Parameterized Verification of Synchronization in constrained reconfigurable broadcast networks

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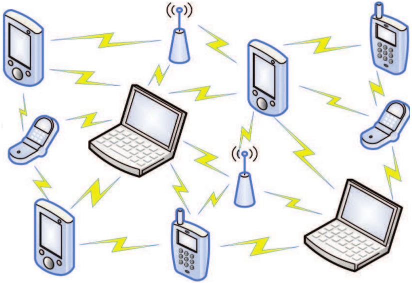
Inria & IRISA

**joint work with Balasubramanian A.R. (CMI)
and Nicolas Markey (CNRS & IRISA)**

TACAS @ ETAPS 2018

Formal Models for Ad Hoc Networks



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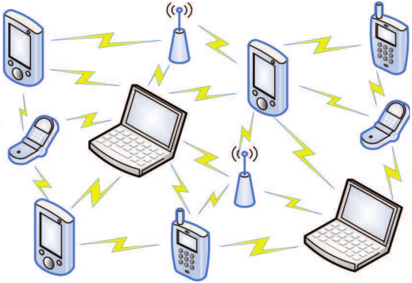

important features

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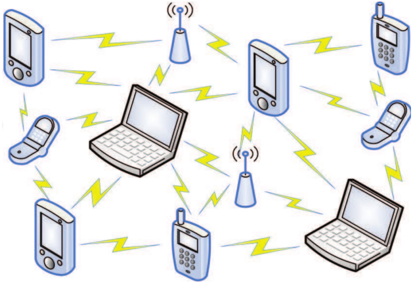

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broadcast networks with evolving communication topology

Reconfigurable Broadcast Networks

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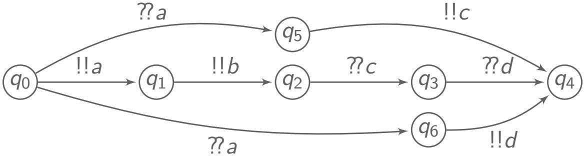

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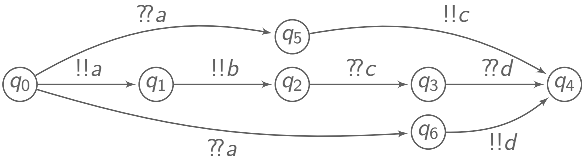

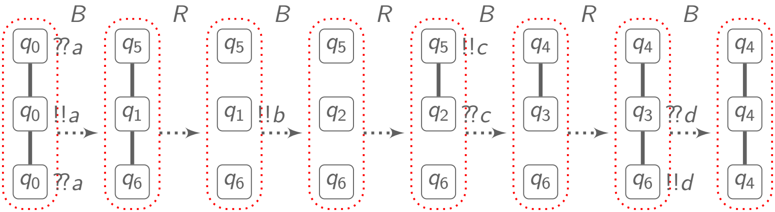

Parameterized Verification of Broadcast Networks

parameter: initial configuration (number of nodes and topology)

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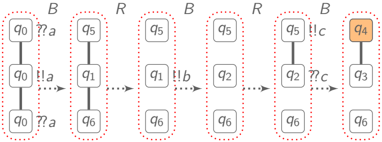
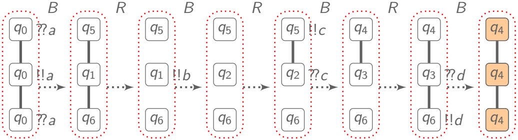


- Synchronization



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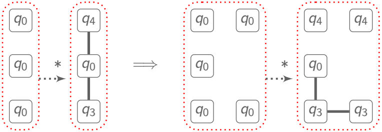
- synchronization: forward and backward iterative pruning of broadcast protocol, using algorithm for reachability

Monotonicity property: if a configuration is reachable, then all configurations obtained by duplicating some nodes are also reachable



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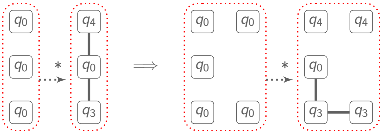

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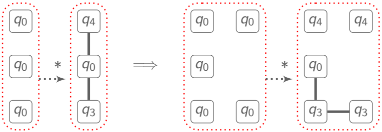


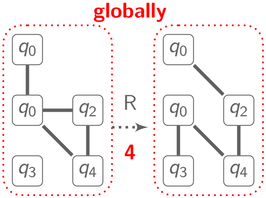
Rk: both problems are undecidable under fixed topology

Limitation: arbitrary reconfigurations between communications

this talk: towards realistic representation of mobility

Constraints on reconfiguration

Quantifying reconfiguration: number of modified links



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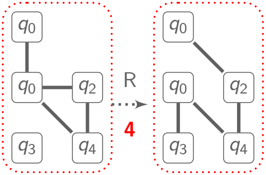
globally



k-bounded execution

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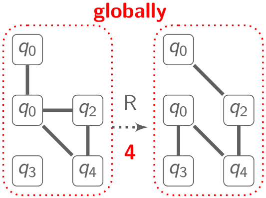

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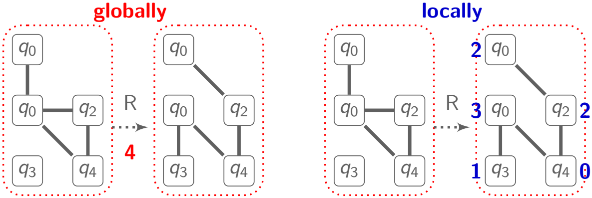

