Controlling a population of identical NFA

Nathalie Bertrand

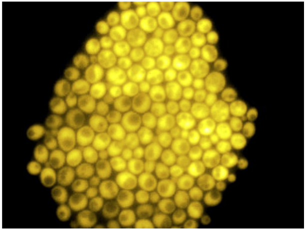
Inria Rennes

joint work with Miheer Dewaskar (ex CMI student), Blaise Genest (IRISA) and Hugo Gimbert (LaBRI)

SynCoP & PV workshops @ ETAPS 2018

Motivation

Control of gene expression for a population of cells



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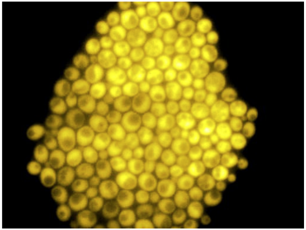

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- drug injections affect all cells
- response varies from cell to cell
- obtain a large proportion of cells with desired gene expression level

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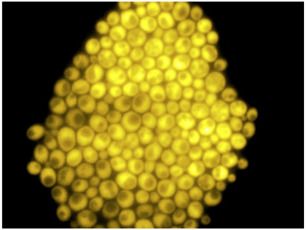

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- arbitrary nb of components
- full observation

- uniform control
- non-det. model for single cell
- global reachability objective

Problem formalisation

- population of N identical NFA
- uniform control policy under full observation
- resolution of non-determinism by an adversary

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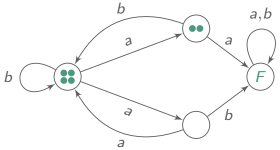

config: # copies in each state

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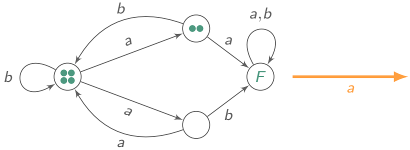

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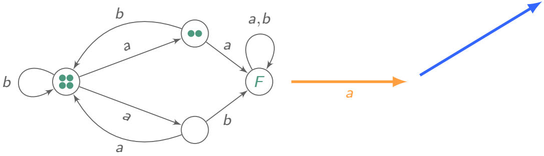

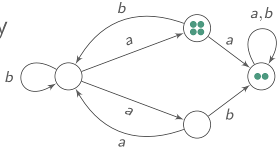

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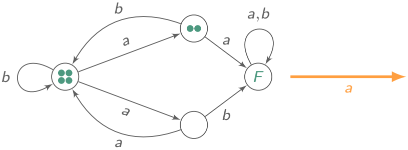

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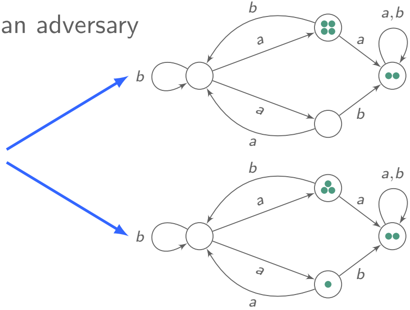

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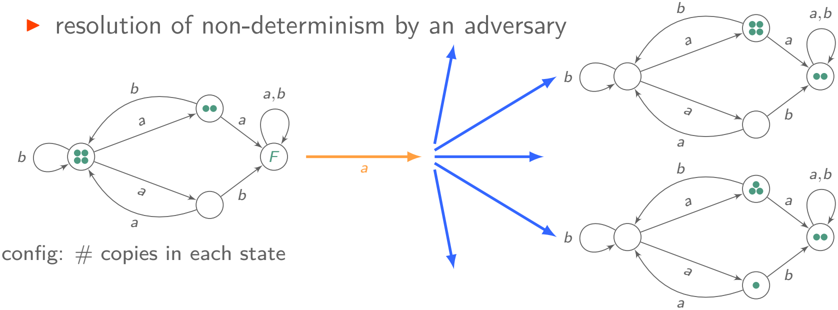

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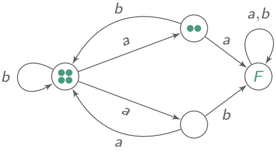

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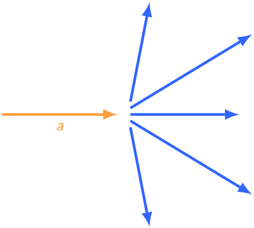

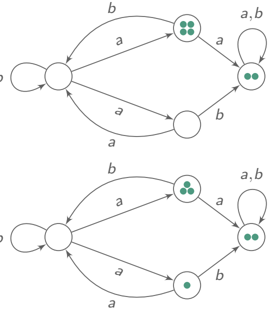

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Question can one control the population to ensure that for all non-deterministic choices all NFAs simultaneously reach a target set?

