A Logical Generalization of Formal Concept Analysis

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Introduction

Concept Analysis has a wide and increasing range of applications

- data analysis
  ex: program understanding [Snelting et al.]
- knowledge extraction (learning, datamining)
  ex: learning from positive and negative examples [Kuznetsov]
- information systems, information retrieval
  ex: software component retrieval [Lindig]

$\Rightarrow$ we are especially interested in information systems

Information Systems (IS) with FCA

[Godin, Missaoui, April] boolean and hierarchical IS are unsatisfactory for organizing and retrieving information

- no free combination of querying and navigation
- flat or tedious organization
- no consistency with updates

⇒ FCA is an interesting alternative

- *concept*: unifies query (*intent*) and directory (*extent*)
- *lattice*: automatic and consistent organization

Need for extending FCA . . .

. . . because of the wide range of applications

- data analysis
  ex: attributes → cubes [Chaudron and Maille]
- learning
  ex: attributes → conceptual graphs [Kuznetsov]
- information system
  We wish to describe objects and ask queries in an arbitrary logical language (in place of mere attributes)

⇒ Logical Information System based on Logical Concept Analysis

Towards Logical Concept Analysis (LCA)

FCA

- object description/query = set of attributes
- subsumption: $\supseteq$
- operations: $\cap$, $\cup$

LCA

- object description/query = formula of a language $\mathcal{L}$ (infinite)
- subsumption: $\dot{\models}$ (logical deduction)
- operations: $\dot{\vee}$, $\dot{\wedge}$ (disjunction and conjunction)
- $\langle \mathcal{L}; \dot{\models}, \dot{\vee}, \dot{\wedge} \rangle$ *has to be* a lattice

$\rightarrow$ LCA can be seen as an abstraction of FCA
because $\langle 2^{\mathcal{A}}; \supseteq, \cap, \cup \rangle$ forms a lattice

LCA: logical context

- logical context
  $\mathcal{O}$: set of objects
  $\langle \mathcal{L}; \dot{\models}, \dot{\vee}, \dot{\wedge} \rangle$: logical language
  $i \in \mathcal{O} \rightarrow \mathcal{L}$: maps a description to each object

- Galois connection
  - the most accurate formula satisfied by a set of objects
    $\sigma(O) \doteq \dot{\bigvee}_{o \in O} i(o)$, $\forall O \subseteq \mathcal{O}$
  - all objects satisfying a formula (its extent)
    $\tau(f) = \{o \in \mathcal{O} | i(o) \dot{\models} f\}$, $\forall f \in \mathcal{F}$

LCA: a context example

object : function

formula : types with unknowns

subsumption : type instanciation

disjunction, conjunction : type anti-unification and unification

function	description
+	(int,int) -> int
+.	(float,float) -> float
random	() -> int
trunc	(float) -> int
sqrt	(float) -> float