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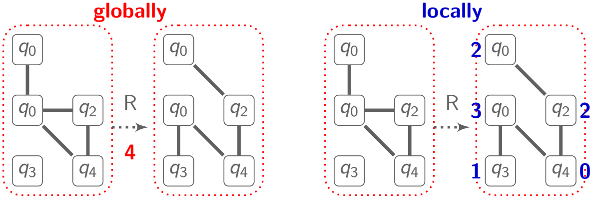

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locally-*k*-bounded execution

Reachability under reconfiguration constraints

There exists a reaching execution iff
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Proof idea: interleave several copies of a witness execution
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Rest of the talk: synchronization

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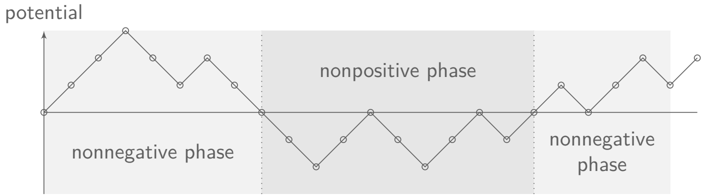

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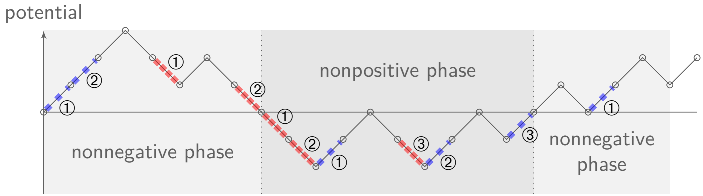

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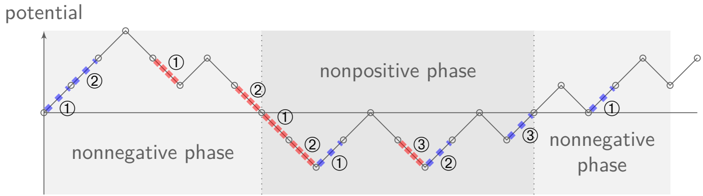

Correspondence between repeated broadcasts, and repeated link modifications
Interleave duplicates of initial execution to strictly alternate $+1$ and -1 steps

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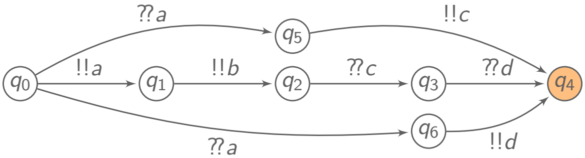
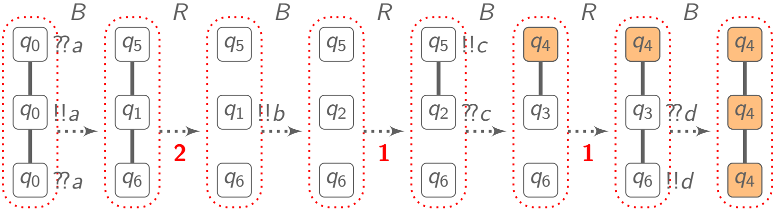


unspecified receptions lead to a sink



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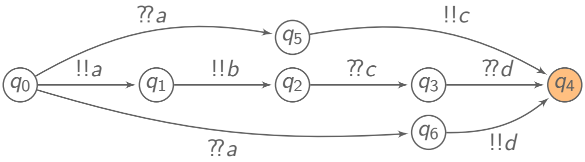

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In any synchronizing execution

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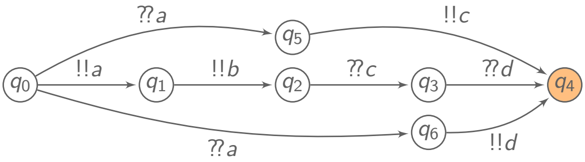

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In 1-bounded executions: ♯ broadcast steps $\geq 1+$♯ reconfigured links

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Corollary
The parameterized synchronization problem is in PTIME when restricting to either 1-locally-bounded, or $f(n)$-bounded executions.

Focus on k-constrained synchronization

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$\Rightarrow$ at most one neighbour per node
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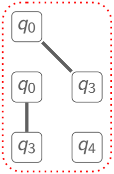

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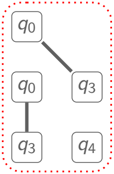

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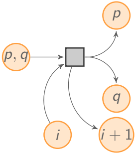
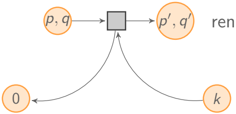
reconfiguration





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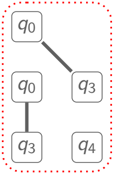

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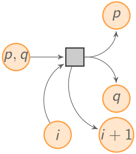

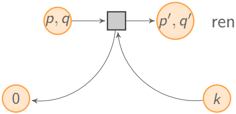

rendez-vous

initialisation generate tokens in places q_0 and (q_0, q_0)

simulation reconfigurations and rendez-vous communications

termination emptying tokens from target place

Summary of results

Constraining reconfigurations in broadcast protocols

Contributions

- equivalence of k-bounded and k-bounded-on-average
- non-equivalence of 1-bounded with unconstrained reconfiguration
- undecidability of synchronization under k-bounded assumption
- decidability assuming rendez-vous communications

Techniques

- duplication of executions and clever interleaving
- encoding of 2-counter machine
- reduction to reachability in Petri net

Future work

Quantitative impact of reconfiguration restrictions

- minimum number of nodes in a witness execution
- mininum number of steps to synchronize

Long-term objective: realistic model of mobility in ad hoc networks

- representing with physical constraints: speed of nodes vs communication rate
- exploiting statistical analysis of real behaviours