Population control

Fixed N: build finite 2-player game, identify global target states, decide if controller has a winning strategy for a reachability objective

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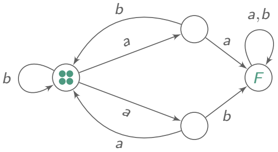
Challenge: **Parameterized control**

$$\forall N \ \exists \sigma \ \forall \tau \ (\mathcal{A}^N, \sigma, \tau) \models \Diamond F^N?$$



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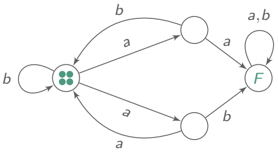

This talk

- decidability and complexity
- memory requirements for controller σ
- admissible values for N

Monotonicity property and cutoff

Monotonicity property: the larger N, the harder for controller

$$\exists\sigma\ \forall\tau(\mathcal{A}^N,\sigma,\tau) \models \Diamond F^N \quad\Longrightarrow\quad \forall M \leq N\ \exists\sigma\ \forall\tau(\mathcal{A}^M,\sigma,\tau) \models \Diamond F^M$$

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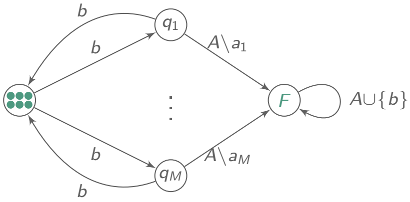


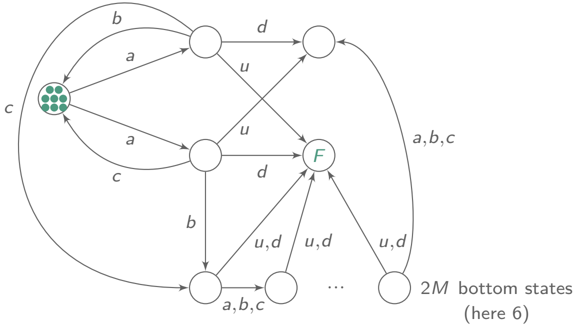
$A = \{a_1, \cdots, a_M\}$
unspecified edges lead to a sink state

winning σ if $N < M$
play b then a_i s.t. q_i is empty

winning τ for $N = M$
always fill all q_i's

cutoff is M

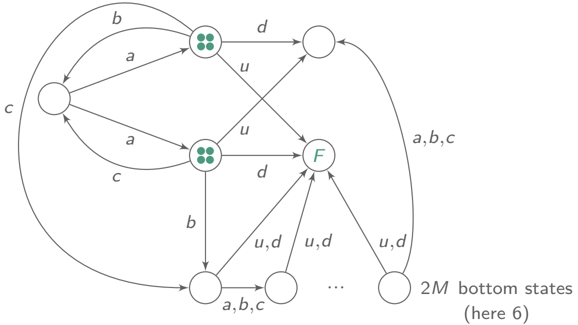
Lower bound on the cutoff



- $\forall N \leq 2^M, \ \exists \sigma, \ \mathcal{A}^N \models \forall_\sigma \Diamond F^N$
  accumulate copies in bottom states, then u/d to converge
- for $N > 2^M$ controller cannot avoid reaching the sink state

Cutoff $\mathcal{O}(2^{|\mathcal{A}|})$

Lower bound on the cutoff



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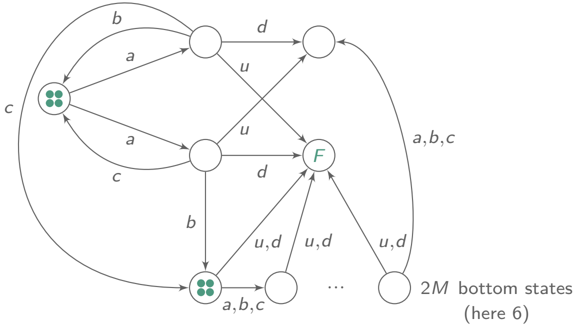