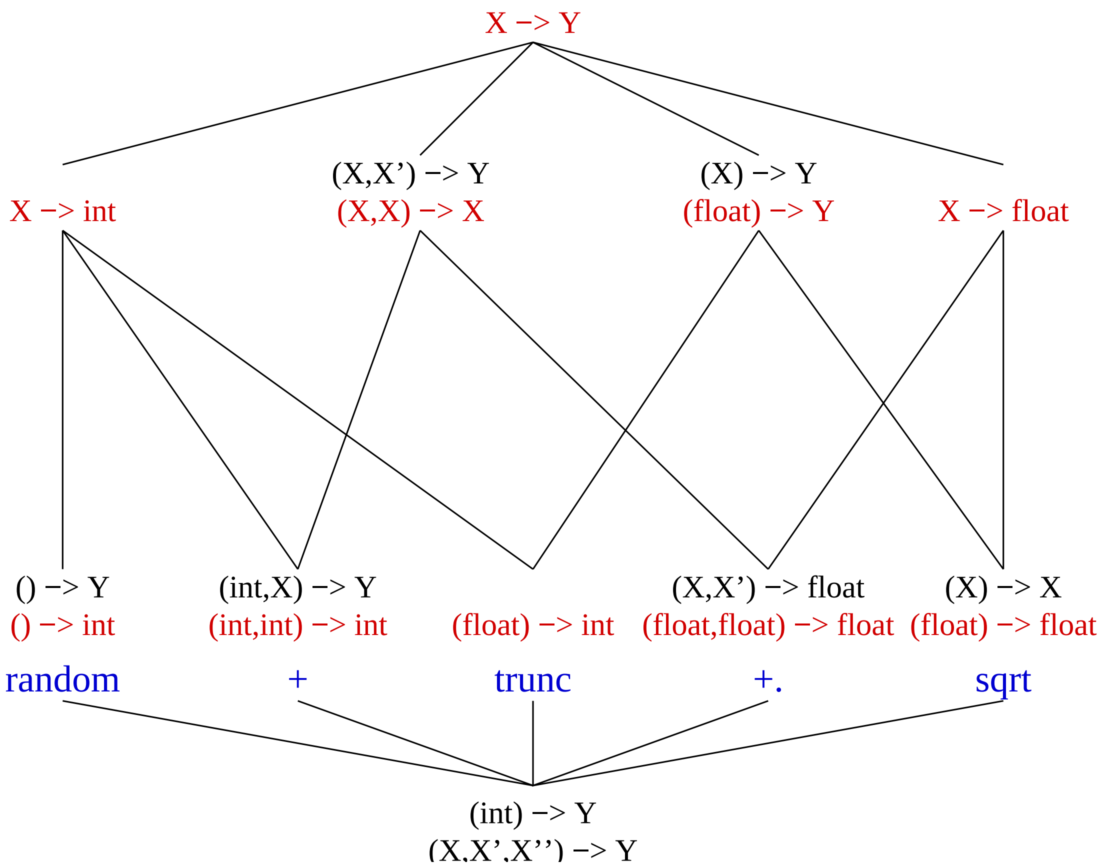
LCA: concept lattice

- concept : pair (extent,intent) = (O, f)
  with $\sigma(O) = f$ and $\tau(f) = O$

- specialization/generalization ordering
  $(O_1, f_1) \leq (O_2, f_2) :\Longleftrightarrow O_1 \subseteq O_2 \iff f_1 \dot{\models} f_2$
  $\longrightarrow \langle \mathcal{C}; \leq \rangle$ is a complete lattice

- labelling
  - by objects: $\gamma(o) = (-, \sigma(\{o\}))$
  - by formulas: $\mu(f) = (\tau(f), -)$

LCA: concept lattice example



() –> Y ==> X –> int

(X) –> Y <==> (float) –> Y

(X,X') –> Y <==> (X,X) –> X

(int,X) –> Y <==> (int,int) –> int

(X,X') –> float <==> (float,float) –> float

(X) –> X <==> (float) –> float

concept intents

objects

LCA: contextualized logic (1/2)

- $f \dot{\models} g$: logical deduction (true in every context)
- $f \dot{\models}^K g$: contextualized deduction (true in the context K)

  definition: $f \dot{\models}^K g :\iff \tau(f) \subseteq \tau(g) \iff \mu(f) \leq \mu(g)$

  ex:
  () -> Y $\dot{\models}^K$ X -> int
  (X,X') -> Y $\dot{\models}^K$ (X,X) -> X

- $f \dot{\models} g \implies f \dot{\models}^K g$: the contextualized deduction *extends* the logical one

Important theorem: $\langle \mathcal{L}/_{\dot{=}^K} \,;\, \dot{\models}^K \rangle$ is isomorphic to $\langle \mathcal{C} \,;\, \leq \rangle$ by the labelling mapping μ

$\Rightarrow$ *it gathers all facts about logic $\mathcal{L}$ and a context K in a full logical way*

LCA: contextualized logic (2/2)

Applications

- data analysis: $f \dot{\models}^K g \iff \mu(f) \leq \mu(g)$
  $\rightarrow$ choice between $\langle extents; \subseteq \rangle$, $\langle intents; \dot{\models} \rangle$ and $\langle formulas; \dot{\models}^K \rangle$ for representing concepts
- datamining/learning: the rule $f \rightarrow g$ stands iff $f \dot{\models}^K g$
  $\rightarrow$ generalizes *implications* between attributes
- information systems: x is a navigation link from f to g iff $f \dot{\wedge} x \doteq^K g$ (see paper for more details)
  $\rightarrow$ objects are accessed through *several concise paths* (completion)

$\Rightarrow$ the contextualized logic enables to take into account properties that are true in a context while false in general

Conclusion

Prototype of a Logical File System based on LCA

- bibtex references described by authors, title, year, keywords, . . .

```
% cd author:Wille/keywords:"concept analysis" &
- "conceptual graph"/year:1990..1999
```
(querying)

```
% ls -v /author:*
```
(navigating)

```
% mv -r .  location:Darmstadt
```
(updating)

Future work for Logical Information Systems

- datamining/learning by navigating rather than an exhaustive production of rules
- terminology, taxonomy (what is true in every context of a domain)
- composite objects and relationships between objects