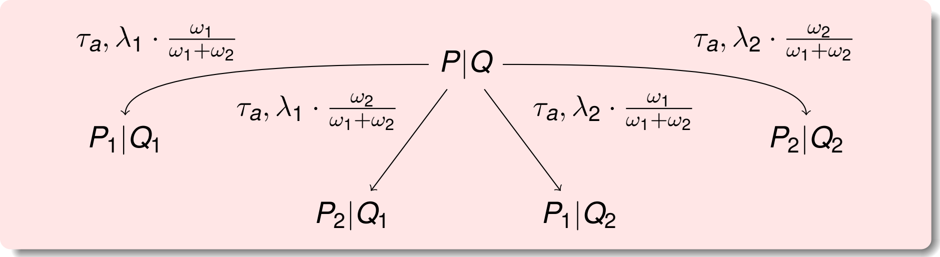
- Two synchronizations can occur with rates:

$$\lambda \cdot \frac{\omega_1}{\omega_1 + \omega_2} \qquad \lambda \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

- The overall sum of the synchronization rates is the same as the one of the output, i.e. it does not depend on the number of available (input) partners.

StoCCS: Transitions rates





STOCCS: Stochastic semantics - 1st attempt

Binary Synchronisation:

$$\frac{P \xrightarrow{\tau_a} \mathscr{P} \quad P \xrightarrow{a} \mathscr{P}_i \quad P \xrightarrow{\bar{a}} \mathscr{P}_o \quad Q \xrightarrow{\tau_a} \mathscr{Q} \quad Q \xrightarrow{a} \mathscr{Q}_i \quad Q \xrightarrow{\bar{a}} \mathscr{Q}_o}{P|Q \xrightarrow{\tau_a} \mathscr{P}|\chi_Q + \chi_P|\mathscr{Q} + \frac{\mathscr{P}_i|\mathscr{Q}_o}{\oplus \mathscr{P}_i} + \frac{\mathscr{P}_o|\mathscr{Q}_i}{\oplus \mathscr{Q}_i}}$$

Next states of $P|Q$ after τ_a, i.e. after a synchronisation over channel a, are:

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STOCCS: Stochastic semantics - 1st attempt

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4. the next states of P after a in parallel with the next states of Q after $\bar{a}$.

StoCCS: Stochastic semantics - 1st attempt

Theorem

$\mathcal{R}_{StoCCS}$ is fully stochastic and image finite.

Theorem

The proposed semantics coincides with the one proposed by Klin and Sassone.

StoCCS: Stochastic semantics - 1st attempt

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The proposed semantics coincides with the one proposed by Klin and Sassone.

Problem

The proposed semantics does not respect a standard and expected property of the CCS parallel composition.

The $|$ operator is not associative!

STOCCS: Stochastic semantics, 1st attempt

A counterexample for associativity

For instance:

$$\overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) \xrightarrow{\tau_a}$$

$$(\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 \xrightarrow{\tau_a}$$

StoCCS: Stochastic semantics, 1st attempt

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$$\overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) \xrightarrow{\tau_a} [P|(Q_1|a^{\omega_2}.Q_2) \mapsto \frac{\lambda\cdot\omega_1}{\omega_1+\omega_2}, P|(a^{\omega_1}.Q_1|Q_2) \mapsto \frac{\lambda\cdot\omega_2}{\omega_1+\omega_2}]$$

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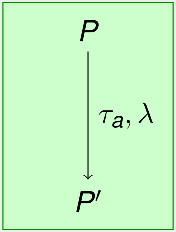
Theorem (From Klin and Sassone - KS08)

STOCCS *parallel composition is associative up-to stochastic bisimilarity if and only if the rate of a synchronisation is determined as the product of the two rates of the involved actions.*

Computing the rate of a synchronization



Computing the rate of a synchronization

$$P \xrightarrow{\tau_a, \lambda} P'$$

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Computing the rate of a synchronization

$$P \xrightarrow{\tau_a, \lambda} P' \qquad a^{\omega}.Q \xrightarrow{a, \omega} Q \qquad P|a^{\omega}.Q \xrightarrow{\tau_a, \lambda'} P'|Q$$

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If $\overline{\omega}$ is the total weight of a in P:

$$\lambda' = \lambda \cdot \frac{\overline{\omega}}{\overline{\omega} + \omega}$$

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If $\overline{\omega}$ is the total weight of a in P:

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This is the key point to guarantee associativity of parallel composition in CCS-like synchronizations.

StoCCS: stochastic semantics, 2nd attempt

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- interactions between input in P with output in Q.

StoCCS: stochastic semantics

Theorem

In StoCCS parallel composition is associative up to rate aware bisimilarity, i.e. for each P, Q and R, $P|(Q|R) \sim (P|Q)|R$

Stoπ: Stochastic π-Calculus

Input, Output and Synchronisation:

$$\frac{}{\overline{a}b^{\lambda}.P \xrightarrow{\overline{a}b} [P \mapsto \lambda]} \ (\textsc{Out}) \qquad \frac{}{a(x)^{\omega}.P \xrightarrow{ab} [P[b/x] \mapsto \omega]} \ (\textsc{In})$$

$$\frac{\begin{array}{ccc} P \xrightarrow{\tau_a(b)} \mathscr{P} & P \xrightarrow{ab} \mathscr{P}_i & P \xrightarrow{\overline{a}b} \mathscr{P}_o \\ Q \xrightarrow{\tau_a(b)} \mathscr{Q} & Q \xrightarrow{ab} \mathscr{Q}_i & Q \xrightarrow{\overline{a}b} \mathscr{Q}_o \end{array}}{P|Q \xrightarrow{\tau_a b} \frac{\mathscr{P}|Q \cdot \oplus \mathscr{P}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{P|\mathscr{Q} \cdot \oplus \mathscr{Q}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{\mathscr{P}_i|\mathscr{Q}_o}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{\mathscr{P}_o|\mathscr{Q}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i}} \ (\textsc{Sync})$$

The other rules are the expected ones.

Outline. . .

Summing Up

- We have introduced Rate Transition Systems and have used them as the basic model for defining stochastic behaviours of processes.
- We have introduced a natural notion of bisimulation over RTS that agrees with Markovian bisimulation.
- We have shown how RTS can be used to provide the stochastic operational semantics of PEPA and CCS.
- We have discussed the generalization of the approach to π-calculus and (in another paper) MarCaSPiS.

Future Work

- Use RTS to model other formalisms
- Use the RTS approach as general framework for modelling other PA semantics (non-deterministic, truly-concurrent, probabilitistic,. . .)
- Consider alternative semantics synchonisation rates:
  - based on *phase type* distributions
  - based on *Interactive Markov Chains*
- Develop tools directly for RTS rather than for CTMC.

Thank you for your attention!

If interested read our ICALP-C 2009 paper
or
the full version available on the web
(e.g. from Michele Loreti's home page).