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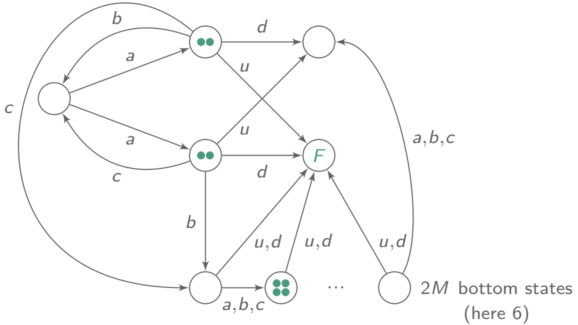

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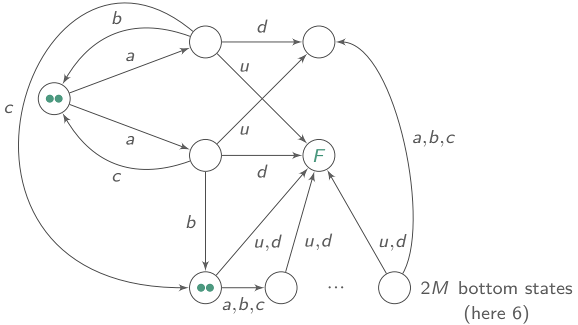

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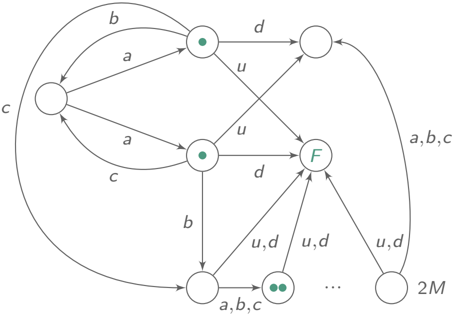

2M bottom states (here 6)

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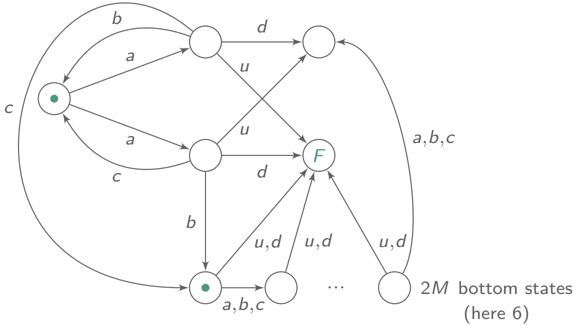

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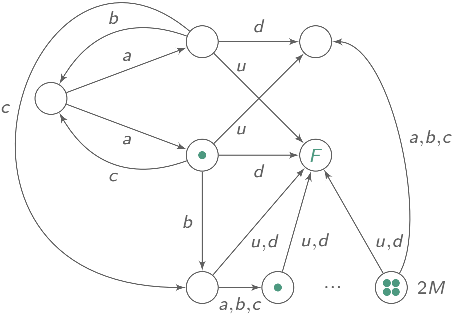

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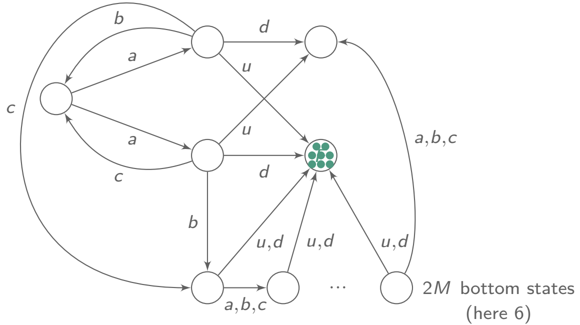

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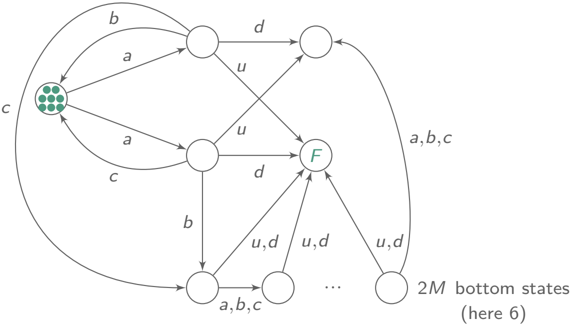

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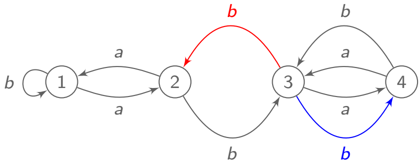
Combined with a counter, cutoff is even doubly exponential!

A natural attempt: the support game



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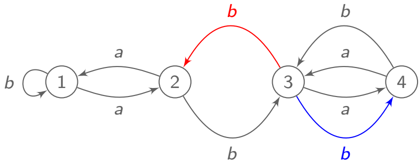

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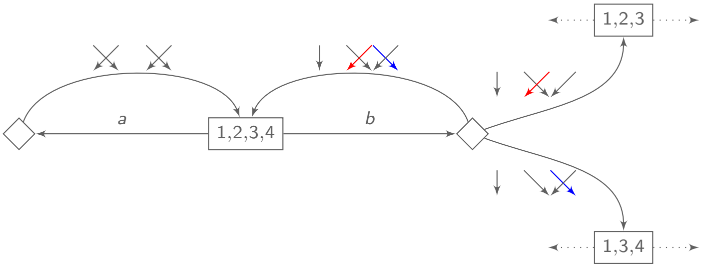

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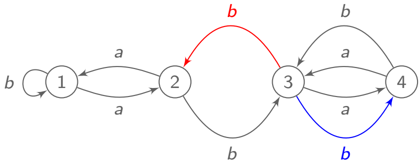

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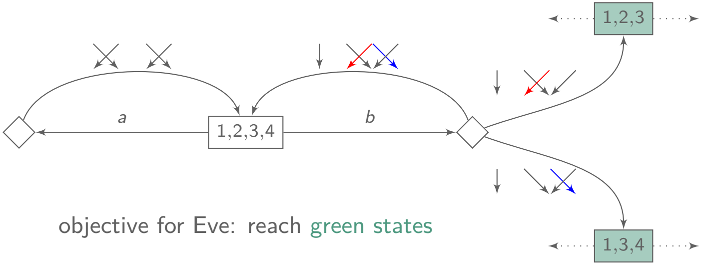

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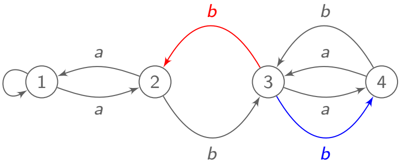

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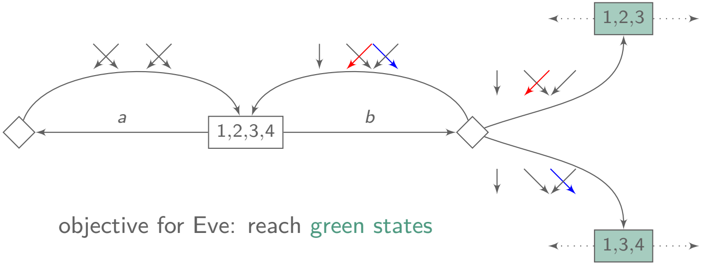

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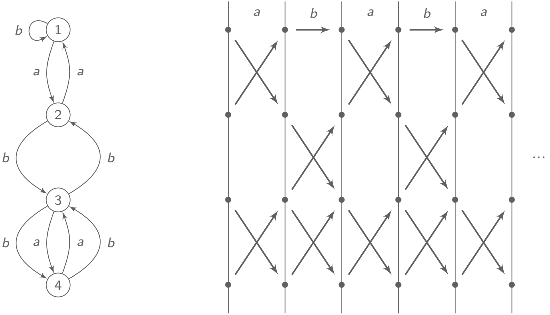
If Eve wins support game then controller has a winning strategy for all N

Support game is not equivalent to population game

- controller alternates *a* and *b* ;
- adversary must always fill 2 and 4 in the *b*-step



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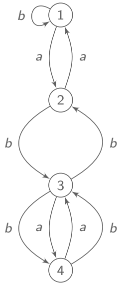

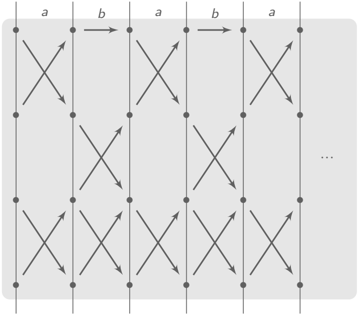

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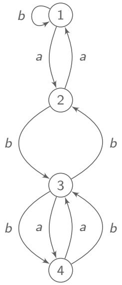

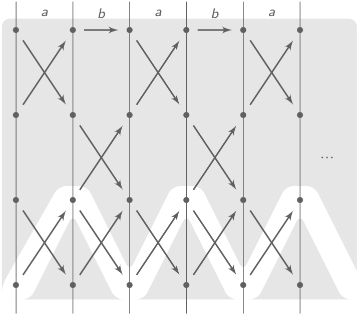

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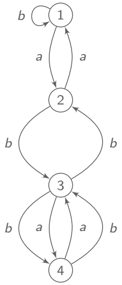

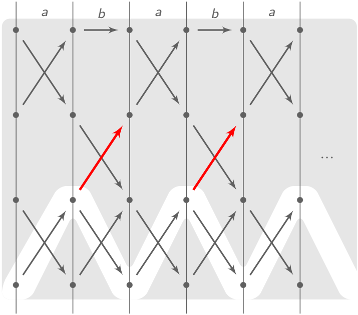

infinitely many leaks from the white flow

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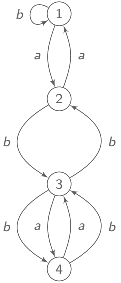

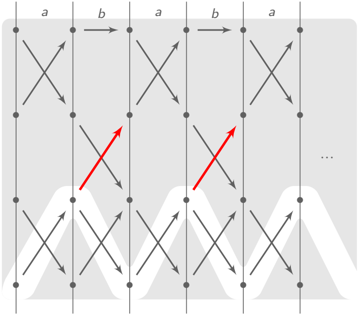

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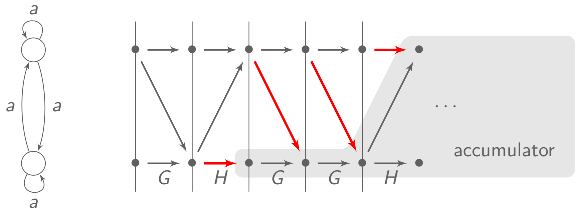
Above play from support game is not realisable in population control

- Controller wins with $(ab)^\omega$!
- Eve loses the support game

Capacity game: refining winning condition of support game



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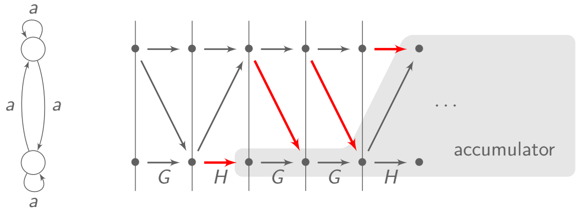

Finite capacity play: all accumulators have finitely many entries
Bounded capacity play: finite bound on # entries for accumulators

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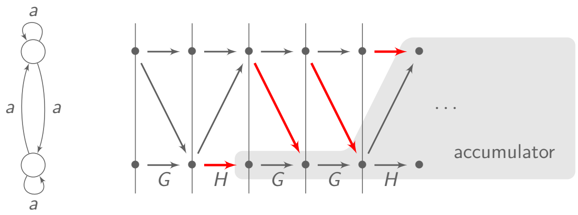

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- corresponds to realizable plays
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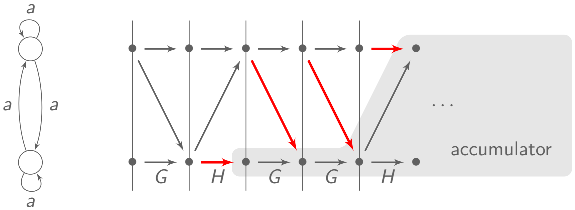

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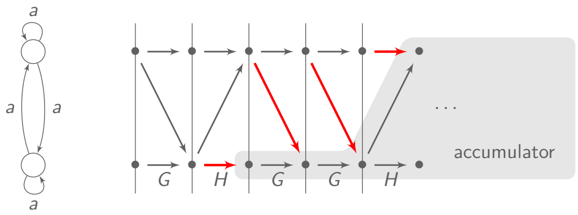

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Eve wins capacity game iff Controller has a winning strategy for all N

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The set of plays with infinite capacity is ω-regular

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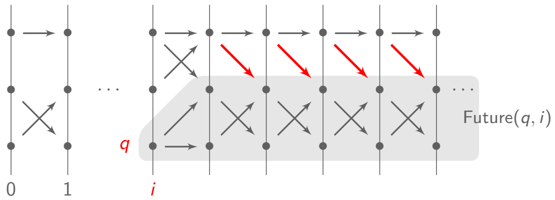

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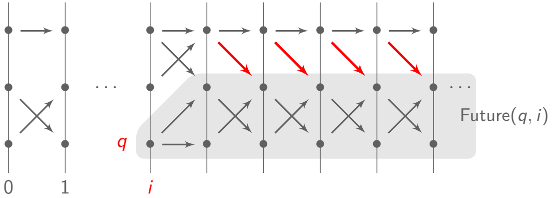

Non-deterministic Büchi automaton

1. guesses a step i, and state q
2. checks that the accumulator Future(q, i) has infinitely many entries

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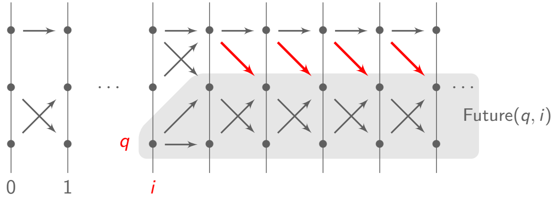

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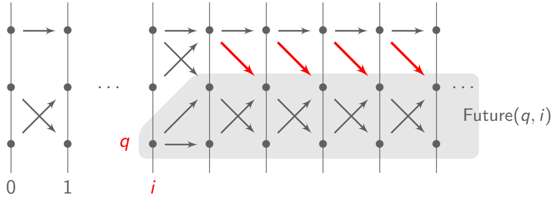

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2EXPTIME decision procedure in the size of NFA $\mathcal{A}$

Solving the capacity game in EXPTIME

Ad-hoc deterministic parity automaton with

$\#$ states $=$ simply exponential in $|\mathcal{A}|$ $\quad$ $\#$ priorities $=$ polynomial in $|\mathcal{A}|$

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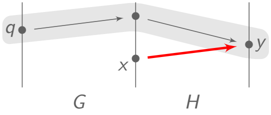
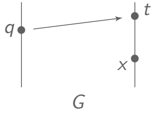


$x \to y$ enters accumulator Future(q)



G separates pair (t, x)

- entries arise from separated pairs
- tracking transfer graphs separating new pairs is sufficient

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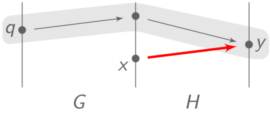

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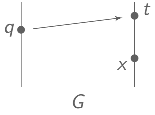

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Parity game:
capacity game enriched with tracking lists in states
priorities reflect how the tracking list evolves (removals, shifts, etc.)

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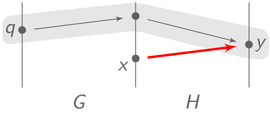

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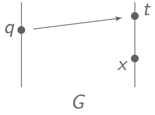

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capacity game enriched with tracking lists in states
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Parity game is equivalent to capacity game.

Complexity of the population control problem

Theorem:
The population control problem is EXPTIME-complete.

Upper bound :

- population control problem $\equiv$ capacity game
- capacity game $\equiv$ ad hoc parity game
- solving parity game of size exp. and poly. priorities

Lower bound : encoding of poly space alternating Turing machine

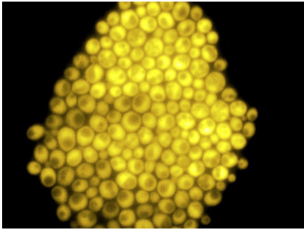
Summary of results

Uniform control of a population of identical NFA

- parameterized control problem: gather all copies in F
- (surprisingly) quite involved!
- tight results for complexity, cutoff, and memory
  - complexity: EXPTIME-complete decision problem
  - bound on cutoff: doubly exponential
  - memory requirement: exponential memory (orthogonal to supports) is needed and sufficient for controller

Back to motivations

Control of gene expression for a population of cells



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Back to motivations

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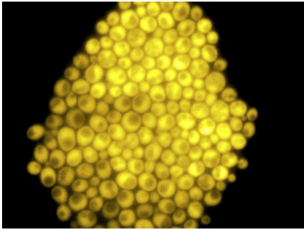

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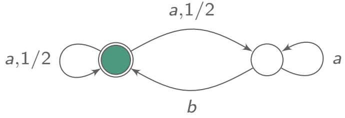
- need for truely probabilistic model
  $\rightarrow$ MDP instead of NFA
- need for truely quantitative questions
  $\rightarrow$ proportions and probabilities instead of convergence and (almost)-sure

$$\forall N \max_{\sigma} \mathbb{P}_\sigma(\mathcal{A}^N \models \Diamond \text{ at least 80\% of MDPs in } F\ ) \geq .7?$$

Probabilistic population

Discrete approximation of probabilistic automata



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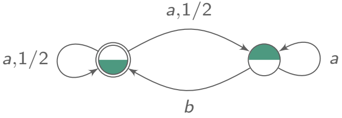

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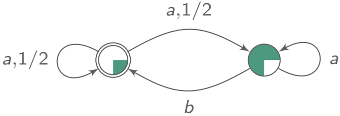

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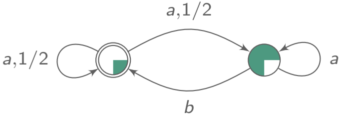

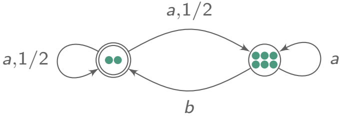

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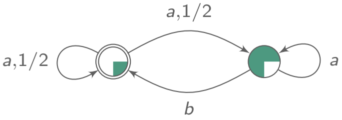
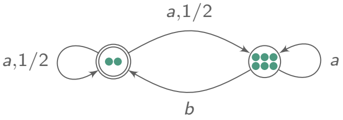
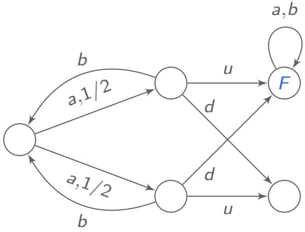




Gap: optimal reachability probability not continuous when $N \to \infty$



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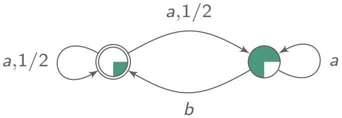

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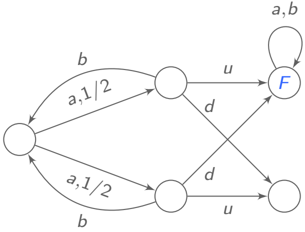

- $\forall N, \exists \sigma,\ \mathbb{P}_\sigma(\Diamond F^N) = 1.$
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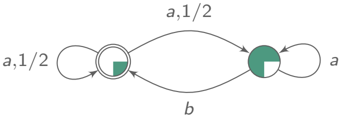

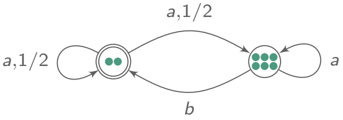

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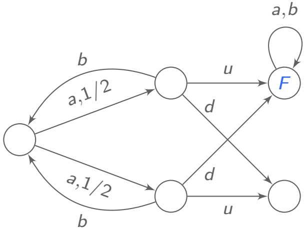

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Good news? hope for alternative **more tractable** semantics for PA

$\epsilon\nu\chi\alpha\rho\iota\sigma\tau\acute{\omega}$!